Soil Mechanics II
土の力学II

Hiroyuki Tanaka
田中洋行
Soil Mechanics

- Geotechnical Engineering
- Meaning of “Geo” The earth, or Ground
- Geology
- Geo-sciences, chemistry, graphy, so on.

- This lecture is proceeding based on “土質力学入門”, written by Prof. 三田地利之.
Shear strength
Evaluation of strength

Concrete

Compression
Small tensile strength

Steel

Bending Moment

Page number of the text book: P 119
Criteria for Soil

Vertical Force: $N$
Shear Force: $S$
Normal Stress: $\sigma_n = N/A$
A: Cross Area
Shear Stress: $\tau = S/A$

$\tau = c + \sigma_n \tan \phi$

Coulomb’s Criteria

Normal Stress: $\sigma_n = N/A$
A: Cross Area
Shear Stress: $\tau = S/A$

Boundary Criteria

$\tau = c + \sigma_n \tan \phi$

C: cohesion, $\phi$: friction angle

P129
Shear and Normal Stresses

\[ \sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha - \tau_{xy} \sin 2\alpha \]

\[ \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha \]

We can find \( \alpha \) for \( \tau = 0 \)

When \( \tau = 0 \), we call this plane “Principal” Plane.

Principal stresses: The maximum and minimum principal stresses: \( \sigma_1 \) and \( \sigma_3 \)

\( \sigma \) and \( \tau \) are changed according to an angle
Mohr’s Stress Circle

Principal Stress

σ₁

σ₃

σ₁

σ₃

σ

τ

τ

σ

σ

Principal Stress

P122-123
図7・4 任意の面上の応力とモールの応力円
How to draw the Mohr’s circle

Positive
σ: compression
τ: anti-clockwise
Measuring parameters $c, \phi$

• **Laboratory Test**
  – Sampling from a borehole
  – Direct Shear Test
  – Triaxial (Unconfined compression) Test

• **In situ Test**
  – No sample
  – Vane Test
  – Standard Penetration Test (N value)
Direct Shear Test

Merit: Easily understand
Demerit: stress and strain are not uniform
Control of drainage is difficult

\[ \tau = c + \tan \phi \]
Triaxial Test

Deviation stress \((\sigma_1 - \sigma_3)\)

- **Tri:** Three
- **Axial:** Axis

No shear stress because of water

Principal plane

Lateral Pressure, Cell pressure

\[ \sigma_2 = \sigma_3 \]
σ₁
σ₃
α
τ
Plane acting the maximum principal stress

\[ \tau = c + \sigma \tan \phi \]

Failure Envelopement

Failure point

Pole

\[ \sigma_1 \]

\[ \sigma_3 \]

\[ \alpha \]

\[ \sigma_1 \]
Failure criterion of Mohr and Coulomb

In stead of using $\sigma$ and $\tau$, the failure criterion is presented by $\sigma_1$ and $\sigma_3$

$$\sigma_1 - \sigma_3 = \frac{(\sigma_1 + \sigma_3) \sin \phi + 2c \cdot \cos \phi}{2}$$
Three conditions by Drainage

- **Unconsolidated Undrained (UU)**
  - Consolidation
  - Shear
- **Consolidated Undrained (CU)**
- **Consolidated Drained (CD)**

**Principle of Effective Stress**

The behavior including the strength is governed by the effective stress.
\[ \sigma' = \sigma - u \]
\( \sigma' \): Effective stress, \( \sigma \): Total stress
Performance of UU Test

\( \sigma' \): Effective stress

\( \sigma \): Total stress

Failure envelop: \( \phi = 0 \)

Cu or Su
Apparent cohesion, or undrained shear strength

Fully Saturated

\( \sigma' \): Effective stress

\( \sigma \): Total stress
Unconfined Compression Test

Sometimes called Uniaxial Test

\[ \sigma_1: \text{At failure, we call this strength } q_u \]

unconfined compression strength

\[ \sigma_3 = 0 \]

In practice, \( c_u \) is called “cohesion”, or apparent cohesion.

