# Existence of Chern-Simons modified gravity based on discovery of many galaxies without dark matter

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**ABSTRACT**

The existence of dark matter has been assumed to explain the flat rotation curves of galaxies. Recently, however, many studies have reported galaxies without dark matter. Therefore, we need to reconsider dark matter theory. Although modified Newtonian dynamics (MOND) (which is another candidate to explain flat rotation curves without dark matter) can describe spiral galaxies well, it is inapplicable to recent observations. Here, we focus on considering the rotational motion of galaxies on the basis of Newtonian dynamics with a relativistic correction, and we find a new empirical formula for flat rotation curves. The simple formula is derived from Chern-Simons (CS) modified gravity. We may be able to understand the dynamics of all galaxies since the helicity $C_2$ in CS theory can take different values. In this study, we calculate CS gravitational acceleration and compare it with MOND gravitational acceleration. Furthermore, we compute the log variance ratio between CS data and MOND data. Consequently, we discover that CS data fits the observed gravitational acceleration better than MOND data for a large radius where flat rotation curves appear. It also means that we can explain the flat rotation curves of galaxies without dark matter. Rotational dynamics regarded as MOND or dark matter phenomena may be interpreted as CS vortex phenomena.

**Key words:** Chern-Simons modified gravity – modified Newtonian dynamics – dark matter

## 1 INTRODUCTION

There is a mystery regarding the flat rotation curves of galaxies. Galaxies rotate as if they contain an invisible mass. To explain this phenomenon, the existence of dark matter has been assumed (Rubin et al. 1978). Although X-rays observed in an elliptical galaxy offer its evidence, a black hole may be an X-ray source in such a galaxy (Haaften et al. 2019). In terms of gravitational lensing in a cluster of galaxies, we cannot decide whether it is a proof of dark matter because the amount of dark matter estimated by gravitational lensing was frequently less than expected (Meneghetti et al. 2020). In addition, many galaxies without dark matter have been discovered (Dokkum et al. 2018; Gui et al. 2020; Montes et al. 2020). Thus, we need to reconsider dark matter theory (Lelli 2014; Firmani et al. 2001).

Modified Newtonian dynamics (MOND) (Milgrom 1983; McGaugh 2008; Smolin 2017) may provide another way to solve this fundamental problem regarding the universe. Although MOND is a phenomenological theory without dark matter (McGaugh & Lelli 2016; McGaugh 2008), it is inapplicable to non-rotating elliptical galaxies. In contrast, we are interested in Chern-Simons (CS) modified gravitational theory (Jackiw & Pi 2003; Konno et al. 2007, 2009; Alexander & Yunes 2009), which deals with a vortex system including a singularity. In this study, we discovered a new empirical formula which describes flat rotation curves. The formula corresponds with the helicity $C_2$ of the space-time structure in CS theory. Hereafter, we use CS$^*$ to indicate the empirical CS. In principle, CS$^*$ may be able to describe flat rotation curves for any type of galaxy from elliptical to spiral since $C_2$ can take from zero to an arbitrary value (Konno et al. 2008). We show that CS$^*$ is the most suitable theory for understanding galactic dynamics.

By using the new empirical formula, we calculated gravitational acceleration and fitted observed data for spiral galaxies. Moreover, we computed the log variance ratio between CS$^*$ and MOND, $k = \delta^2_{\mathrm{CS}^*}/\delta^2_{\mathrm{MOND}}$, for small values of gravitational acceleration from baryonic matter: $g_{\mathrm{bar}} \leq 10^{-10}\mathrm{m/s}^2$ (that is, $r > 5 \times 10^{21}$ m in the case of the Milky Way galaxy). We show that CS$^*$ can be applied to the problem of flat rotation curves. This is consistent with the discovery of galaxies without dark matter.

This paper is organized as follows. In §2, we outline the limitations of dark matter and MOND. In §3, we describe the relationship between the new empirical formula and CS theory. Each theory is compared in §4, which comprises the discussion and conclusion.

