北京建筑大学
BEIJING UNIVERSITY OF CIVIL ENGINEERING AND ARCHITECTURE

中国科学院寒区旱区环境与工程研究所
CAREERI

Creep of frozen soils

Prof. Jilin Qi

College of Civil and Transportation Engineering,

Beijing University of Civil Engineering and Architecture

ACKNOWLEDGEMENTS

- Prof. Wei Wu; Prof. Jianming Zhang, Dr. Xiaoliang Yao; Dr. Guofang Xu; Dr. Fan Yu; Dr. Songhe Wang; Dr. Ling Ma; M. Sc.Wei Hu; Mr. Mengxin Liu
- National Natural Science Foundation of China (Nos. 41172253 and 41201064)
- The European Community through the program “People” as part of the Industry–Academia Pathways and Partnerships project CREEP (PIAPP-GA-2011-286397).
- Chinese Academy of Sciences (100-Young Talents Program granted to Dr. Jilin Qi)
- The Importation and Development of High-Caliber Talents Project of Beijing Municipal Institutions granted to Dr. Jilin Qi (CIT&TCD20150101)

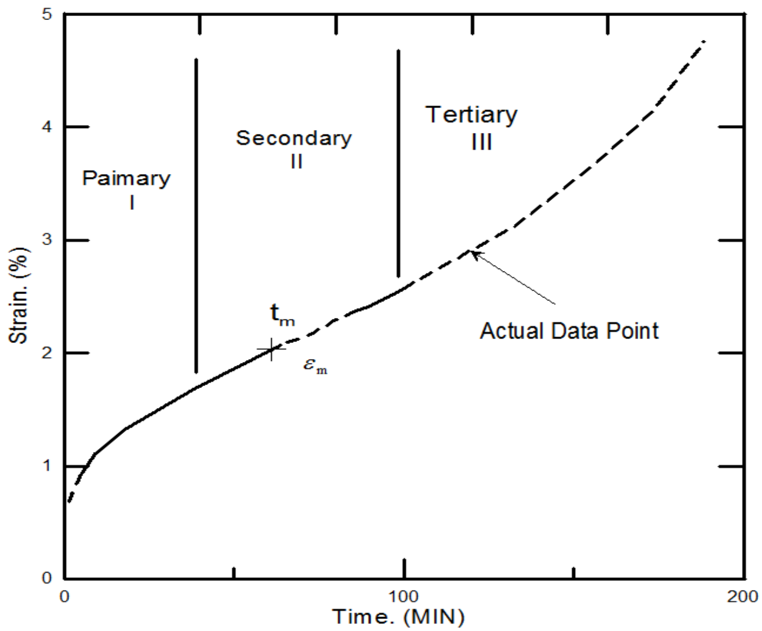
AGENDA

- Factors influencing creep of frozen soils
- Creep of Warm Frozen Soils: Field Observations
- State of The Art: Creep Models for Frozen Soils
- Our attempts on constitutive modeling
- Concluding Remarks

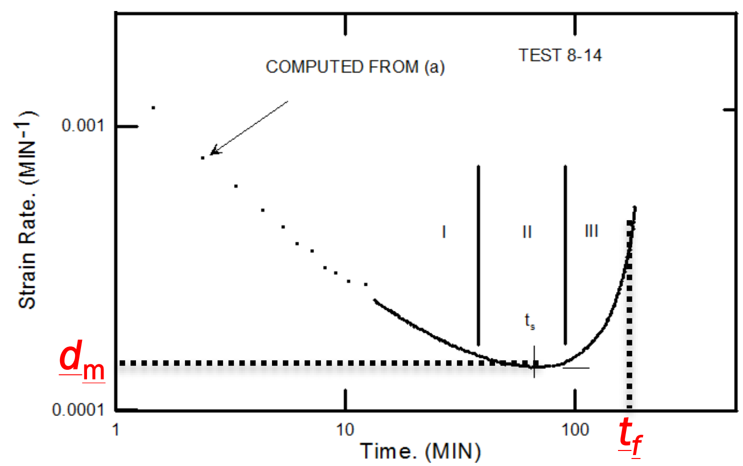
AGENDA

- Factors influencing creep of frozen soils
- Creep of Warm Frozen Soils
- State of The Art: Creep Models for Frozen Soils
- Our attempts on constitutive modeling
- Concluding Remarks