\[ C_u = \frac{q_u}{2} \]

Failure envelop: \( \phi = 0 \)

\( \sigma \): Total stress

P140-142
Young Modulus, $E_{50}$ and Sensitivity

\[ E_{50} = \frac{\sigma_{50}}{\varepsilon_{50}} \]

\[ \text{Sensitivity} = \frac{q_u}{q_r} \]
Performance of CU Test

After consolidation, $\sigma_3 = \sigma'_3$

Pore pressure generated by shearing

Undrained shear strength $S_u$
After consolidation, \( \sigma_3 = \sigma'_3 \)

If Normally consolidated

For Japanese clays, \( su/p = 0.3 \sim 0.35 \)
Performance of CD Test

$\sigma'_3$ does not change during shearing

Total and effective stresses are always the same because of no excess pore water pressure
Effective stress and Total stress analysis

• The effective stress analysis (ESA) seems more reasonable.

• Permeability is high (sandy soil), the ESA is applicable. Sand

• For clayey soil, effective stress or pore water pressure is unknown. Total stress analysis, in another word, $\phi=0$ method Clay
Shear strength for Total stress analysis

- Undrained shear strength
- For low OCR, i.e., negative dilatancy, undrained shear strength (UC or UU test) always is smaller than the drained shear strength.
- For long consolidation, the increase in the undrained shear strength can be expected (CU test)
$S_u$ measured by in situ test

Vane test
Dilatancy

• Volume change during shearing
• Performance is changed under undrained or drained conditions
  – Undrained:
    • No volume change $\Delta V = 0$
    • Pore water pressure change $\Delta u$
      – Positive: Negative  Negative: Positive
  – Drained:
    • No Pore water pressure $u = 0$
    • Volume change $\Delta V$
      – Positive: Expand  Negative: Compressive
Critical Void Ratio

Void ratio, \( e \)

\( e_{\text{crit}} \)

- Small
- Dense, \( \gamma \): unit weight large
- Large
- Loose, \( \gamma \): small

Dilatancy

- Positive
- Drained: Volume change
- Expands

- Negative
- Undrained: Pore water pressure
- Compresses
Pore water Pressure

Excess pore water pressure:  Deviator Stress: shear

\[ \Delta u = B\{\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)\} \]

Skempton’s A and B Coefficient:  B=1 for saturated soil, A: dependent on Dilatancy
Dilatancy for sand and clay

**OCR**: Over-Consolidation Ratio

\[ \text{OCR} = \frac{p_c}{p_{vo}} \]

- **Sand**
  - \( e < e_{\text{crit}} \): large, loose
  - \( e > e_{\text{crit}} \): small, dense
  - **Drained**: Volume change
    - **Positive**: Expand
    - **Negative**: Compression
  - **Undrained**: Pore water pressure
    - **Positive**: Skempton’s A: Low or negative
    - **Negative**: A: High

- **Clay**
  - **Drained**: Volume change
    - **Positive**: Expand
    - **Negative**: Compression
  - **Undrained**: Pore water pressure
    - **Positive**: OCR: 1 ~ 2
      - Slightly Normally
    - **Negative**: OCR: High
      - Heavily OverConsolidated

P 144-147, 150-151
Drained and Undrained strength

\[ \tau \]

\[ c' \]

\[ \phi' \]

CD

CU: Negative Dilatancy

CU: Positive Dilatancy
Liquefaction

If $\sigma'_3 = 0$, fully liquefied

$\sigma'_3 = \sigma_3 - u$

constant
increase

$\sigma_{1d}$: 変動 $\sigma_{3d}$: 一定

P147-149
Counter measurements for liquefaction

• Densification
  – Positive dilatancy. No positive water pressure
  – Vibration

• Lowering the ground water table
  – No water

• Stabilized with cement
  – Cohesion $\tau=c'+(\sigma-u)\tan\phi$
Earth Pressure
Earth Pressure
Relating the movement of a wall

Active Earth Pressure
Earth pressure at rest
Passive Earth Pressure

Retaining wall

P160, 169
How to calculate the earth pressure?