## 2 DARK MATTER AND MOND

Dark matter is one of the strong candidates for explaining flat rotation curves. In dark matter theory, spiral galaxies contain several times more dark matter than luminous (baryonic) matter. However, it is currently unclear whether or not this theory is applicable to elliptical galaxies (Romanowsky et al. 2003). For example, the deprivation of dark matter was recently observed in the elliptical galaxy NGC 7507 (Lane et al. 2015). In addition, many elliptical galaxies without dark matter have been discovered (Dokkum et al. 2018; Gui et al. 2020). Further studies may solve this problem, the deprivation or the absence of dark matter (Bidin et al. 2012, 2015). In contrast, MOND is a phenomenological theory that can also describe the flat rotation curves of spiral galaxies. This model was first proposed by Milgrom (Milgrom 1983), and McGaugh developed the following gravitational acceleration formula (McGaugh & Lelli 2016; McGaugh 2008) :

$$g_{\rm MOND} = \frac{g_{\rm bar}}{1 - e^{-\sqrt{g_{\rm bar}/g_\dagger}}} \tag{1}$$

where $g_\dagger$ is a universal acceleration constant and $g_{\rm bar}$ is gravitational acceleration from baryonic matter. Figure 1-(a) shows the results obtained with this equation and the observed gravitational acceleration of spiral galaxies. We theoretically consider MOND data (McGaugh & Lelli 2016). In Figure 1-(a), MOND provides a good description of flat rotation curves. Equation (1) must also be applicable to (non-rotating) elliptical galaxies because it is a universal modification of Newtonian dynamics. This universality contradicts recent discoveries of galaxies without dark matter.
Therefore, although dark matter theory and MOND are generally used to explain flat rotation curves, they may not provide information about elliptical galaxies (except for certain rotating galaxies). Here, we describe flat rotation curves from Newtonian dynamics with a relativistic correction. In the next section, we show that a new empirical formula is applicable to various galaxies because of a relationship with CS theory.

## 3 NEW EMPIRICAL FORMULA

In this study, we newly added a function to Newton's equation of motion. When we consider rotational galactic motion based on this expansion, the next equation of rotational speed $v_{\rm rot}$ is assumed

$$v_{\rm rot} = \sqrt{\frac{GM}{r} + \frac{C_{\rm rot}}{2}} \tag{2}$$

where $M$ is the mass of a galaxy, $G$ is the universal gravitational constant and $r$ is the radius from the center of a galaxy. The first term indicates rotational speed derived from Newtonian mechanics. We assumed that $C_{\rm rot}$ is a function of $r$. We analyzed flat rotation curves with this equation. First, we assumed a standard Laurent expansion for $C_{\rm rot}$:

