Topological Crystals and the quantum effects:as a new Paradigm

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Outline

(1) Introduction

(2) Crystal Growth Covalent wires

(3) Topological Crystals

Ring, Möbius, Figure8, Hopf-link Crystals

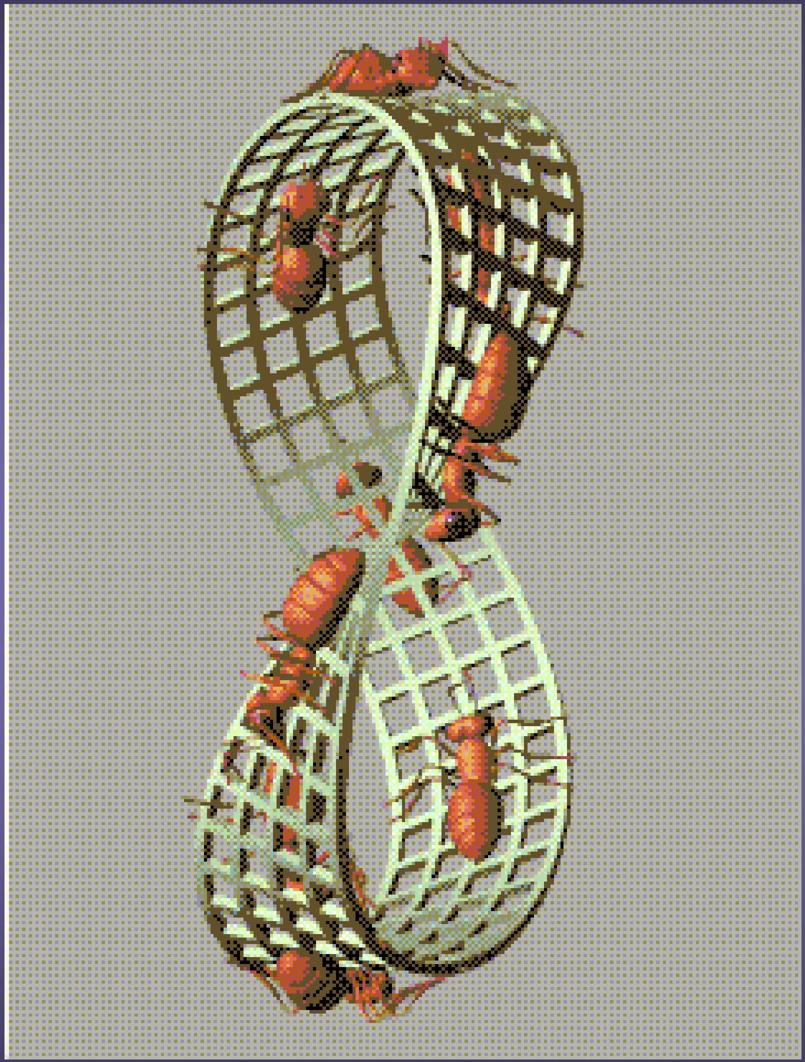
Mechanism

(4) Classification of Topological Crystals

(5) Electronic properties: AB effect, Frolich supercon.

(6) Summary

Introduction: Escher's ants



Möbius Strips II (1963)
by M. C. Escher

Ants return to the original point after two laps on Möbius Strips

No distinction between the Obverse and the Reverse

↓

An exotic one-sided world !!

↑

from Artists to Scientists

Ants ⟶ Electrons, Spins
Photons, CDWs, Super

Strips ⟶ Lattice, Crystal

Is it Possible ?

What happens ?

Our Goal

1. Topological Crystallography (Ladder)

2. Quantum topological effects in the topological crystals (Ants)

Macroscopic quantum coherenc
in nontrivial topological space

Topological gauge fields

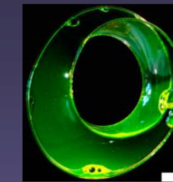
Berry Phase

Strategy

Is it Possible ?

Soap-film Möbius strip

Goldstein, Moffatt, Pesci, Ricca PNAS (2010)



For synthesize of topological crystals

Requirements

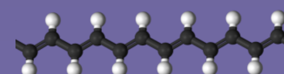
★ One-dimensional wire

Too flexible

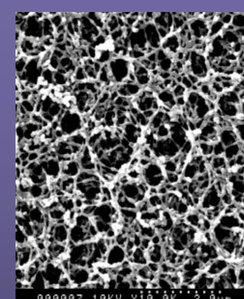
Organic polymer

Protein

DNA



$(CH)_x$

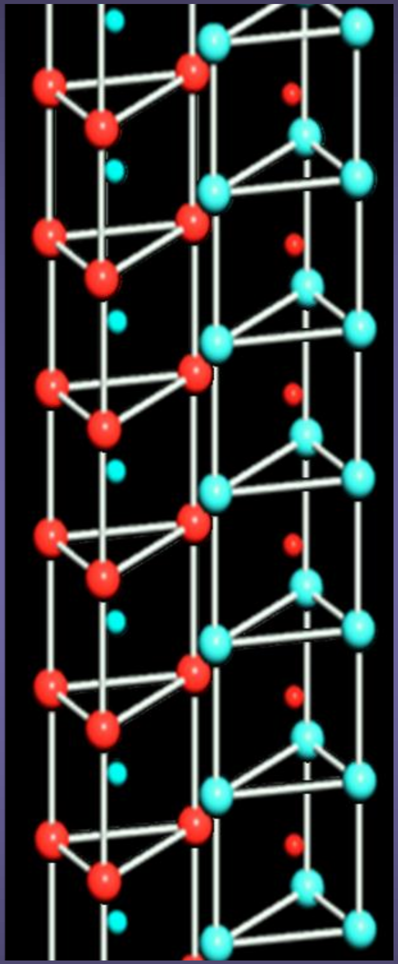


Network

★ Rigidity

Inorganic polymer → MX_3, MX_2

Inorganic Covalent wires



Requirement for twisting and bending materials such as Mobius crystals

chain

Semi-flexible !

Typical materials

NbS_3 $NbSe_3$

TaS_3 $TaSe_3$

TiS_3 $ZrTe_3$

⋮ ⋮

MX_3

Covalent : Intra-chain

Van der waals force: Inter-chain

MX_3 : CDW and Superconductors

Researches in Hokkaido Univ. 35 Years

Discovery of $NbSe_3$ and TaS_3: Yamaya, Sambongi, Tsutsumi (1977)

Memory Effect of CDW in $NbSe_3$: Ido Oda, Okajima, Sambong (1986)

Ring Crystals in $NbSe_3$: Kawamoto, Okajima, Yamaya, Tanda (1999)

Möbius Crystals in $NbSe_3$ Tanda Tsuneta Inagaki Okajima Yamaya Hatakenaka

Nature 417 397 (2002)

Hopf-link crystals Matsuura, Yamanaka, Hatakenaka, Matsuyama, Tanda

PRB (2006)

Topological quantum Effects in MX_3: Tsuboa, Matsuura, Kumagai, Tanda

PRB (09,10,11,12)

Chiral CDW in $TiSe_2$: Ishioka, Oda, Ichimura, Tanda PRL (2010,2012)

MX_2 Nanotubes (03,04), Topology change(09,10,11,12)

Ring Crystals

Seamless!!

1999

Synthesize and Condition

Chemical Vapor Transportation

Closed Quartz Tube

<10^{-7} Torr(initial vacuum)

10 days

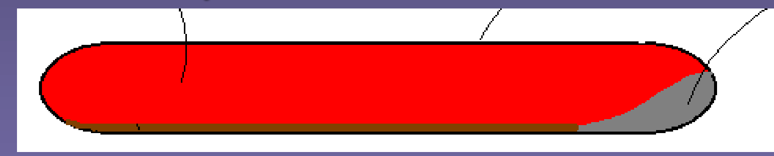
Nb, Ta (99.999%),

Se (99.9999%)

Furnace Temperature 600℃~800℃

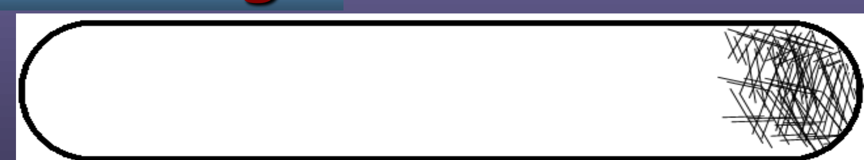
Starting materials (Nb, Se) is sealed in an evacuated ampoule.

1. heating



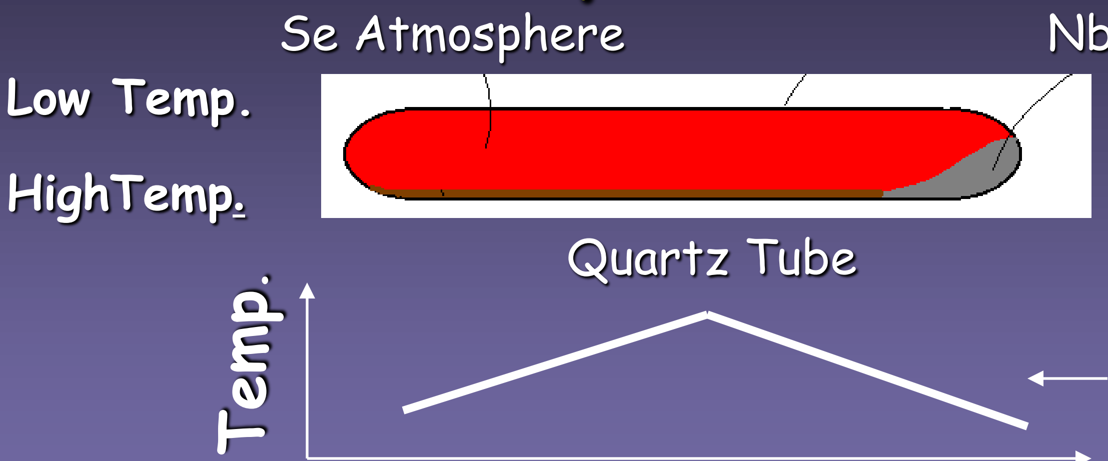
2. transportation

3. final stage



After a few weeks of heating, products are transported to the lowest-temperature end where vapor pressure is also the lowest.

Nonequilibrium condition



Large Temperature gradient (more than 150℃)

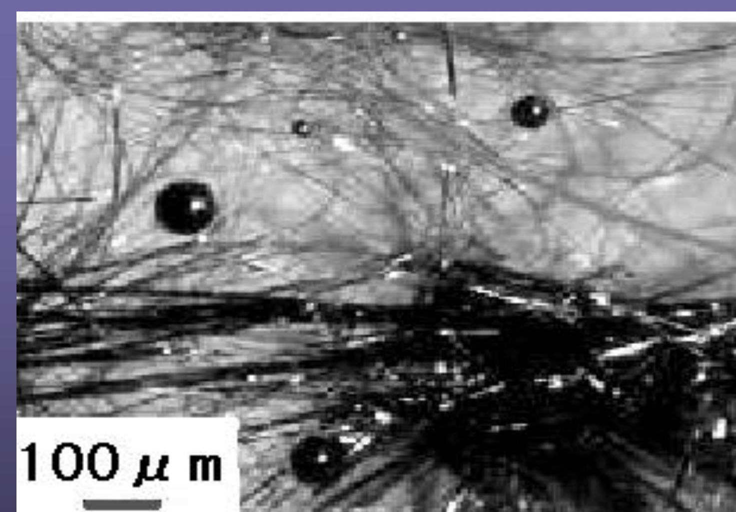
Se circulates through

Vapor , mist , liquid droplet

Earth system make

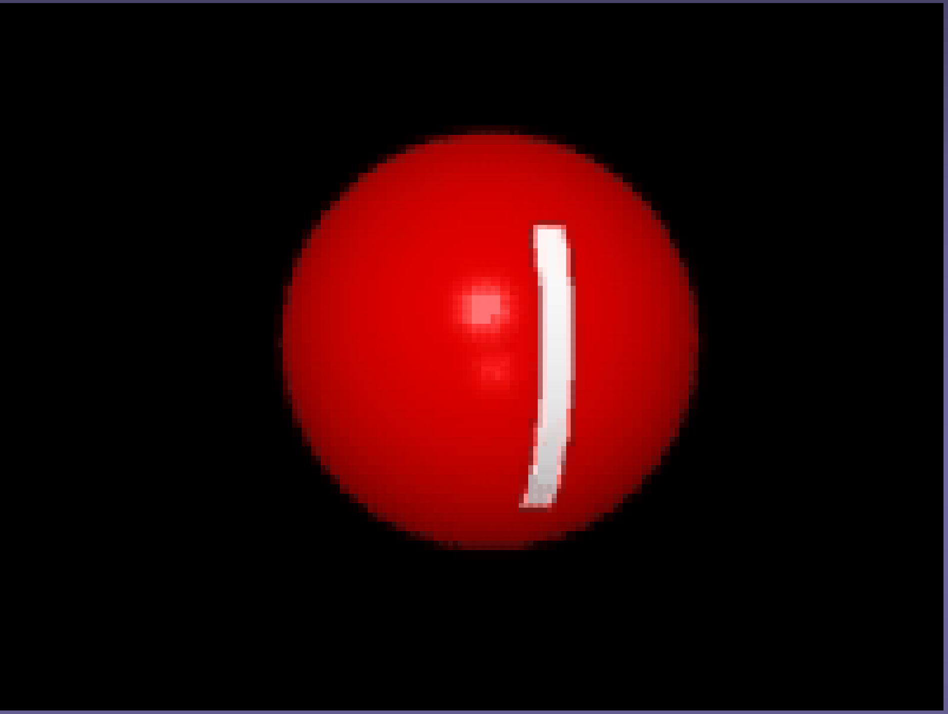
Life polymer

Analogous method

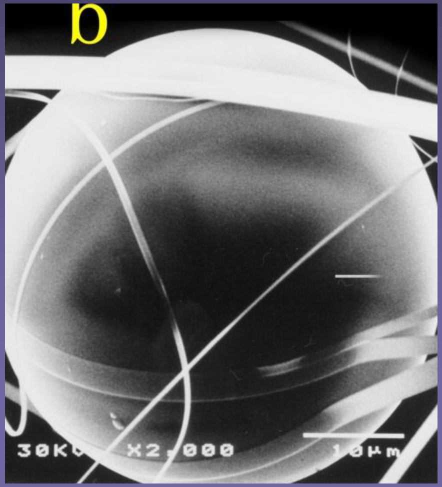


These droplet need for formation of ring crystals.

