16. Overview: ‘The’ strength, or strength’s’?

What is soil strength? In usual contexts, it means the peak strength, or the maximum stress (or stress ratio) that soil can bear, mobilised at large strains. However, soils’ stress-strain relationships exhibit a variety of strain-hardening/softening types, particularly at large strains, depending very much on soil types, density, drainage conditions, deformation modes and other factors. It is difficult in quite a few situations to define a unique strength (‘the’ strength) without considering relevance to a particular engineering problem in interest.
17. Peak shear strength

17-1. Drained strength and undrained strength

First of all, make sure that you understand the relationship between total and effective stress paths under drained and undrained conditions, discussed in Week 3.

Consider triaxial compression of soil obeying the Mohr-Coulomb failure criterion:

\[ q = \frac{6 \sin \phi'}{3 - \sin \phi'} \quad (\rightarrow \text{Week 4}) \]

\[ q_u = 2c_u \]

\[ q_u = 2c_u \]

Undrained path: Moderately contractive

Very contractive

\[ \Delta u \]

Drained path

\[ 3 \quad (\rightarrow \text{Week 1}) \]

Half the maximum value of \( q \) (\( q_u \)) is the undrained shear strength, \( c_u \)

(see the right diagram for why it should be half. Recently, it is encouraged to express it as \( S_u \)).

The strength defined in terms of effective stress (the envelope) and the undrained shear strength are related via the pore water pressure development, \( \Delta u \).
Now imagine that isotropic pressure is applied to the soil, increasing the total stress by $\Delta p$.

**Long-term strength**: The strength available after any change in the total stress is converted into a change in the effective stress via consolidation and dissipation of pore water pressure.

**Short-term strength**: The strength available before the change in the total stress is converted into a change in the effective stress.

Because the effective stress does not change, $c_u$ does not change. The $c_u$ value (only) appears to be independent of the total stress.
Note the following:

- If you look at the short-term strength in terms of the total stress, the strength looks like independent of the stress. That is why it is considered as ‘cohesion’ and the letter “c” was assigned. However, it has nothing to do with the intersect of the failure envelope (called “true cohesion”). That is why it is encouraged to use “S” for “S”trength, in place of c.

- In usual cases, the short-term strength is relevant to clays, while the long-term strength is relevant to sands/gravels. This is simply because of the time required for consolidation and transfer of $\Delta p$ to $\Delta p'$. Sometimes clays are called ‘c-material’ or ‘cohesive material’, while sands/gravels are called ‘$\phi$-material’ or ‘frictional material’. If the behaviour is interpreted in terms of effective stress, such a nomenclature does not make sense.

Cut slope in the London Clay: Why does it stand (Assignment 4)?

(Diagram from Kovacevic et al., 2007)

I was working here!
A direct shear test (shear box test) has the following characteristics:

- Constant vertical stress tests or constant volume (corresponding to undrained conditions) tests may be performed.
- The horizontal stress is not known.

Because we do not know the horizontal stress, we cannot construct Mohr’s stress circle as we did in triaxial tests. It means that we can derive neither $c_u$ (i.e., the radius of the circle) nor $\phi'$ (the slope of a tangent to the circle) in a rigorous way. It involves some assumptions to derive these.

It is often assumed that the shear stress, $\tau_{vh}$, is at the crown of the Mohr’s stress circle (assuming $\sigma_v = \sigma_h$). Hence,

$$c_u = (\tau_{vh})_{max}$$

Or, it is also often assumed that the maximum stress ratio is mobilised along the horizontal plane. Hence,

$$\phi' = \tan^{-1}(\tau_{vh} / \sigma'_v)$$

These two assumptions are not compatible (think why), but both are often invoked.
17-3. Peak shear strength of soils: Drained conditions

Peak shear strength of soils are closely related to dilatancy characteristics. It is observed better under drained conditions.

Experimental findings:

- Higher density and lower confining stress
- Higher peak stress ratio and more dilative behaviour

Below is an interpretation of how dilatancy adds apparent frictional strength.

\[
\tau = \sigma \delta \gamma - \delta \varepsilon
\]

The work done by external forces:

\[
\delta W = \tau \delta \gamma - \sigma (-\delta \varepsilon)
\]

The energy dissipated internally due to friction:

\[
\delta E = \tau_f \delta \gamma
\]

where \( \tau_f \) is the internal friction stress.

From \( \delta W = \delta E \),

\[
\tau_f + \sigma (-\frac{\delta \varepsilon}{\delta \gamma}) = \tau
\]

- True friction stress
- Dilatancy
- Observed in test

Drained simple shear of Leighton Buzzard Sand (Cole, 1967; reproduced from Oda, 1975)

Dilatancy of granular material (Diagrams taken from Wood, 1990)
Do you remember the data from Assignment 2? They were from drained triaxial tests on normally consolidated Soma Silica Sand. Here indeed you see an ‘apparent’ cohesion for the denser condition, caused by dilatancy.

Bjerrum & Simons (1960) showed that corrected $\phi'$ from drained triaxial tests and $\phi'$ from undrained triaxial tests are broadly identical (diagram reproduced from Ishihara, 1988).
17-4. Peak shear strength under undrained conditions: Clays

Under undrained conditions, we do not have the problem of dilatancy's influence on the apparent friction, because the constant-volume condition prohibits dilatancy. However, the suppressed dilatancy leads to changes in the effective stress.

So under undrained conditions, knowing strength comes down to knowing which direction the undrained effective stress path goes, as well as knowing the location of the peak strength envelope.

Effective stress paths of reconstituted Todi Clay (Burland et al., 1996)

We have already studied a model (the Cam Clay Model) to interpret these effective stress path behaviour in relation to overconsolidation. For heavily over-consolidated soils, however, it is difficult to observe effective stress paths converging to the Critical State, as the strain-softening leads to bifurcation (i.e. shear banding) phenomena in most testing methods.
17-5. Peak shear strength under undrained conditions: Sands

Shown in the diagrams are undrained triaxial compression data of clean, Toyoura sand by Verdugo (1992; reproduced from Yoshimine & Ishihara, 1998).

Note the following:

- Dense sand can exhibit extremely high shear strength, while the opposite is true for loose sand (static liquefaction, or flow failure).

- Higher confining stress can work as if equivalent to looser conditions.

The right diagram is an interpretation of these observations. A "phase transformation" (Ishihara et al., 1975) from contractive to dilative behaviour is characteristic of granular soils.
17-6. Sampling, disturbance and laboratory strength

Knowing the effective stress path characteristics under undrained conditions helps understand the disturbing mechanisms involved in sampling. Let us consider normally consolidated clay as an example, in which shear (i.e. changes in \( q \)) causes a reduction in \( p' \).

During sampling (which is considered to be an undrained process for clayey soils), the release of \( p \) is compensated by a reduction of \( u \). Any change in \( p' \) is therefore ideally due to plastic deformation by shearing.
The change in $p'$ is recoverable by re-consolidation in a laboratory triaxial (CU) test, while this is not possible in unconfined compression or unconsolidated triaxial (UU) test. However, sample disturbance may cause permanent changes in the failure envelope in ‘sensitive’ soils. This loss of structure is generally not recoverable, as we see next week.

Factors affecting $p'$ during sampling (Hight et al., 1992)
References