• Rankine’s Method
  – Plastic equilibrium
  – Theoretically, limitation of its application

• Coulomb’s Method
  – Stability of soil mass
  – Trial calculation, more extensive application
Rankine method

\[ \sigma_v = \gamma z; \quad \gamma = \text{unit weight of soil}, \ z = \text{depth} \]

Active State

Passive State
Rankine method

\[ \sin \phi = \frac{\sigma_p - \gamma z}{\sigma_p + \gamma z} \]

\[ K_p = \frac{\sigma_p}{\gamma z} = \frac{1 - \sin \phi}{1 + \sin \phi} = \tan^2 \left(45 + \frac{\phi}{2}\right) \]

Using half angle formulae

\[ K_a = \frac{\sigma_a}{\gamma z} = \tan^2 \left(45 - \frac{\phi}{2}\right) \]

Important Value: \( \phi = 30^\circ \) \( K_a = 1/3, K_p = 3.0 \)
Earth Pressure at Rest

For conventional ground

\[ \sigma_v > \sigma_h \]

\[ K_o = \frac{\sigma^h}{\sigma_v} \]

- \( K_o \): Coefficient of earth pressure at rest

Depending on OCR. For NC (OCR=1) \( K_o = 1 - \sin \phi \)
Earth Pressure acting on the wall

Total Pressure: \( P_a = \gamma_t H \tan^2 \left(45^\circ - \frac{\phi}{2}\right) H \frac{1}{2} = \frac{1}{2} \gamma_t H^2 \tan^2 \left(45^\circ - \frac{\phi}{2}\right) \)
Rankine method $\phi=0$

$C = \frac{q_u}{2}$

$\sigma_a = \gamma z - 2c$

$\sigma_p = \gamma z + 2c$

For Clay
With surcharge

\[ h = \frac{q}{\gamma_t} \]

\[ q \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \gamma_t H \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \]
Change in properties or existence of water table

Earth pressure
(Effective stress)

Hydraulic pressure
Coulomb’s Method

\[ \delta : \text{Friction between the ground and the wall in Rankine’s theory, cannot take account.} \]
Water Level

γ = 20 kN/m³
γ'(γ_{sub}) = 10 kN/m³

Sheet pile

Active Earth Force

Passive Earth Force

Check for Embedded Depth:

\[ 2 \times l_a < 3 \times l_p \]
The embedded depth is enough?

Active Earth Pressure

\[ P_{a1} = \frac{20 \times 3}{2} = 30 \text{kN/m} \]

\[ l_{a1} = \frac{2}{3} \times 3 = 2 \text{m} \]

Passive Earth Pressure

\[ P_{a2} = 20 \times 9 = 180 \text{kN/m} \]

\[ l_{a3} = 3 + \frac{9}{2} = 7.5 \text{m} \]

\[ P_{a3} = \frac{30 \times 9}{2} = 135 \text{kN/m} \]

\[ l_{a3} = 3 + \frac{2}{3} \times 9 = 9 \text{m} \]

Center of Gravity

\[ l_p = 9 + \frac{2}{3} \times 3 = 11 \text{m} \]

\[ P_p = \frac{90 \times 3}{2} = 135 \text{kN/m} \]

\[ P_p \times l_p = 135 \times 11 = 1485 \text{ kN} < P_a \times l_a = 30 \times 2 + 180 \times 7.5 + 135 \times 9 = 2625 \text{ kN} \]

Unstable
Stability of slope
Evaluation of the slope

Safe or Danger?