Formation of Ring Crystals



a growing crystal can eat its own tail.

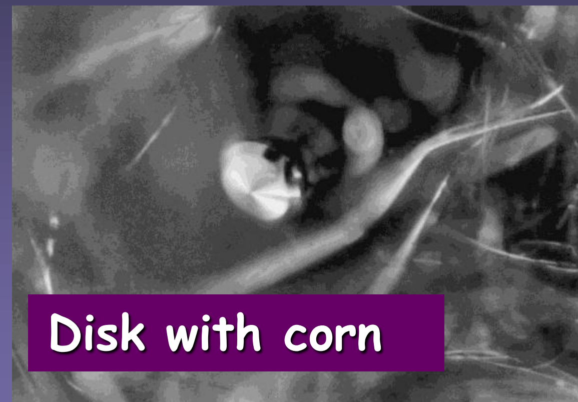


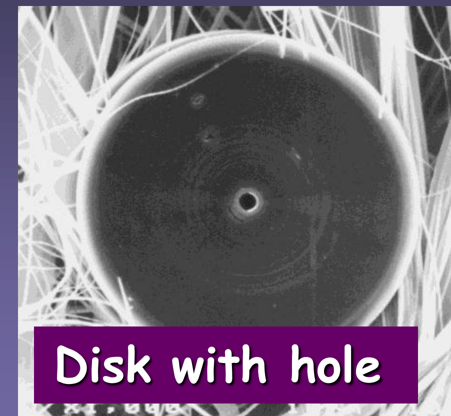
The ribbon-shaped $NbSe_3$ crystals grown in the viscous Se droplet are bent due to Se surface tension.

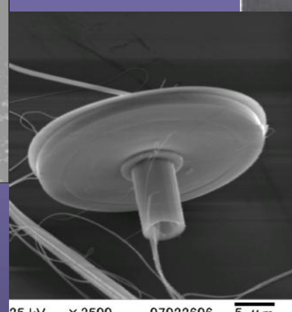
NbSe3 fiber circulate on the equator of Se droplets during growth

Seamless ring

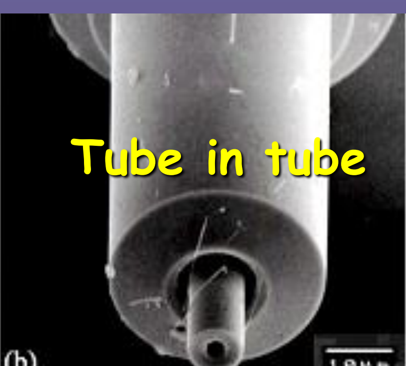
Disks and Tubes Crystals







Hole radius is less than 1 μm !!



CDW Correlation length is 5 μm , which is larger than hole radius!

This system is ideal matter in observation of Interference effect or AB effect of CDW

Figure-8 Crystals (2π-twist)

Knot with Framing
in Math. term

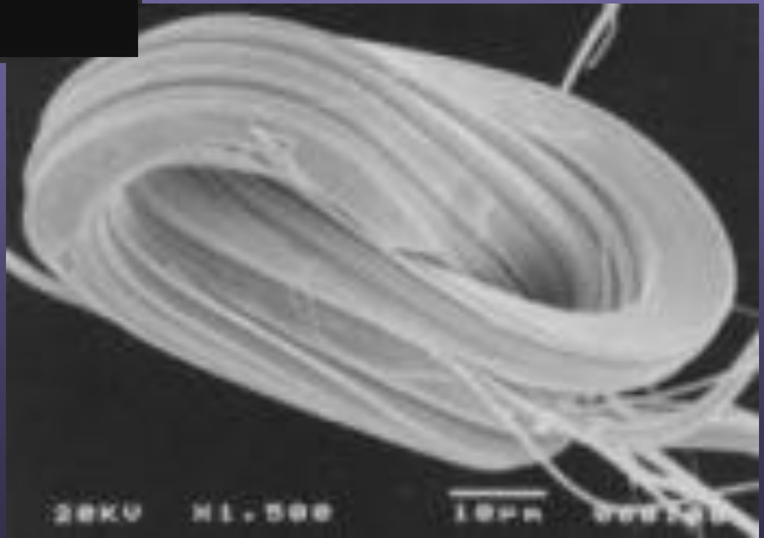
$+2\pi$-Twist -Band
-2π-Twist -Band

No evidence of breaking of Chiral Symmetry

Growing

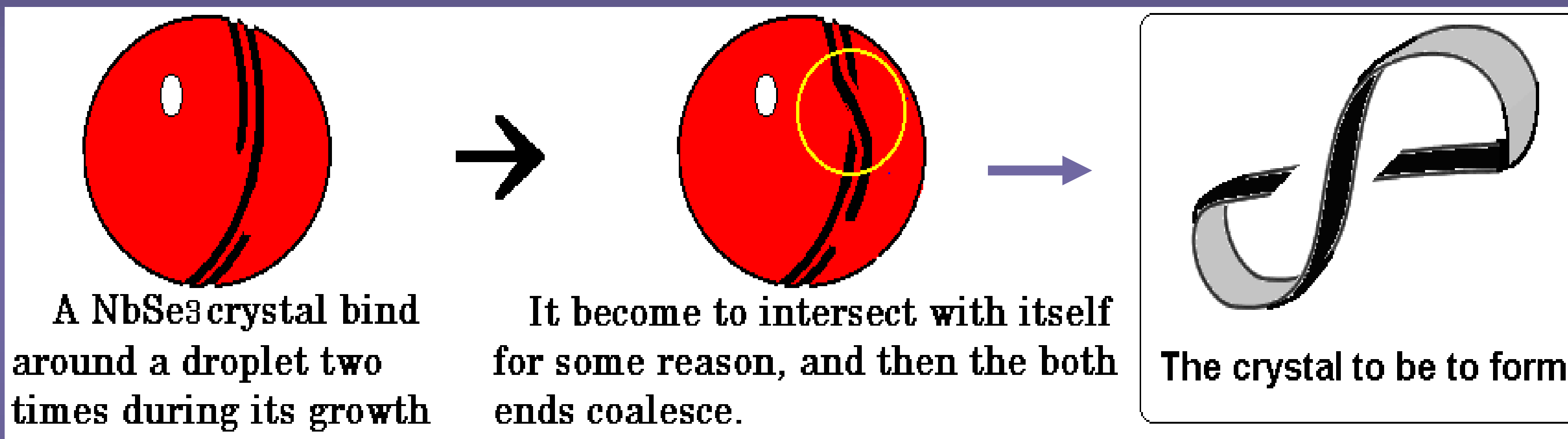
Self-crossing Maximum Crystals

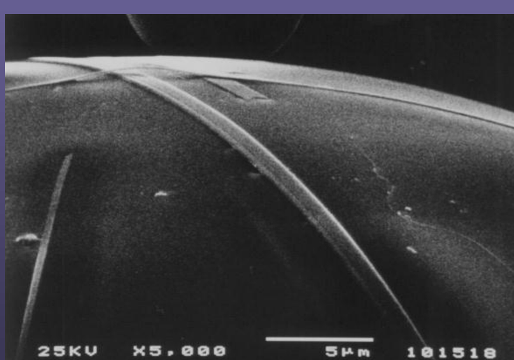
Hertman-Nirenberg Theorem (2002)

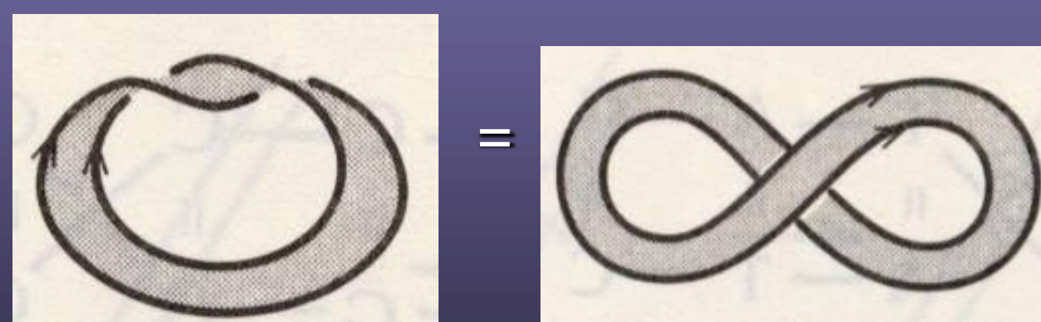


2002 Nature

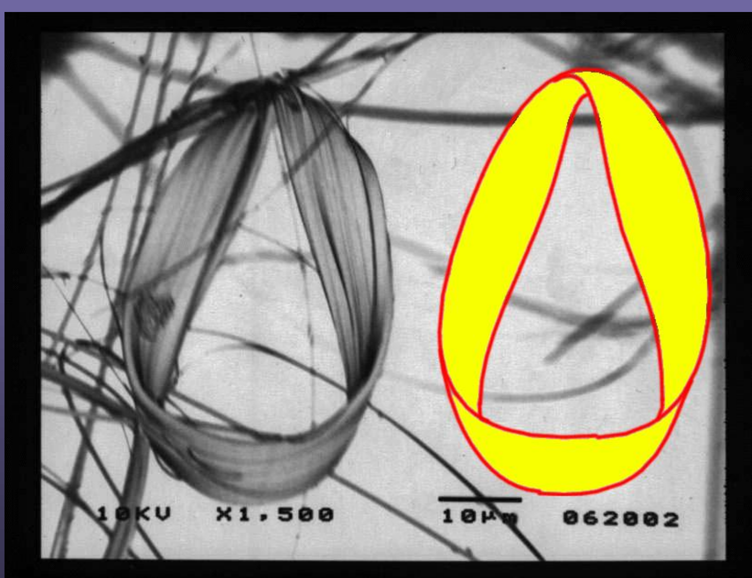
Mechanism of formation of Figure 8 Crystals (2π twist)



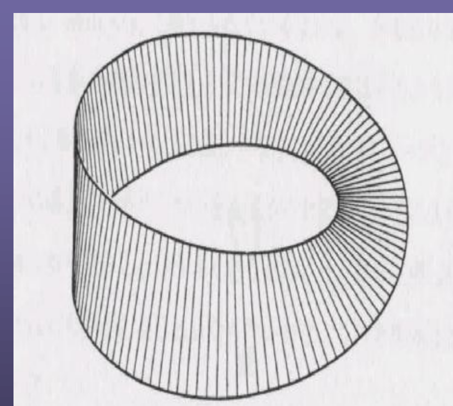


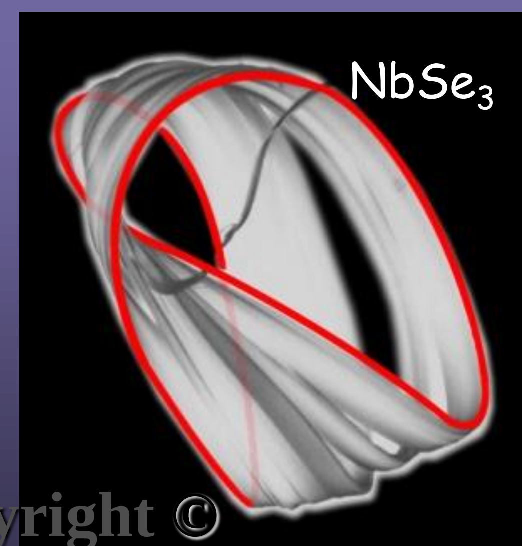


Möbius Crystals (π twist)