CREEP CURVES FOR FROZEN SOILS



Strain vs. time

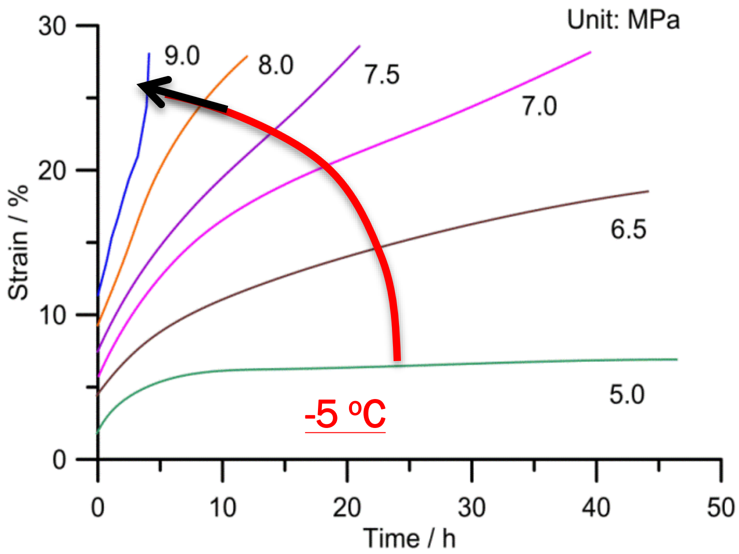


Strain rate vs. time

TYPICAL creep curves for frozen soil (Ting 1983)

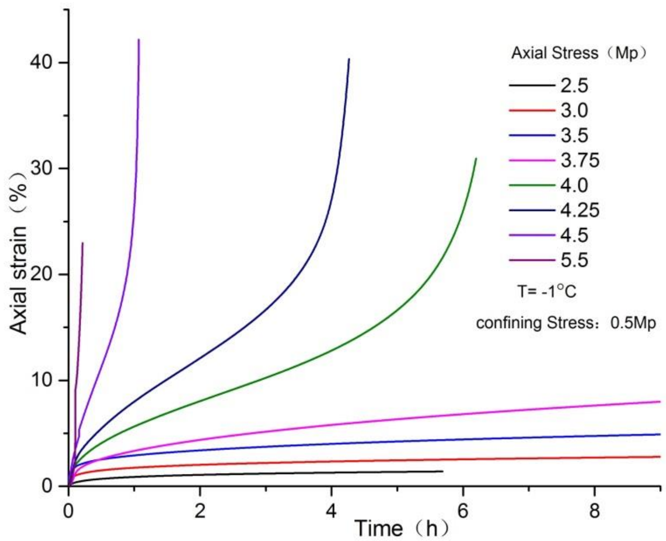
Frozen soil *displays features very similar to those of unfrozen soil*