$$C_{\rm rot} = \sum_{n=-\infty}^{\infty} a_n r^n \tag{3}$$

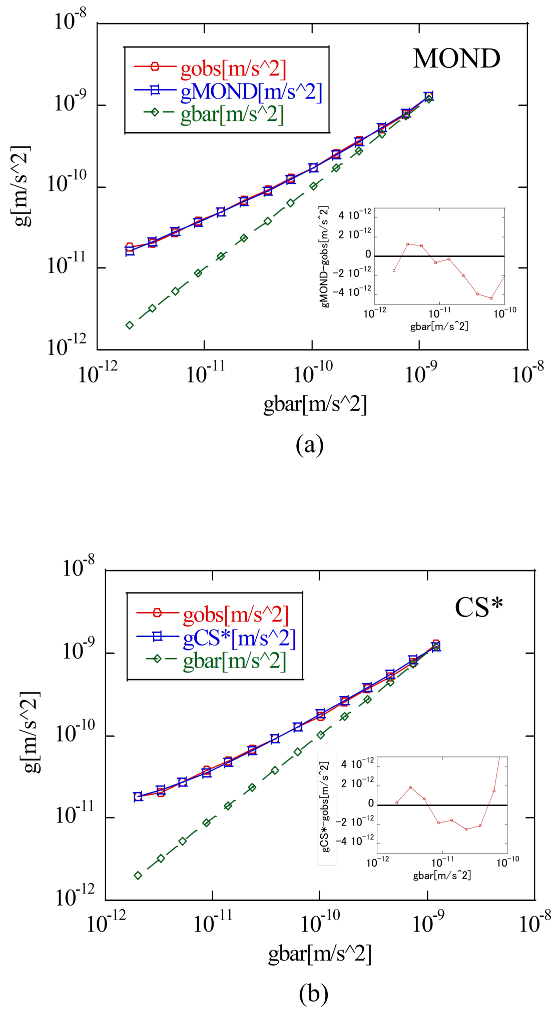


**Figure 1.** (a) The horizontal axis represents $g_{\rm bar}$ in log scale and the vertical axis represents $g_{\rm obs}$ or $g_{\rm MOND}$ in log scale. The red line shows $g_{\rm obs}$, which is the observed gravitational acceleration of spiral galaxies. The blue line shows $g_{\rm MOND}$, which was calculated with Equation (1). (b) The horizontal axis represents $g_{\rm bar}$ in log scale and the vertical axis represents $g_{\rm obs}$ or $g_{\rm CS^*}$ in log scale. The red line shows $g_{\rm obs}$ as in Figure 1-(a). The blue line shows $g_{\rm CS^*}$, which we calculated with Equation (4), (5) and (6). Each figure shows that MOND and CS* can describe flat rotation curves since each set of calculated data is comparable to observed data. In particular, we focused on seven points from a long distance and confirmed the difference between MOND and CS* in more detail. In the insets, the horizontal axis represents $g_{\rm bar}$ in log scale and the vertical axis represents $g_{\rm MOND} - g_{\rm obs}$ or $g_{\rm CS^*} - g_{\rm obs}$, where $g_{\rm MOND} - g_{\rm obs}$ and $g_{\rm CS^*} - g_{\rm obs}$ are the difference from observed data represented by a thick black line. By computing the log variance ratio between CS* and MOND, $k = \delta^2_{\rm CS^*}/\delta^2_{\rm MOND}$, based on the result, we obtained $k = 0.885$. This indicates that CS* fits the observed data better than MOND for a large radius.

As explained below, we discovered that the following three terms are effective for flat rotation curves:

$$C_{\rm rot} = \alpha r + \frac{\beta}{r} + \gamma \tag{4}$$

where $\alpha$, $\beta$ and $\gamma$ are constant and expressed by $a_n$ in Equation (3). We found $\alpha = 2.4 \times 10^{-17}$ s$^{-1}$, $\beta = -2.1 \times 10^{26}$ m$^2$/s, $\gamma = 3.5 \times 10^5$ m/s for the best fitting. $\gamma$ is the most effective term for $C_{\rm rot}$ because $r$ and $r^{-1}$ work as correction terms: these terms correct at a larger radius and a smaller radius, respectively. In fact, the contribution of the $r^{-1}$ term is 1 to 2% of $\gamma$, thus the new empirical formula is just the Laurent expansion around $\gamma$ up to the first order in $r$. If we compare space-time and fluid, we can treat $\alpha r$ as a forced vortex, and $\beta r^{-1}$ as a free vortex. These two terms are interpreted as the conservation of angular momentum. $\gamma$ is linked to a topological term. The most important $\gamma$ term in Equation (4) is derived from CS theory for the following reasons. CS modified gravity in (3 + 1) dimensions was proposed by Jackiw and Pi (Jackiw & Pi 2003). The effects of the CS action in gravitational fields have been discussed in several studies (Alexander & Yunes 2007a,b). A perturbative study with small angular momentum provides the following velocity (Konno et al. 2008):

$$v_{\rm CS} = \sqrt{\frac{GM}{r} + \frac{C_2}{2} + \frac{r_s C_2}{r} + \frac{r_s^2}{2r^2}\left[C_1 + C_2 + 2C_2 \log\left(\frac{r - r_s}{r_s}\right)\right]} \tag{5}$$

where $r_s = \frac{2GM}{c^2}$ is the Schwarzschild radius, $M$ is the mass of a galaxy, $G$ is the universal gravitational constant, $c$ is the speed of light, $r$ is the radius of a galaxy and $C_1$, $C_2$ are the helicity in CS. At a large $r$, Equation (5) simply becomes Equation (2) and $C_{\rm rot}$ corresponds to the helicity $C_2$, which is an important term in CS. This implies that our assumed Equation (2) is strongly related to CS theory.

Gravitational acceleration of CS* is calculated by the following equation:

$$g_{\rm CS^*} = \frac{v_{\rm CS^*}^2}{r} \tag{6}$$

Figure 1-(b) shows the results of CS* gravitational acceleration with Equation (6). CS* can describe flat rotation curves as well as MOND. In particular, the results were comparable to observed data when we added the $r^{-1}$ term. This indicates the importance of the $r^{-1}$ correction term to explain flat rotation curves. CS* can be applied to various galaxies since the helicity $C_2$ in Equation (5) can take different values including zero. We can understand the motion of galaxies by considering CS gravity.