TaSe3





Bending to Twisting

formation mechanism of Mobius crystals

Difficulties → twisting shear

Question is how the twisting is introduced without actual shear forces

Answer

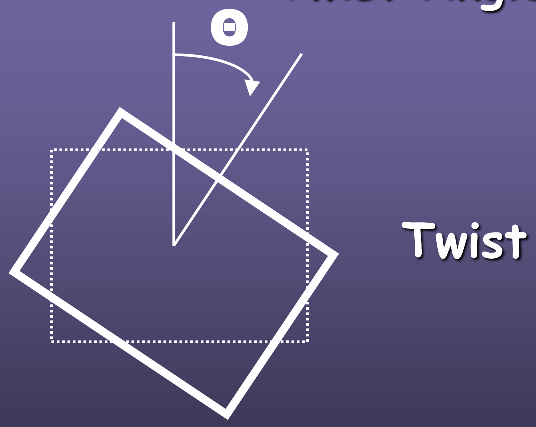
Crystal symmetry → The bending-twisting conversion

Bending ⟷ Twisting

Compliance

Monoclinic , triclinic

Twist Angle: Θ

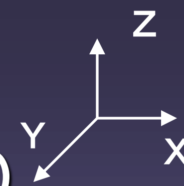


Cross Section

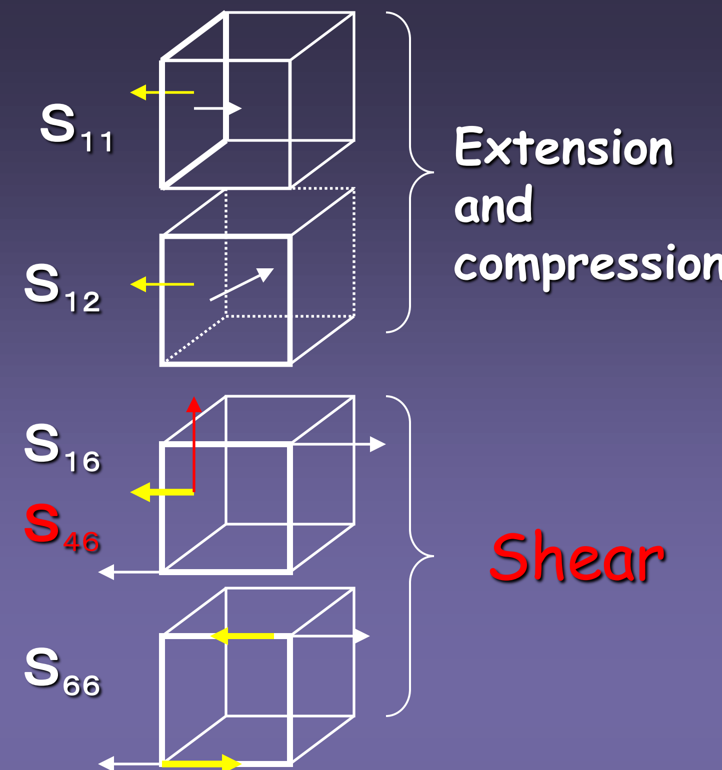
Compliance

Elasticity (Expansion of Hook's law)

$X=(1/K)\times F$ $F=kX$



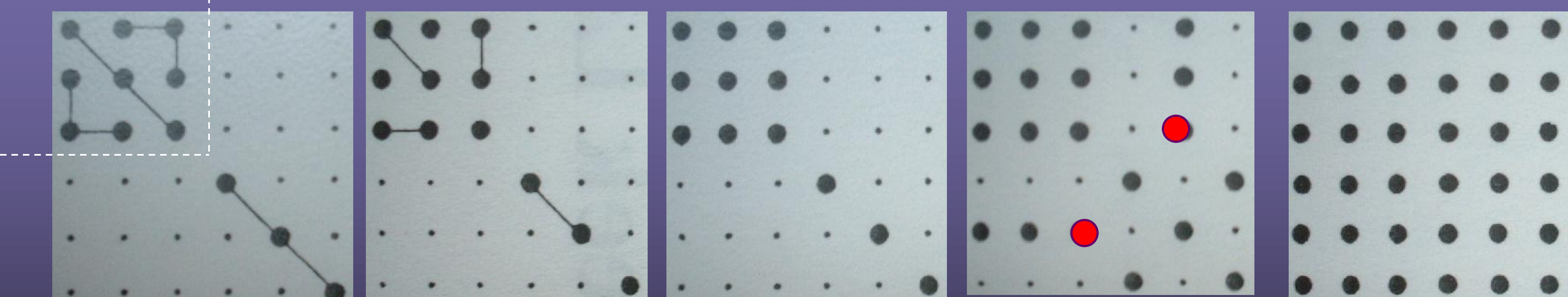
$$
\begin{aligned}
\epsilon_1 &= s_{11}^{E,T}\sigma_1 + s_{12}\sigma_2 + s_{13}\sigma_3 + s_{14}\sigma_4 + s_{15}\sigma_5 + s_{16}\sigma_6 \\
\epsilon_2 &= s_{12}\sigma_1 + s_{22}\sigma_2 + s_{23}\sigma_3 + s_{24}\sigma_4 + s_{25}\sigma_5 + s_{26}\sigma_6 \\
\epsilon_3 &= s_{13}\sigma_1 + s_{23}\sigma_2 + s_{33}\sigma_3 + s_{34}\sigma_4 + s_{35}\sigma_5 + s_{36}\sigma_6 \\
\epsilon_4 &= s_{14}\sigma_1 + s_{24}\sigma_2 + s_{34}\sigma_3 + s_{44}\sigma_4 + s_{45}\sigma_5 + s_{46}\sigma_6 \\
\epsilon_5 &= s_{15}\sigma_1 + s_{25}\sigma_2 + s_{35}\sigma_3 + s_{45}\sigma_4 + s_{55}\sigma_5 + s_{56}\sigma_6 \\
\epsilon_6 &= s_{16}\sigma_1 + s_{26}\sigma_2 + s_{36}\sigma_3 + s_{46}\sigma_4 + s_{56}\sigma_5 + s_{66}\sigma_6
\end{aligned}
$$



S_{ab}:Compliance (tensor)

Stress:σ

Strain:ε

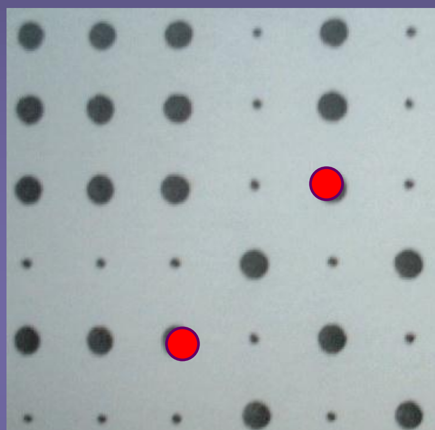


Cubic | Tetragonal | Orthornbic | Monoclinic | Triclinic

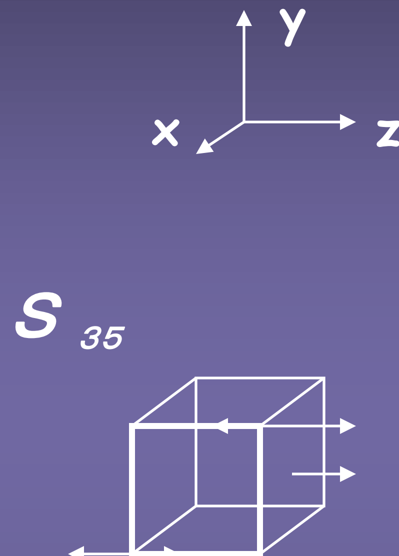
▪ : zero
● : nonzero

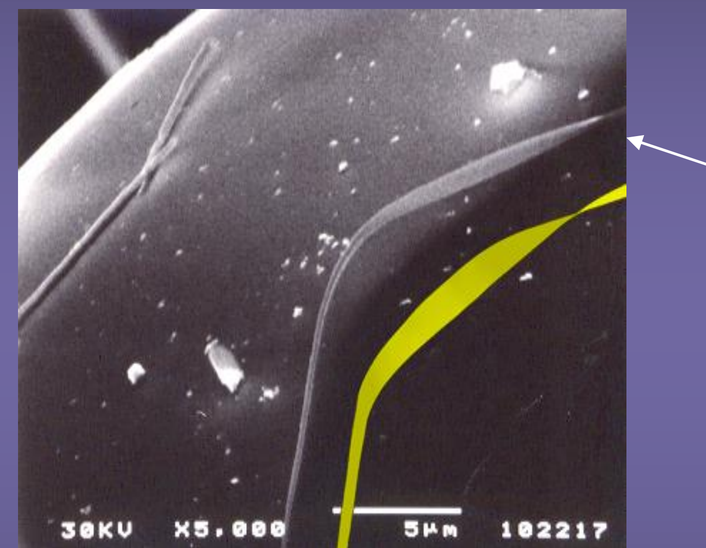
Lower symmetry

Evidence of Twisting



Monoclinic NbSe3

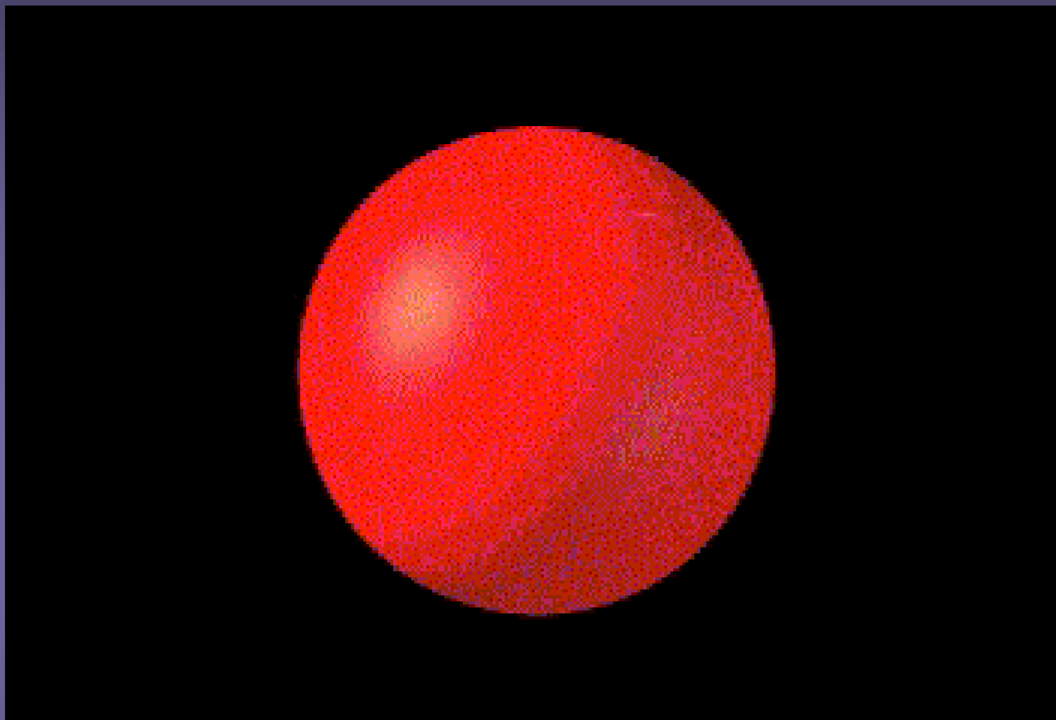




▪: zero
●: nonzero
●: Shear term for Monoclinic

twisting is introduced by bending through this mechanism without actual shear forces during the growth processes along surface

Formation of Möbius Crystals



To form a Möbius strip (1π) , a twist is needed to be introduced during spooling.

Can we introduce the concept of topology into crystals ?

Originally, What's Crystal ?

↓

Bragg reflection !