SF or $F_s$ : Safety Factor = \[ \frac{\text{Resistance}}{\text{Drive Force}} \]

Resistance: Shear Strength

Drive Force: Gravity, Seismic
Stability of infinitive slope

\[ W = 1 \times \cos i \times h \times \gamma_t \]
\[ N = W \times \cos i \]
\[ S = W \times \sin i \quad \text{Driving Force} \]
\[ R = N \times \tan \phi \quad \text{Coulomb’s criteria: Resistance Force} \]

FS = \frac{R}{S} = \frac{\cos^2 i h \gamma_t \tan \phi}{\cos i h \gamma_t \sin i} = \frac{\tan \phi}{\tan i}

FS = 1, \ i = \phi \quad \phi: \text{angle of repose}
Circle Failure method

$\phi=0$: Short term

$$\text{Resistance moment} = (FE \times s_u \text{ or } c_u) \times R$$

$$\text{Drive Moment} = W \times x$$
Circle Failure method
Slice method

\[ SF = \frac{\sum \text{Moment for Resistance}}{\sum \text{Moment for Drive}} \]

Resistance moment = \((FE \times s_u \text{ or } c_u)xR\)

Drive Moment = \(W \times x\)

P196
Search for the smallest SF

Critical Circle
Tailor’s Chart
Stability Factor $N_s$

$$N_s = \frac{\gamma_t H_c}{C}$$

Stability Number

$N_s = \frac{\gamma_t H_c}{C}$

- $\gamma_t$: N/m$^3$ x m
- $H_c$: N/m$^2$
- $C$: No dimension

Slope angle

Toe Failure
Base Failure
Slope Failure
Failure pattern

Base Failure  Toe Failure  Slope Failure
Bearing Capacity

Shallow Foundation
Deep Foundation
Pattern of Failure

Bearing Capacity: Ultimate: $Q_u$
Allowable: $Q_{al} = Q_u / SF$
SF: Settlement, uncertainty for soil parameters
Theoretical Value of Prandtl

- **I Active Zone**
- **II Transition Zone**
- **III Passive Zone**

\[ \alpha \text{: dependent on the roughness of the footing.} \]

- Smooth: \[ \alpha = 45^\circ + \frac{\phi}{2} \]

Important value: \( \phi = 0 \), \( Q_u = 2b q_u \), \( q_u = (2 + \pi)c = 5.14c \)
Calculation of $Q$

$Q = qB = ?$
Terzaghi’s equation

\[ q_u = acN_c + \beta \gamma_1 BN \gamma + \gamma_2 D_f N_q \]

Cohesion \((C_u, S_u)\)  Friction  Surcharge

**Shape Factor**

<table>
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<tr>
<th>基礎形状</th>
<th>連続</th>
<th>正方形</th>
<th>長方形</th>
<th>円形</th>
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<tbody>
<tr>
<td>(\alpha)</td>
<td>1.0</td>
<td>1.3</td>
<td>(1 + 0.3 \frac{B}{L})</td>
<td>1.3</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.5</td>
<td>0.3</td>
<td>(0.5 - 0.2 \frac{B}{L})</td>
<td>0.3</td>
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</table>

Continuous  Square  Rectangular  Circle

**Factors**

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>(N_c)</th>
<th>(N_\gamma)</th>
<th>(N_q)</th>
<th>(N_c)</th>
<th>(N_\gamma)</th>
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**General Failure**  **Local Failure**

\(Q_u = Bq_u\)
Deep Foundation (Piles)
Types of Pile classified by support system

- Skin Friction
- Point Resistance
- Pointed Pile Friction Pile
- Friction Pile

P217
Design of Pile

• Using N value (Standard Penetration Test).
• Empirical equation

\[ R_u = \left( 40 \bar{N} A_p + \frac{1}{5} \bar{N}_s A_s + \frac{1}{2} \bar{N}_c A_c \right) \times 9.8 \]

- \( A_p \): cross sectional area
- \( A_s \): surface area of the sand layer
- \( A_c \): surface area of the clay layer
Standard Penetration Test (SPT)

- **Hammer (Mass = 63.5 kg)**
  - Height = 76 cm

- **Knocking Head**

**Definition of N value**

**How many blows for penetration of 30 cm**


**Pile Group**

\[ R_T = E \cdot n \cdot R_u \]

- **\( R_T \):** Bearing capacity of the pile group
- **\( R_u \):** Bearing capacity of the single pile
- **\( n \):** Number of the Pile
- **\( E \):** Efficiency of the pile group \(<1.0\)
Negative Skin Friction

Conventional case

Reclaimed Ground

Positive Friction

Negative Friction