## 4 DISCUSSION AND CONCLUSION

We compared CS* and MOND by computing the differences between both theories and observed data. Those results are shown in the insets in Figure 1. Gravitational acceleration values based on CS* and MOND fit well at a large radius. To verify the validity of our fitting, we calculated the log variance defined by:

$$\delta_{\rm th}^2 = \frac{1}{N}\sum_{n=1}^{N}(\log g_{\rm obs} - \log g_{\rm th})^2 \tag{7}$$

where $g_{\rm th}$ denotes $g_{\rm CS^*}$ or $g_{\rm MOND}$, and $N$ is number of data. Then, we calculated the log variance ratio between CS* and MOND, $k = \delta_{\rm CS^*}^2/\delta_{\rm MOND}^2$, for small values of $g_{\rm bar} \leq 10^{-10}$ m/s$^2$ (that is, $r > 5 \times 10^{21}$ m in the case of the Milky Way galaxy). As a result, we obtained $k = \delta_{\rm CS^*}^2/\delta_{\rm MOND}^2 = 0.885$. The log variance ratio increases for a smaller radius. However, this deviation is not important here because flat rotation curves appear for galaxies with larger radiuses. That is, the result indicates that a CS* fit can describe flat rotation curves better than a MOND fit. Moreover, CS* has a decisively different feature, namely it can be applied to elliptical galaxies. In that situation, $C_2$ in CS can take zero or a small value. CS* may be a more general theory that solves the problem with MOND.

Figure 2 outlines the differences between dark matter theory, MOND and CS*. Dark matter theory and MOND give us information about the flat rotation curves of spiral galaxies. However, if they were the only theories that can be used to explain flat rotation curves, then paradoxically the dark matter deficiency in elliptical galaxies favors dark matter theory. This strange logic fails if we further consider CS*. In principle, CS* can describe not only spiral galaxies but also elliptical galaxies since the helicity $C_2$ can take different values. We may be able to classify galaxies based on Hubble classification with CS*. The $r^{-1}$ term in $C_2$ that we discovered may be computed from a different form of CS, e.g. dynamical CS (Konno & Takahashi 2014; Yagi et al. 2012). Although it is difficult to find an exact solution, we plan to work on this problem in a future study.

In spite of many efforts, there has as yet been no direct laboratory detection of dark matter particles. On the other hand, if we change our viewpoint and assume that CS helicity exists in the universe, we can obtain a compatible explanation for the flat rotation curves of galaxies. In this study, we confirmed that CS* can describe spiral galaxies as well as MOND without dark matter. With a large radius (where flat rotation curves appear) the log variance ratio $k = 0.885$ implies that CS* fits observed gravitational acceleration better than MOND. In addition, there is a definite difference between CS* and MOND. CS, which is related to the new empirical formula, is a universal theory and can be applied to all galaxies. This theory may provide us some solutions for flat rotation curves. CS structure has the potential to solve the problems posed by many astrophysical phenomena (Canizares et al. 2012; Barrientos et al. 2019) whereas dark matter or MOND is not required in some cases (Lane et al. 2009). Ultimately, it can be applied to condensed matter physics and particle physics (Cisterna et al. 2019). We plan to use CS to research many problems including exploring axial symmetry theory (Bahamonde et al. 2021).

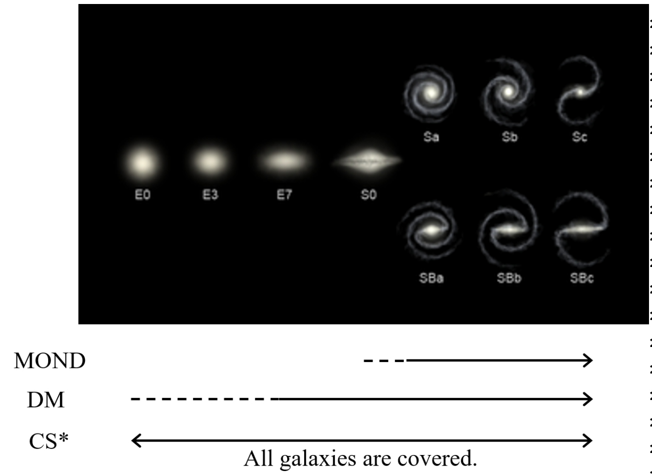


**Figure 2.** Classification of dark matter, MOND and CS* based on Hubble classification. Arrows with solid lines indicate the region in which each model is applicable. The dashed lines indicate the region where each theory may not be applicable. We do not currently know why galaxies rotate so strangely (winding dilemma etc.). MOND can explain flat rotation curves only for spiral galaxies and some rotating elliptical galaxies (Samurović & Vudragović 2019), that is, this theory is not applicable to non-rotating galaxies. Dark matter theory is a strong candidate for solving the problem of flat rotation curves. Recent observations for elliptical galaxies indicate the existence of galaxies with less dark matter than expected. On the other hand, we can understand all galaxies with CS* since universal CS theory has the important helicity $C_2$. Each galaxy may have different (including vanishing) values of $C_2$ depending on its type. Credit: Ville Koistinen.

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