Definition of International Union of Crystallography (1991)

Definition of Crystals

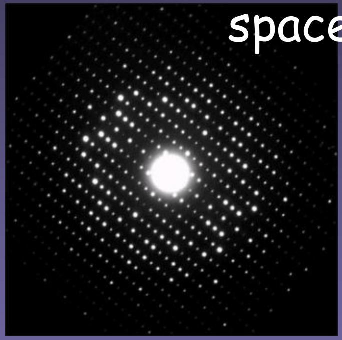
Crystals show Bragg reflection

Snow crystal

Bragg reflection of electron beam/X-ray

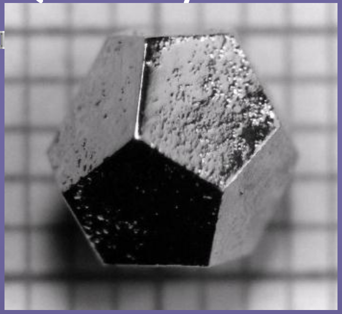
Mathematically, Fourier transform from real space (lattice) to *k*-space (Bragg spots)

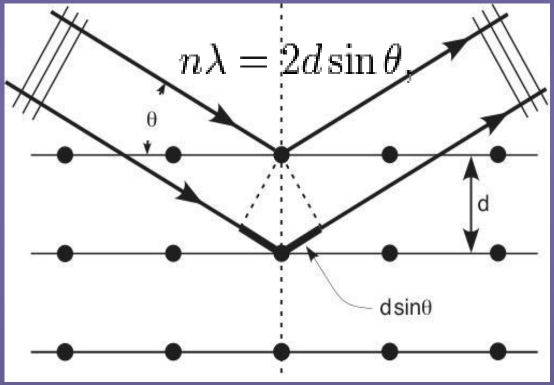
Reciprocal space

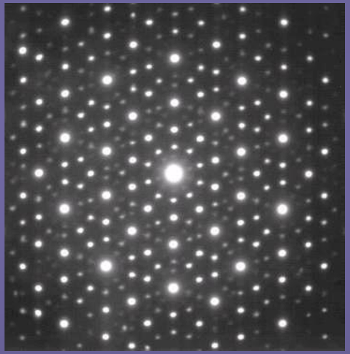


spot

Quasicrystal

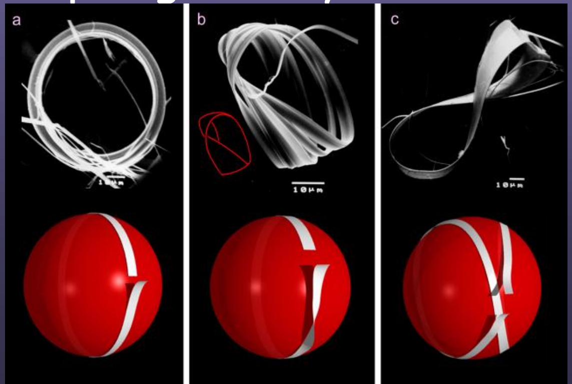






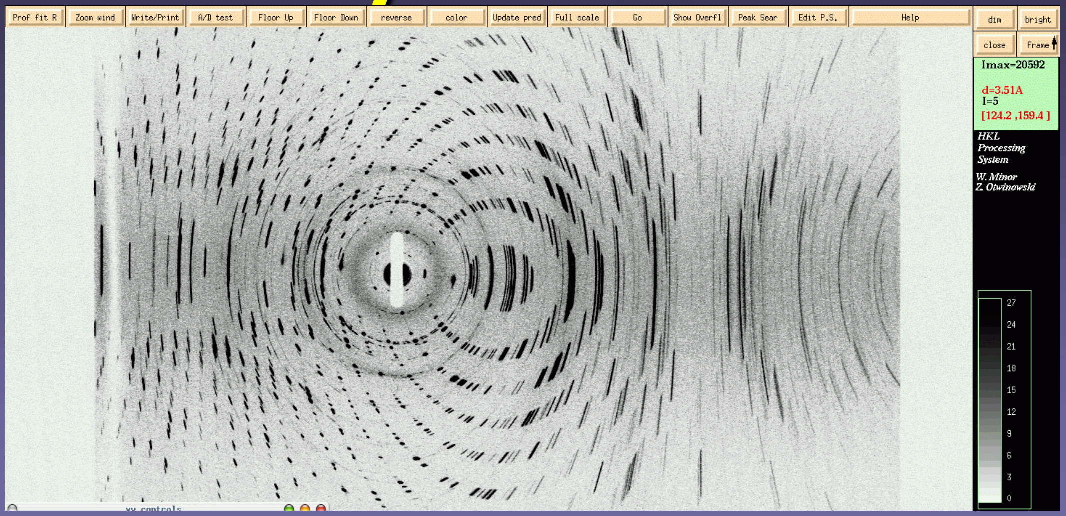
Crystals and quasicrysta
show sharp Bragg spots

Topological crystal



?

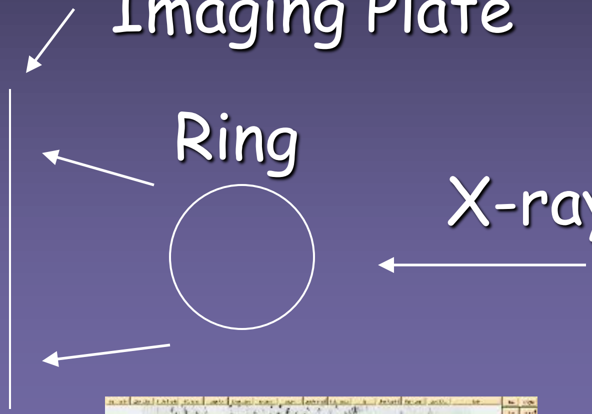
X-ray Diffraction: Ring

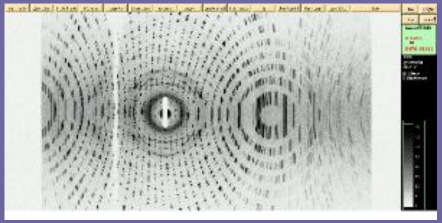


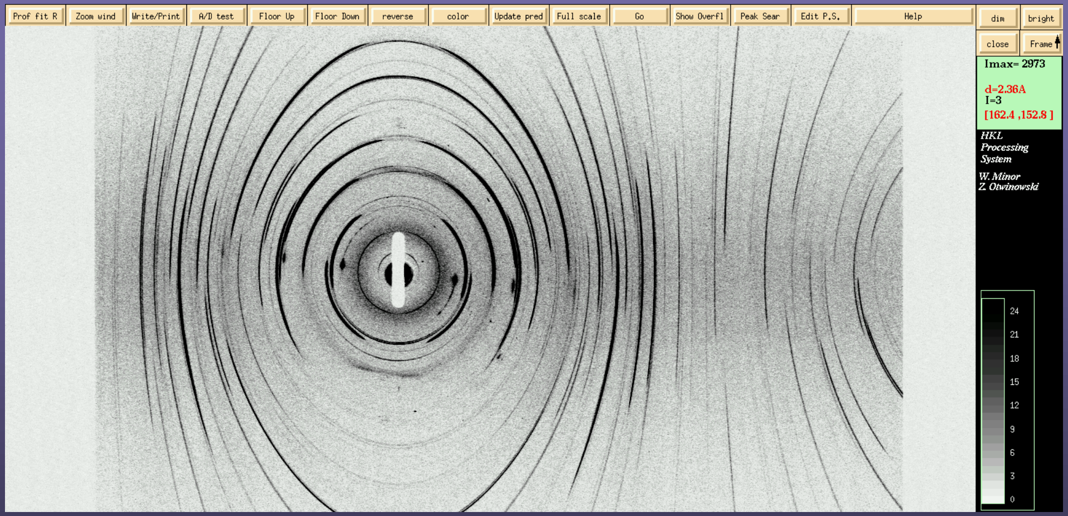
Imaging Plate

Ring

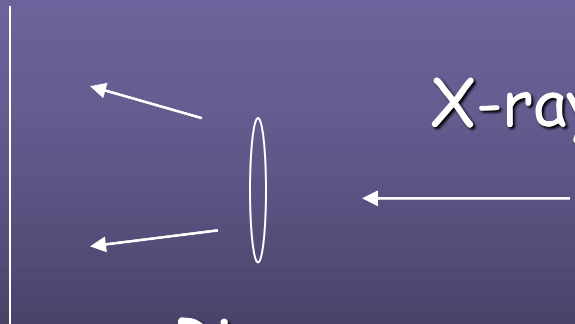
X-ray







X-ray



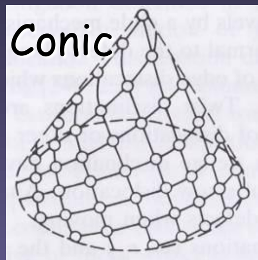
Ring

Bragg Ring

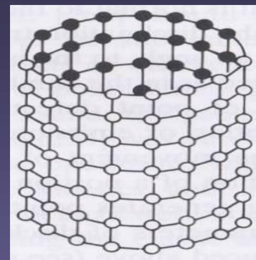
Line is homogeneous

Single Crystals !!

Classification of Topological Crystals



$\omega = 1/2\pi$
$\omega^* = 0\pi$



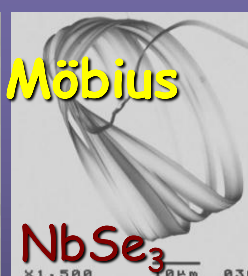
$\omega = 2\pi$
$\omega^* = 0\pi$

ω: Wedge Disclinations
ω^* : Twist

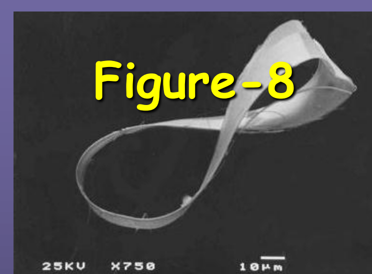
ω^*, ω is independent,respectively, due to the topological defect theory of the crystals, as yet. (by Frank)

Topological Crystals ⟷ **New Definition !** Knot Theory ⟷ **$L_k = (\omega+\omega^*)/2\pi - 1$ Linking Number**

$\omega = 2\pi$
$\omega^* = 0\pi$
$L_k = 0$



$\omega = 2\pi$
$\omega^* = 1\pi$
$L_k = 1/2$



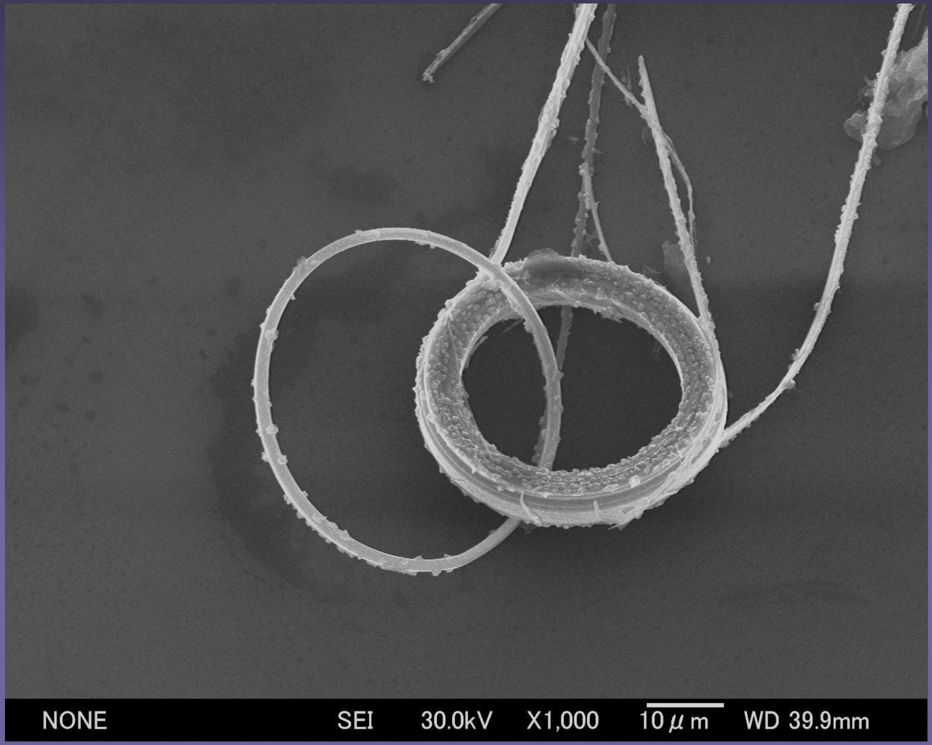
$\omega = 2\pi$
$\omega^* = 2\pi$
$L_k = 1$

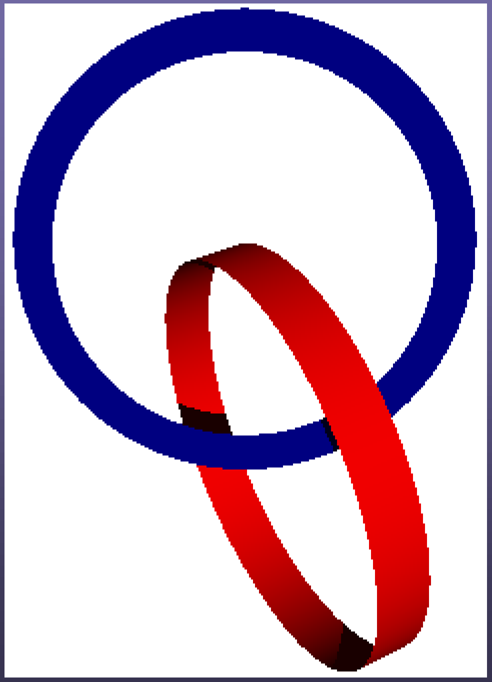
Global Wedge + Global Twisting

Knots Crystals !!

$$\frac{1}{2\pi}\int_{crystal} f\left(\omega, \omega^*\right) dV = L_k$$

Discover Hopf-Link Crystals



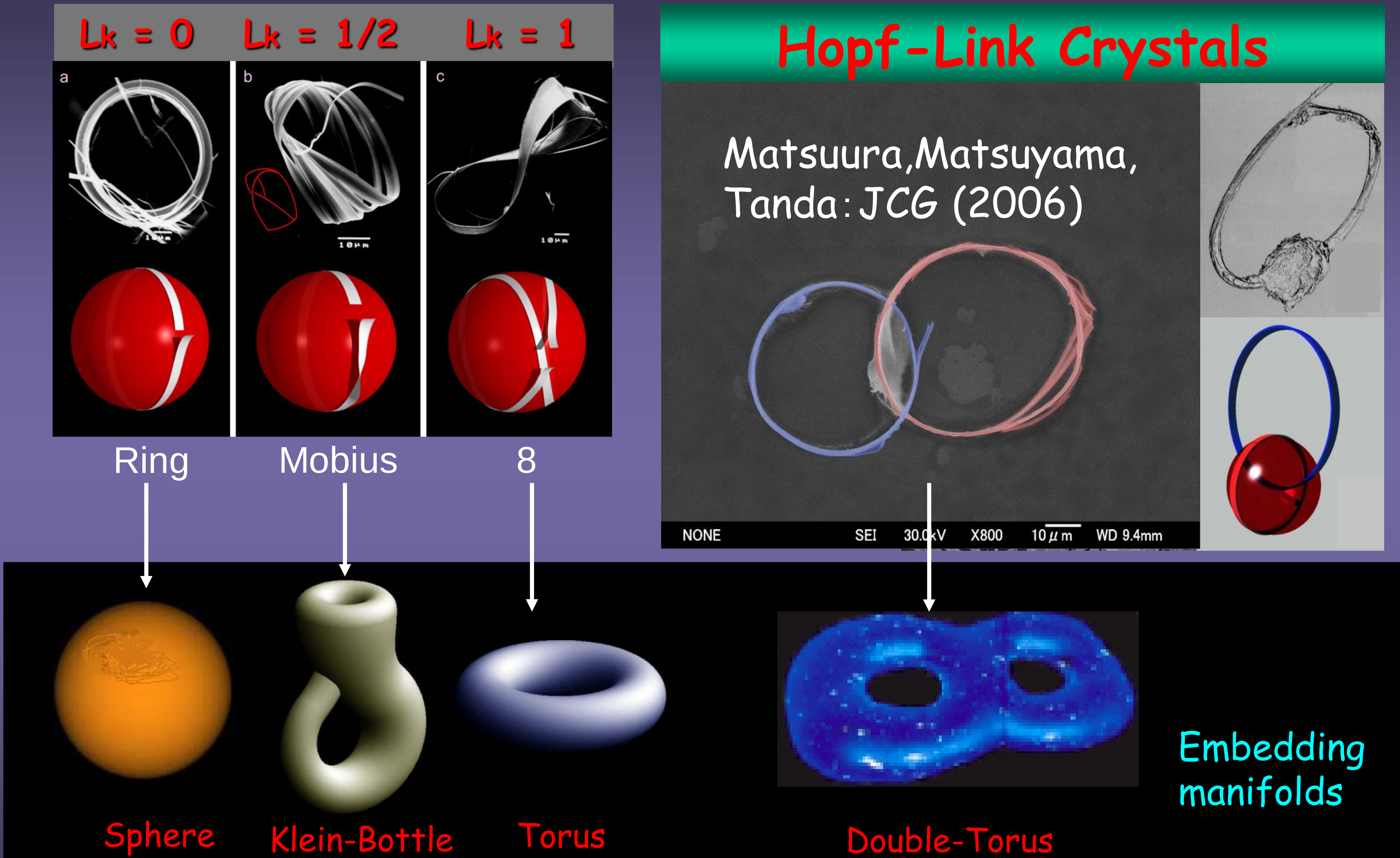


Topologically linked crystals

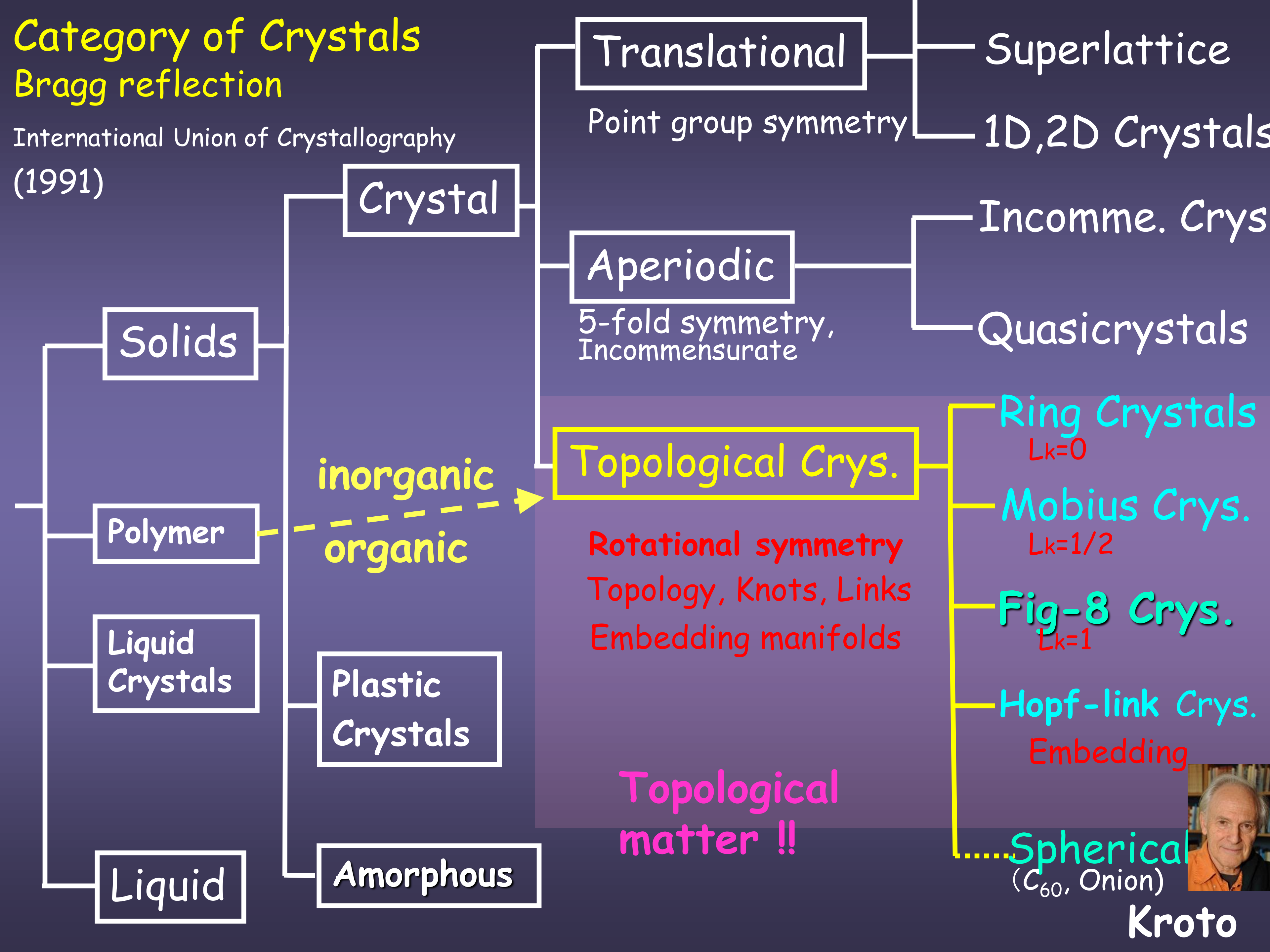
T.Matsuura, M.Yamanaka, N. Hatakenaka,T. Matsuyama, and S. Tanda, Journal of Crystal Growth 297, 157 (2006).

Hopf-link crystals cannot be categorized by the linking number of Knots crystals and usual point groups → New Classification

Classification By Embedding manifolds



We propose New Classification with Embedding manifolds


Category of Crystals
Bragg reflection
International Union of Crystallography
(1991)
Solids
Crystal
Translational
Point group symmetry
Superlattice
1D,2D Crystals
Aperiodic
5-fold symmetry,
Incommensurate
Incomme. Crys
Quasicrystals
Topological Crys.
Rotational symmetry
Topology, Knots, Links
Embedding manifolds
Ring Crystals
Lk=0
Mobius Crys.
Lk=1/2
Fig-8 Crys.
Lk=1
Hopf-link Crys.
Embedding
Spherical
(C60, Onion)
Kroto
Polymer
inorganic
organic
Liquid Crystals
Plastic Crystals
Amorphous
Liquid
Topological matter !!

Our Goal

1. Topological Crystallography (Ladder)

↓

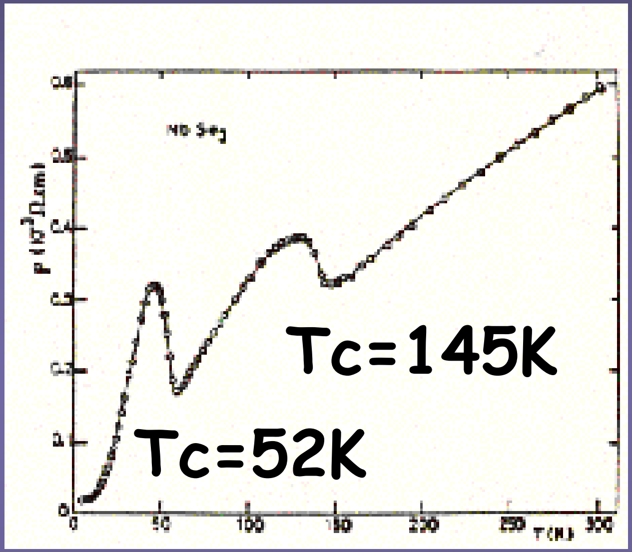
2. Quantum topological effects in the topological crystals (Ants)

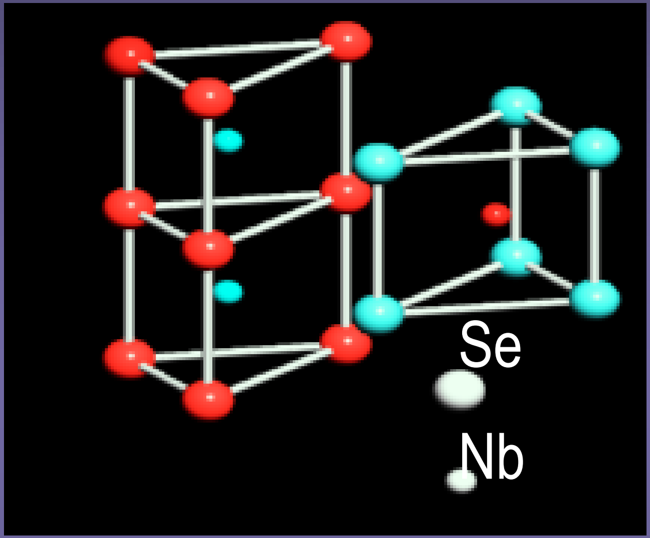
Macroscopic quantum coherence in nontrivial topological space

Topological gauge fields
Berry Phase

$NbSe_3$: Charge Density Waves

CDW: Charge-Density-Waves



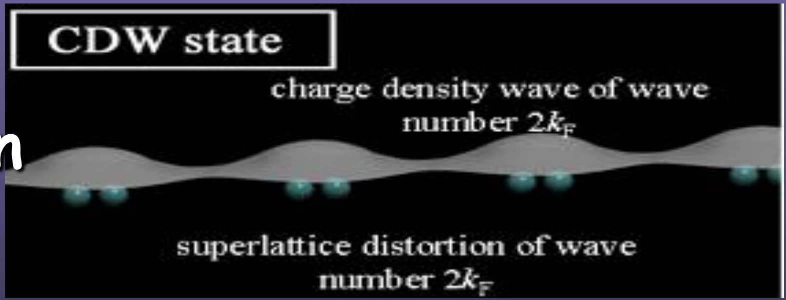


Chain axis

Covalent wire

Fiber structure

Transition

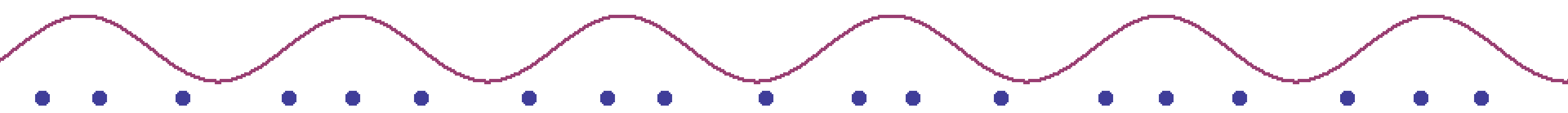


A periodic charge modulation

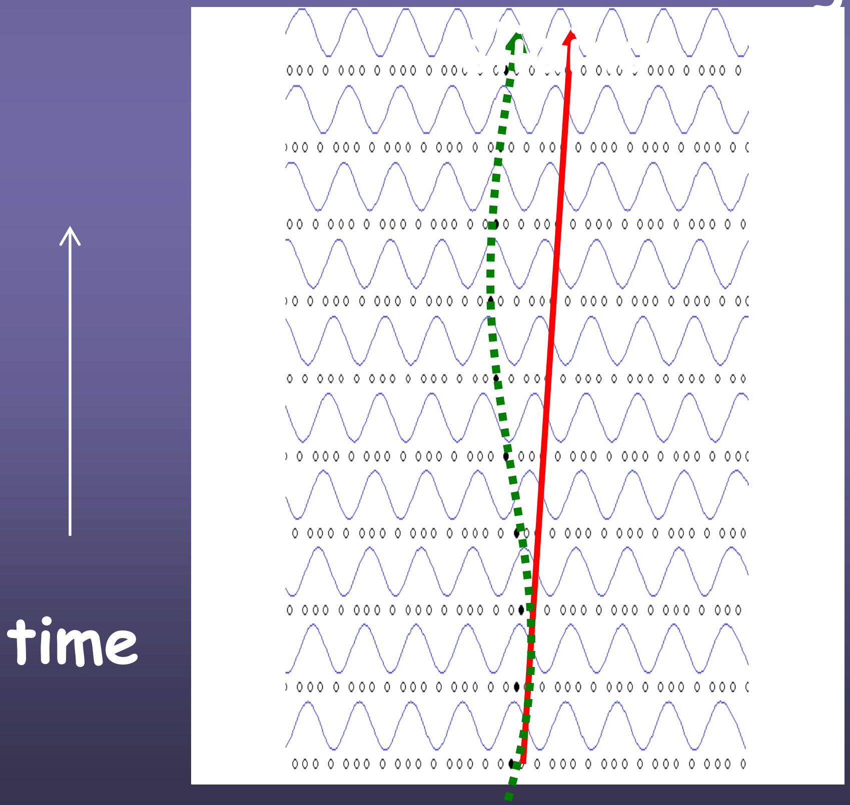
Macroscopic wave function

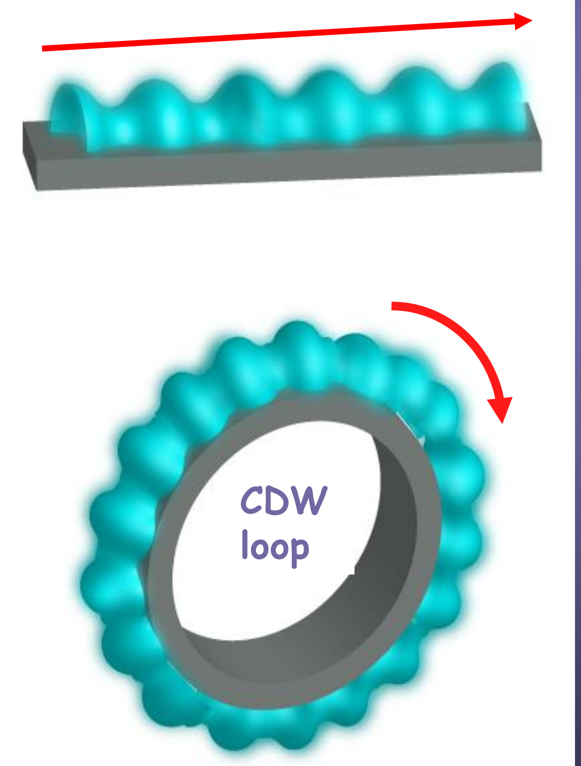
Sliding of Macroscopic wave function

Fröhlich superconductors : phason



CDW sliding





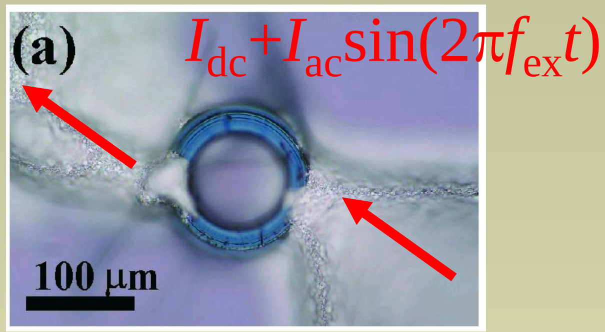
Eternal current

Real space topology

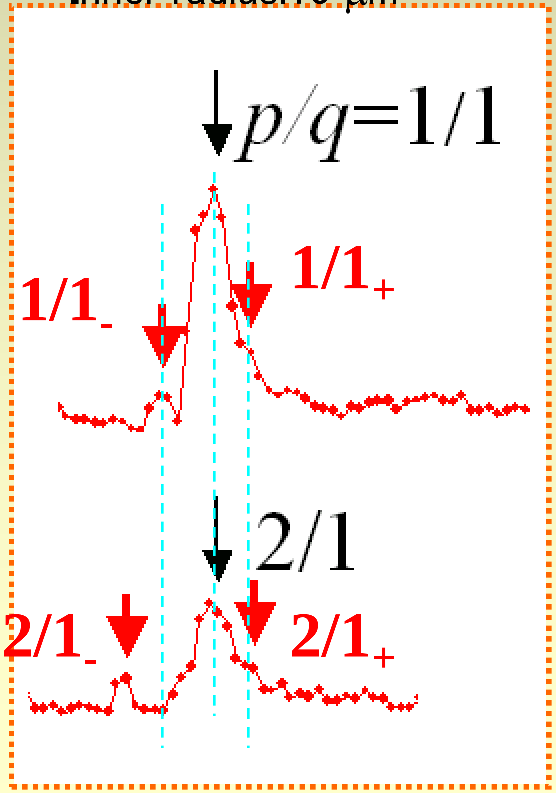
I. Circulating Sliding Current of CDW By Shapiro Steps in the Loop

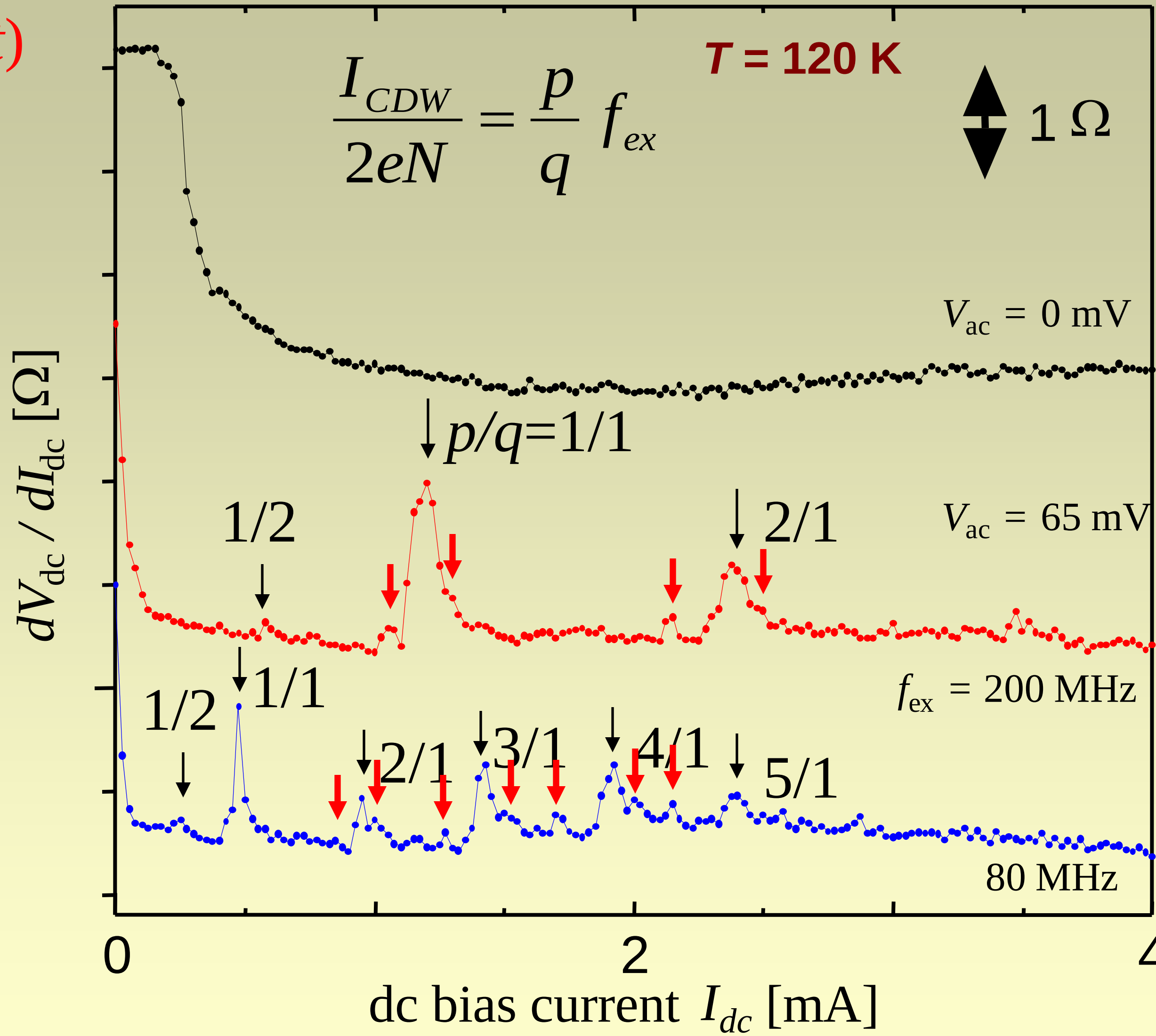
II. AB-Effect of CDW in the Loop

★★ Shapiro peaks in CDW loops



NbSe$_3$ ring
Outer radius : 120 μm
Inner radius:10 μm

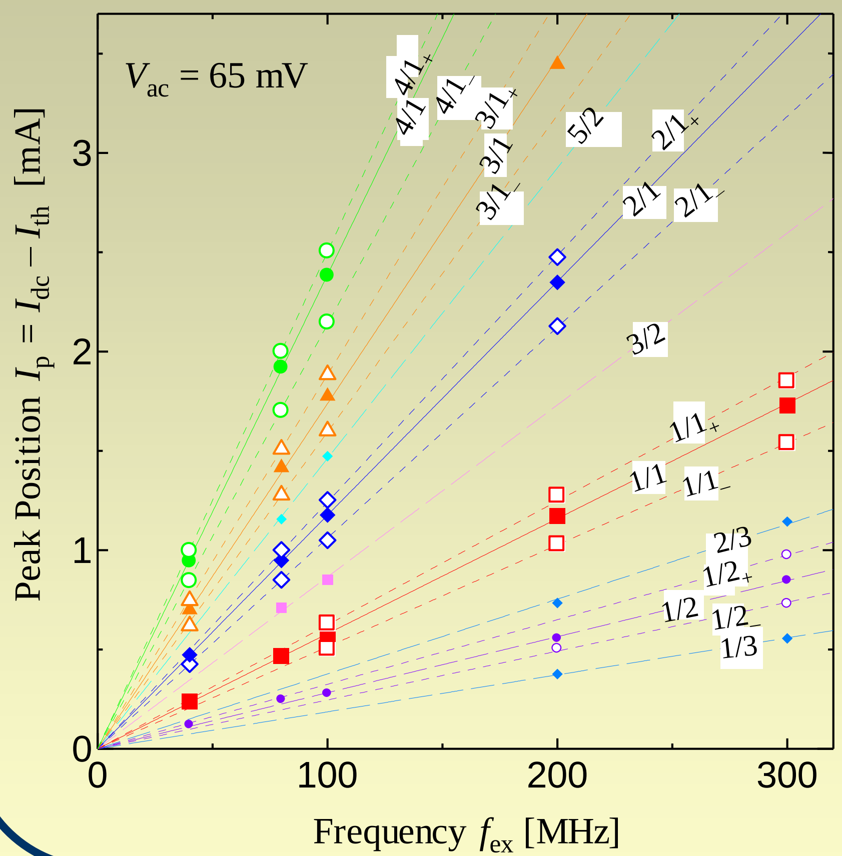




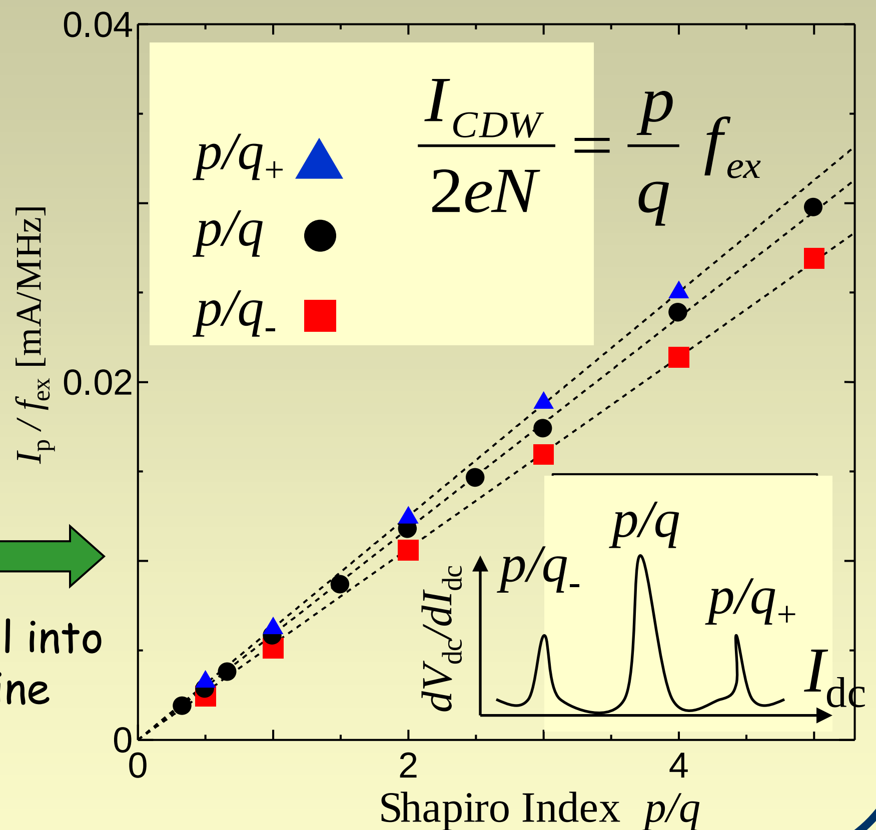
Subpeaks at both sides of Shapiro peaks are observed!

Analysis of peak positions

f_{ex} = 40, 80, 100, 200, 300 MHz

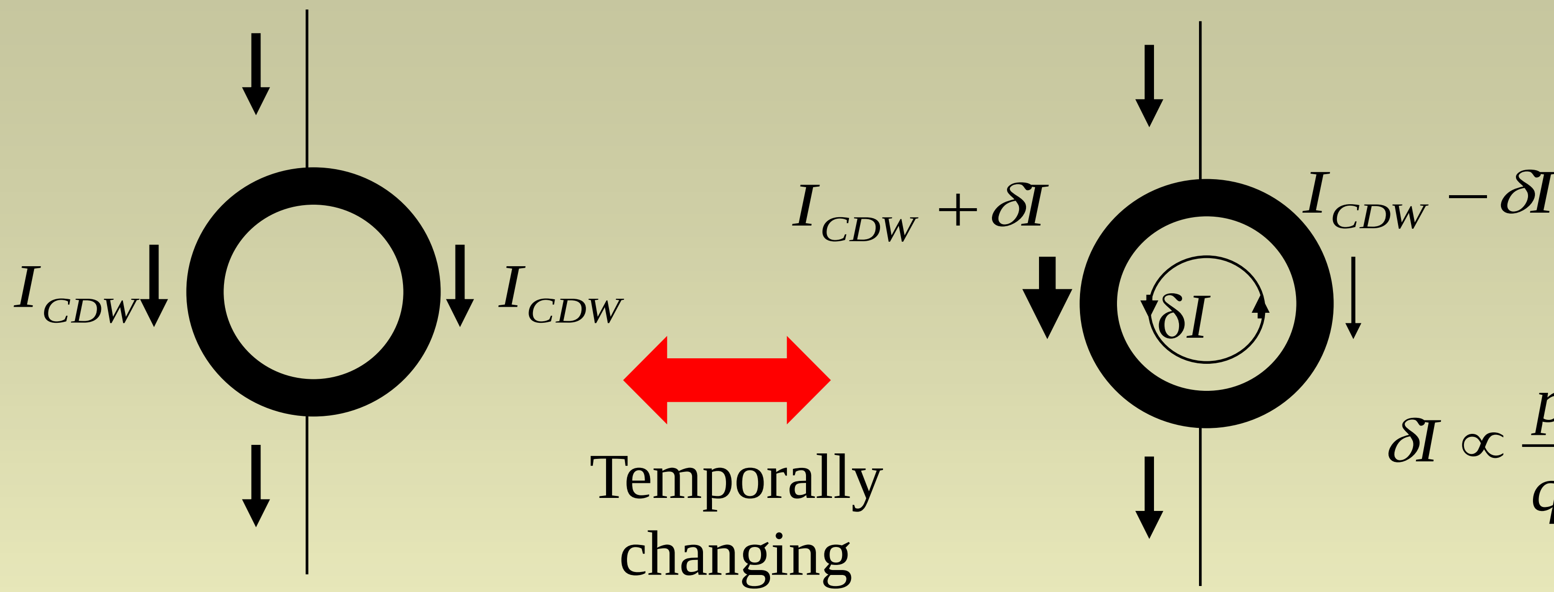


Fall into 3 line



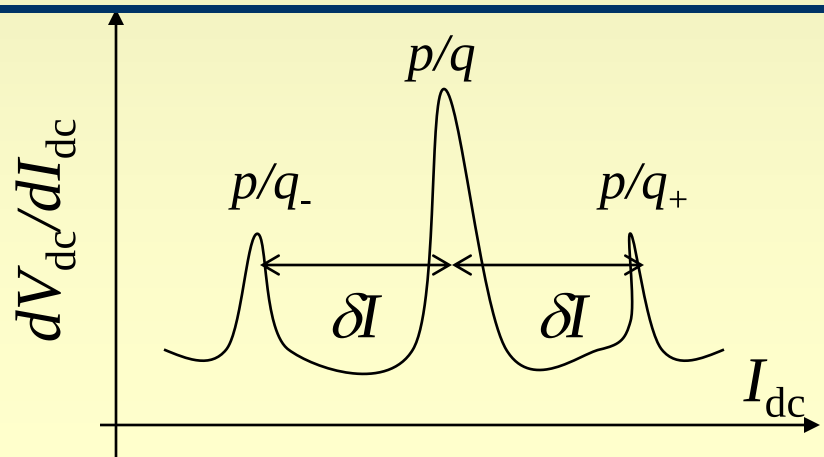
Discovery of Beat peak only in the loop CDW !!

Circulating current



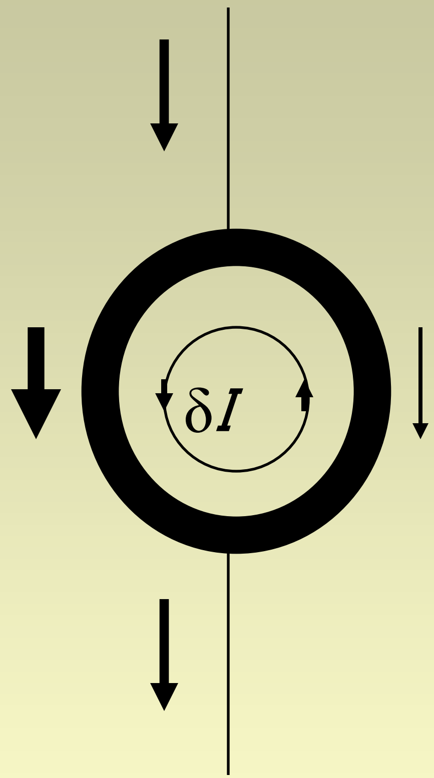
$$\delta I \propto \frac{p}{q} f_{ex}$$

$$\frac{I_{CDW}}{2eN} = \frac{p}{q} f_{ex} \qquad \frac{I_{CDW} \pm \delta I}{2eN} = \frac{p}{q} f_{ex}$$



Additional current must exist in the loop CDW

Damping time of Circulating current



If CDW is circulating, the damping time can be estimated by

Circumference: 10^{-4} m

Phason velocity: 10^{4} m/s

Circumference / Phason velocity = 10^{-8} s

>> $NbSe_3$ Phason damping time: 10^{-11} s

Richard and Chen, Solid State Commun. **86** 485 (1993)

Elongation of damping time !!

→ Precursor of Fröhlich supercurrent

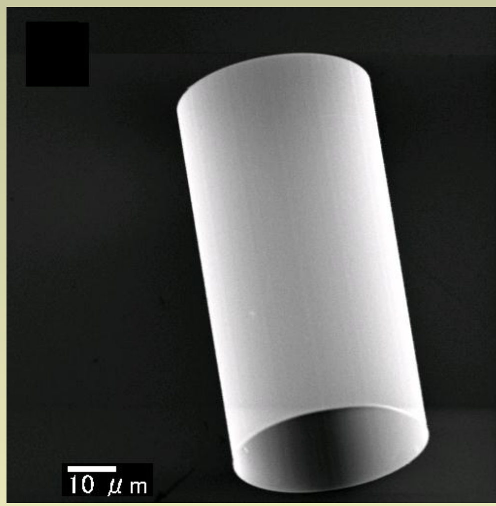
T. Matsuura, K. Inagaki, and S.Tanda, Phys.Rev.B 79, 014304 (2009).

Real space topology

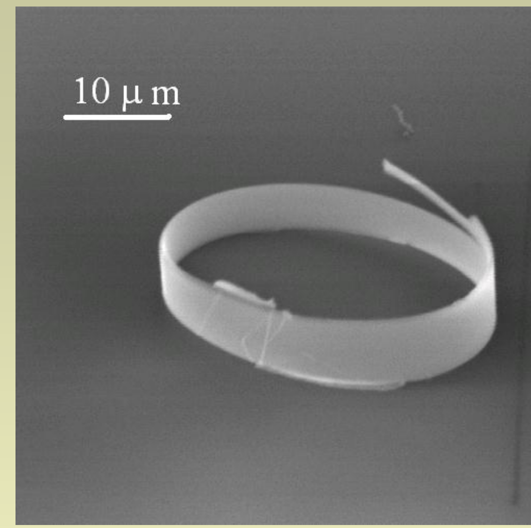
1. **Circulating current of CDW By Shapiro Steps in the Loop**

2. **AB Effect of CDW in the Loop**

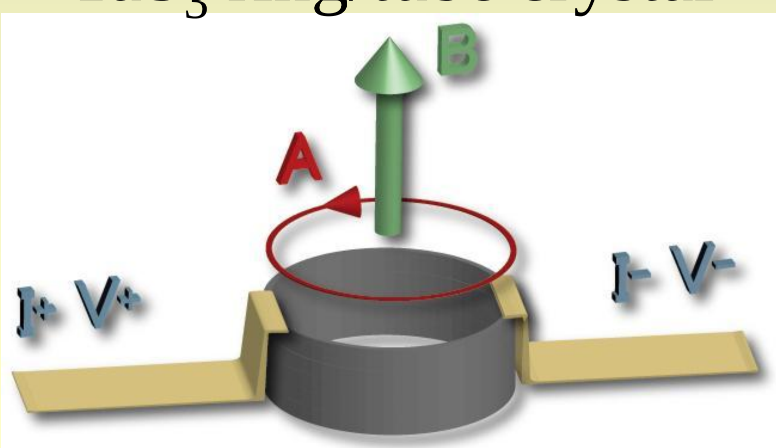
Experimental Setup

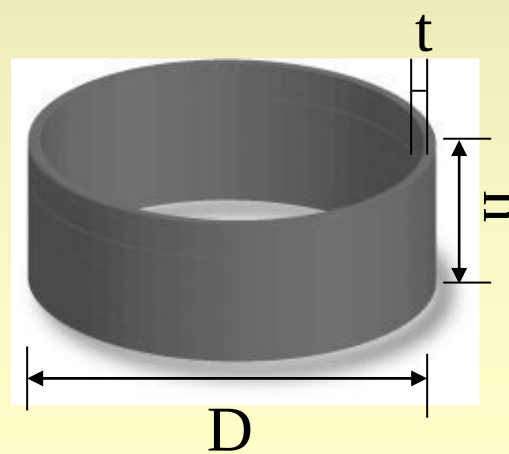


FIB cut



TaS_3 ring/tube crystal



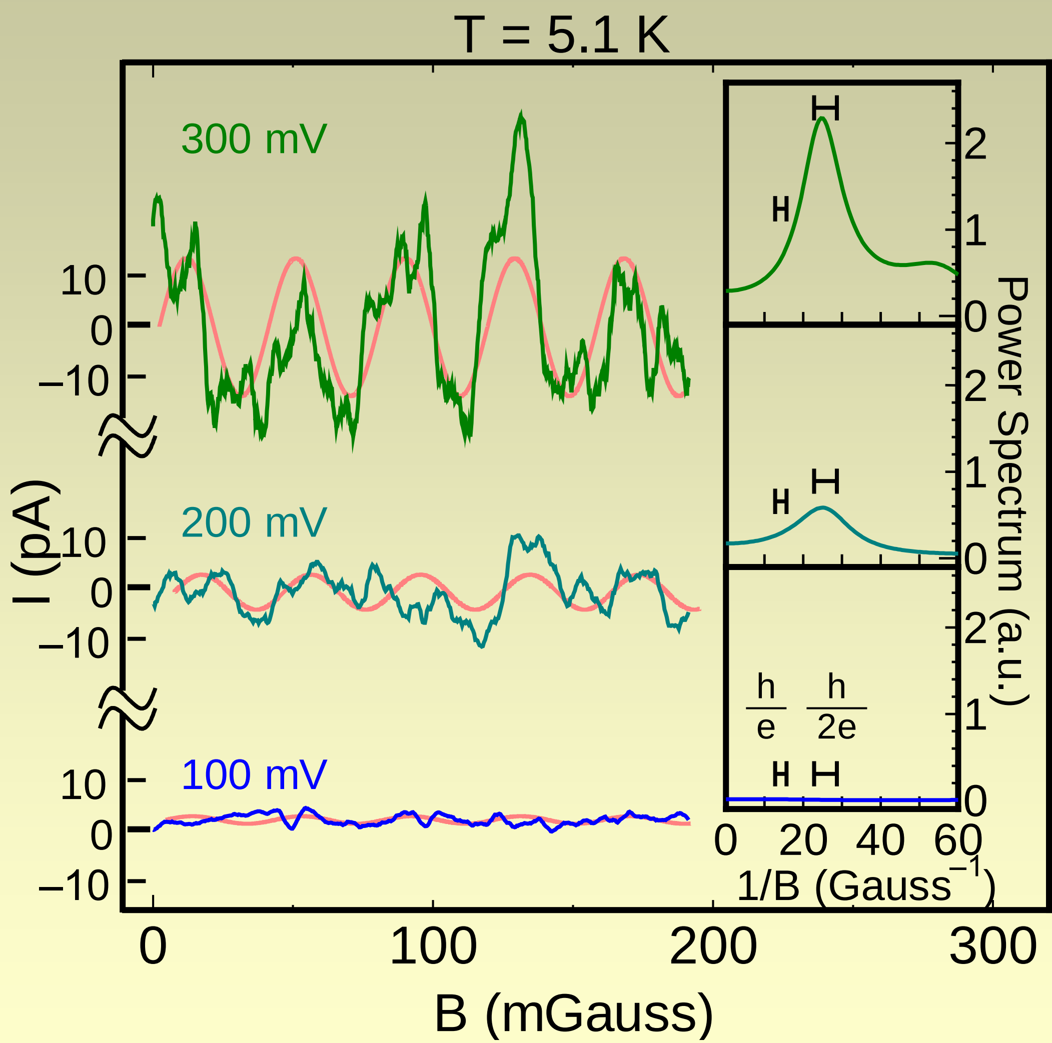


$t \approx 0.1 \mu m$

$h \approx 1 \mu m$

$D \approx 27 \mu m$

Result



Periodic oscillations were observed.

Estimation of unit charge

$$\Delta B = \frac{\Phi}{S} = \frac{h}{e^*} \cdot \frac{1}{\pi r^2}$$

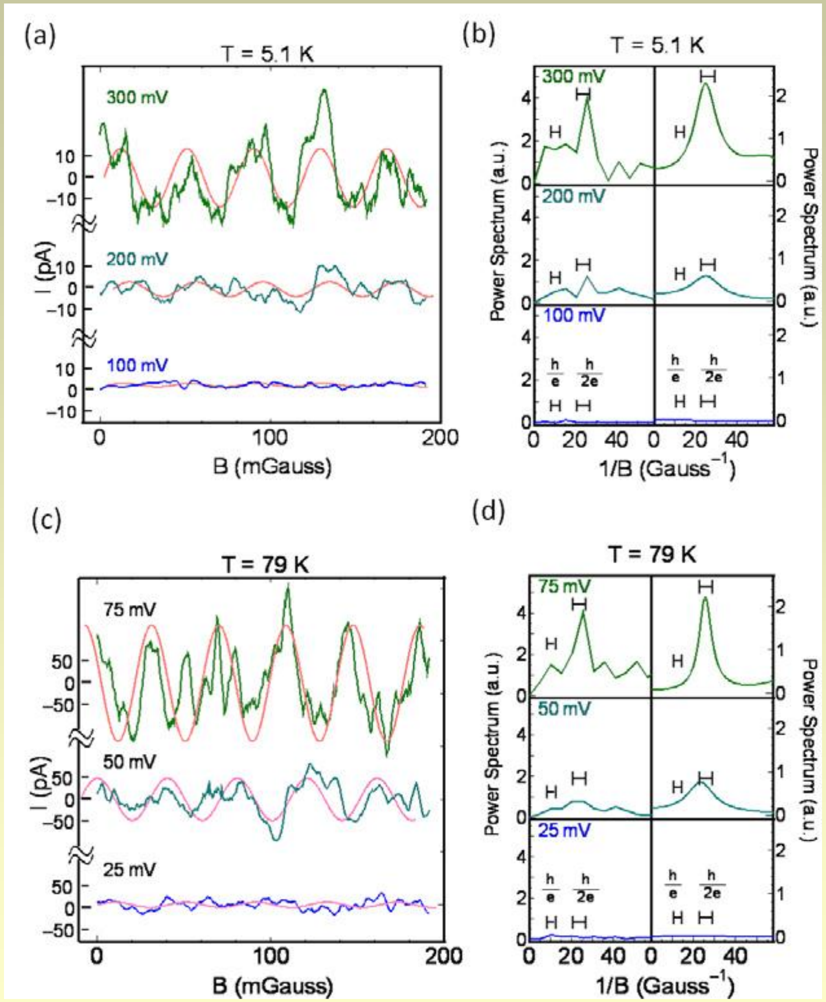
ΔB: The period of the oscillation

S: The area of the ring crystal

Sample	Diameter	Area m^2	Period	Charge
A	27 μm	5.6×10^{-10}	39.7 mGauss	3.0×10^{-19} C
B	17 μm	2.3×10^{-10}	95.2 mGauss	3.1×10^{-19} C

Unit charge corresponds to 2e (=3.2×10^{-19} C)

Quantum coherence at temperature of liquid-nitrogen



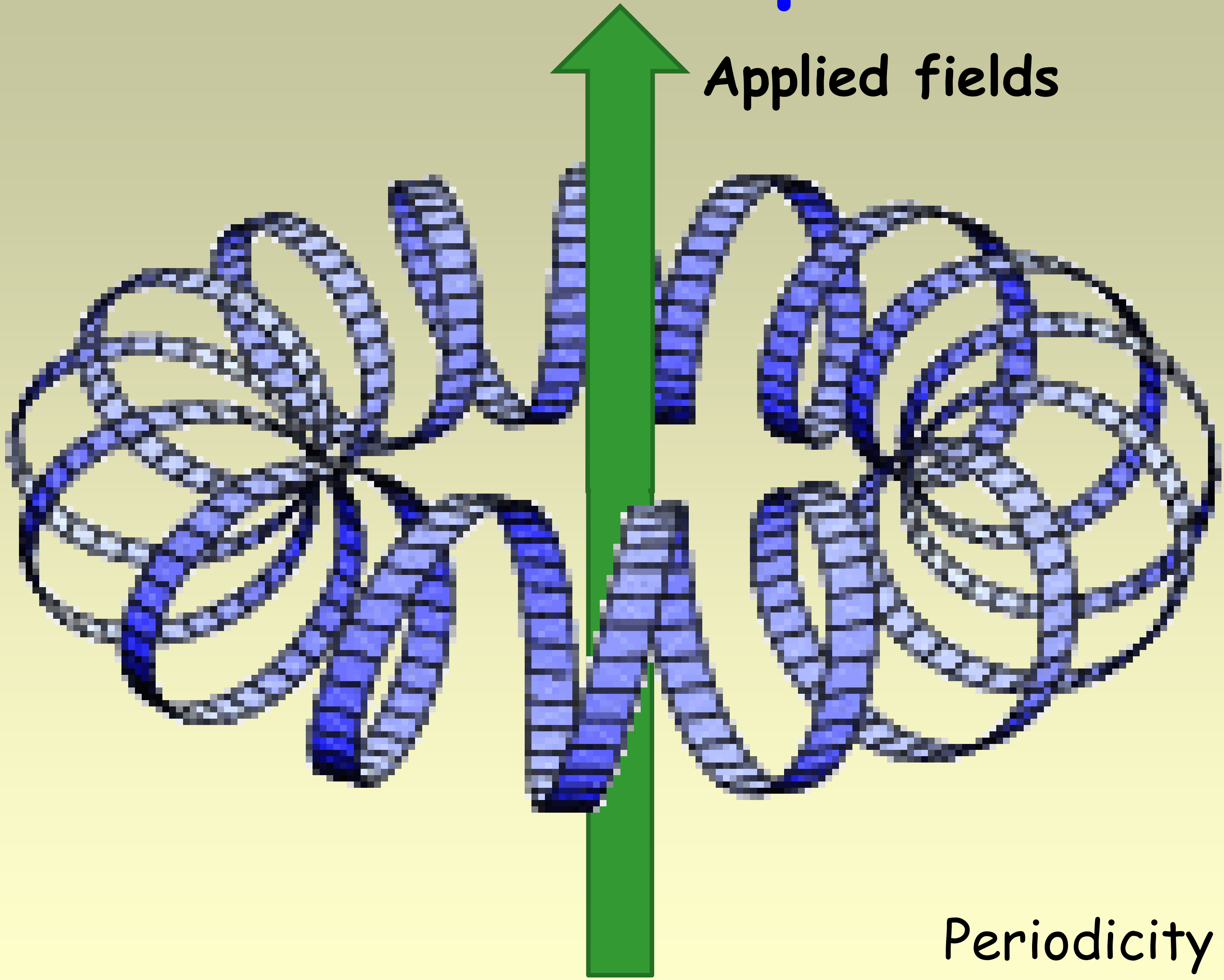
T=5.1K

T=79K

We also observed AB-effect of CDW sliding at 79K in TaS_3 ring crystals !

EPL, 97 (2012) 57011

Quantum Phase Slip in CDW rings



Periodicity $\frac{h}{2e}$

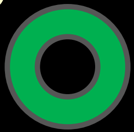
Application of topological crystals

We obtain NbS_3 topological crystals  Tc=340K

, which have room-temperature Macroscopic-wave function (CDW) !

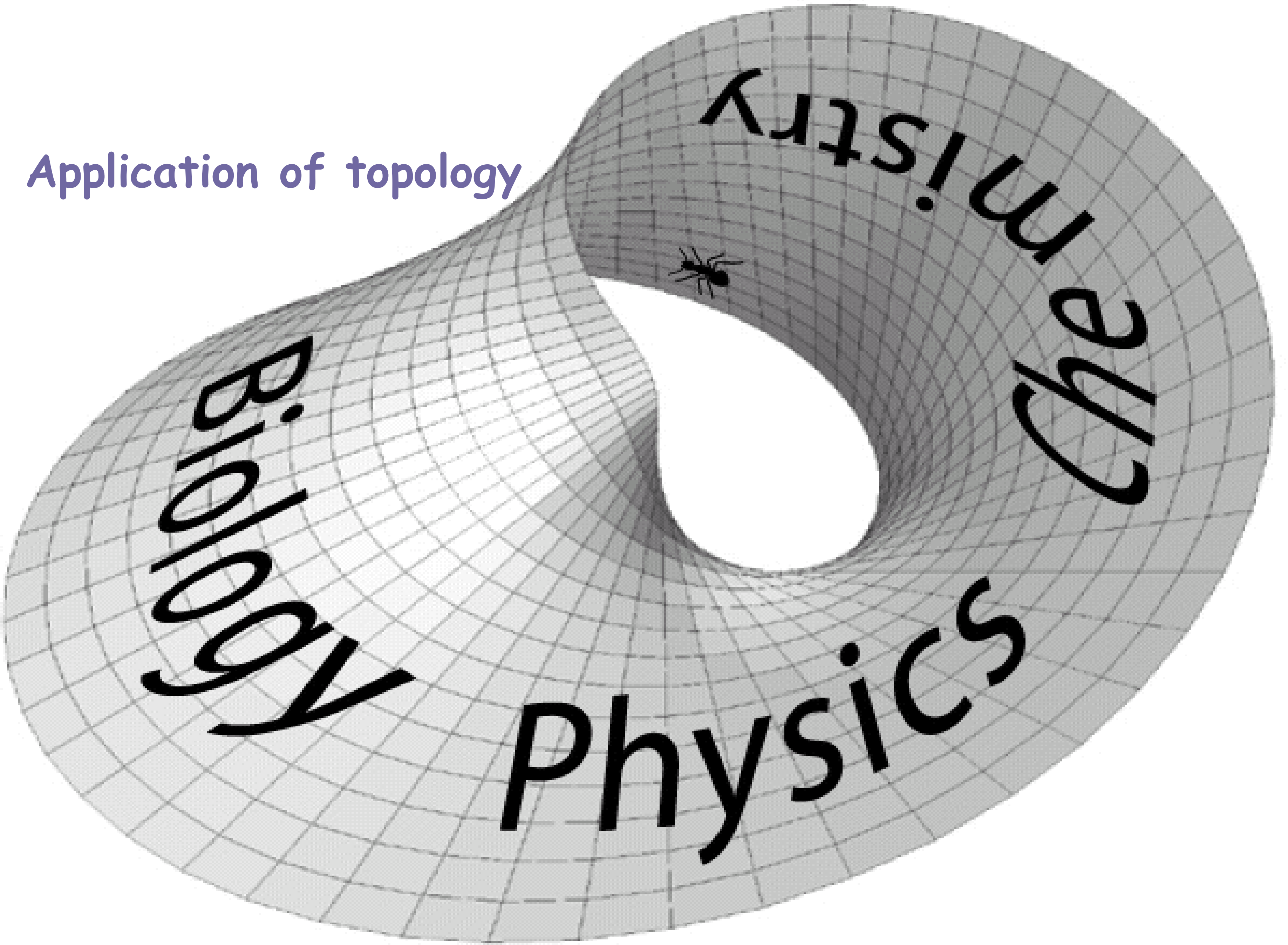
Room-temperature SQUID

Topology change

	Simply connected space : (Cut-ring) 	**Multiply connected space :**  
Crystals	Cycloid crystals (cut-ring) 	Ring, Mobius, 8, Hopf-link **Knots & links** crystals **Embedding** manifold
CDWs	Insulatotor Pinning of CDW at the edge	**Frölich superconductor** **AB effect** of CDW
Superconducting vortices	Abrikosov lattice	**Cylindrical vortex in ring crystals** Knot vortex in Mobius and 8 ?

Topology defines the properties of condensed matter

Application of topology



Thank you for your attention !

色即是空

"Zen": Hakuin (Buddhist) used Mobius strip in 1758

色＝空

色=obverse

空=reverse