STRESS DEPENDENCE



Uniaxial compression Frozen Lanzhou sand

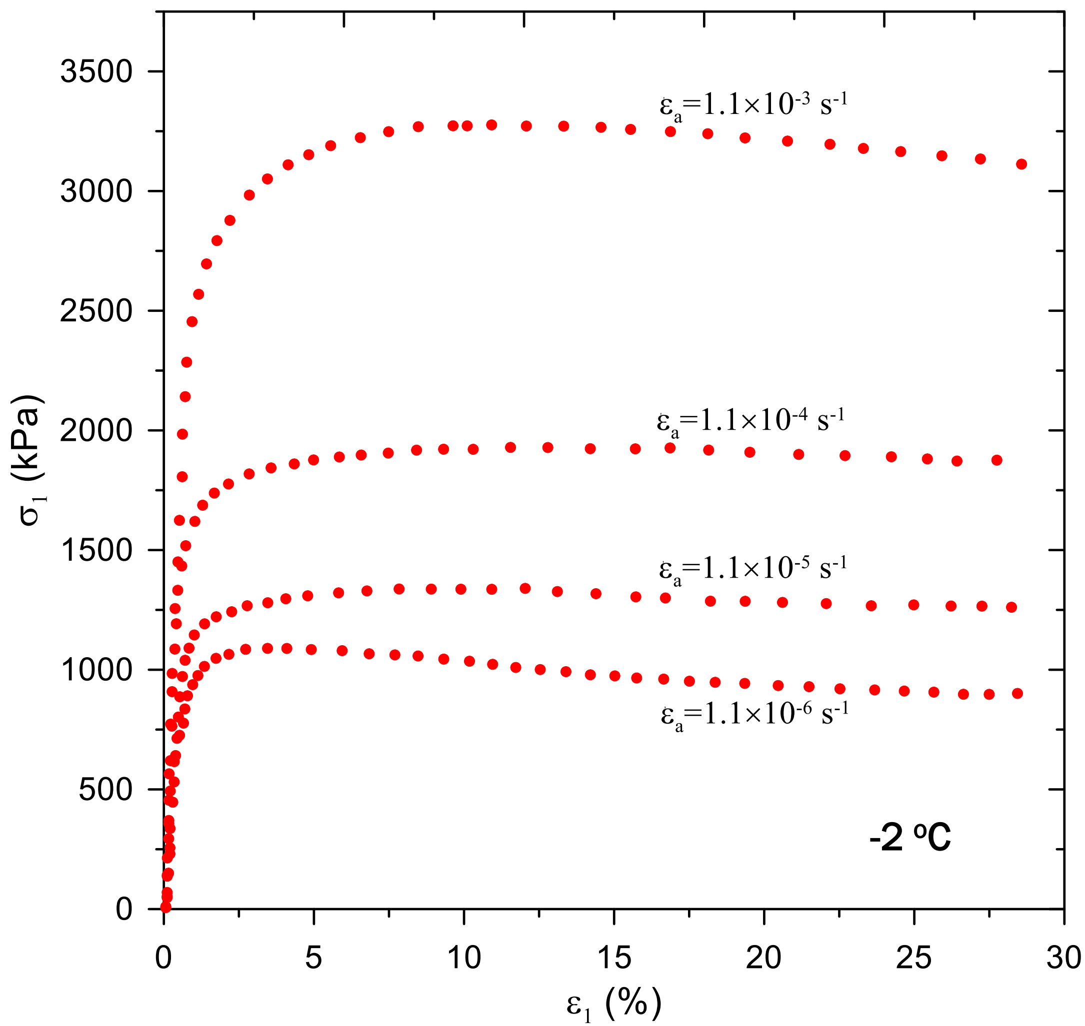
From *Wu and Ma. 1994*



Triaxial compression
Frozen ISO Standard sand

Both very much stress dependent.

STRAIN RATE DEPENDENCE

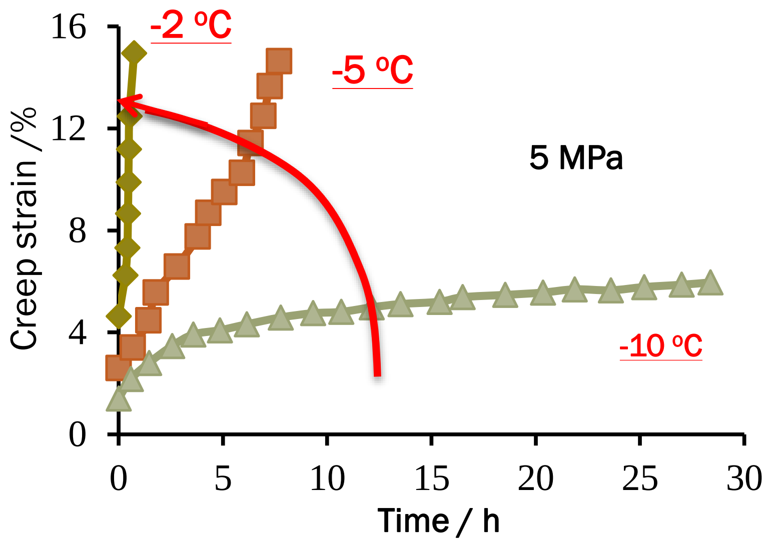


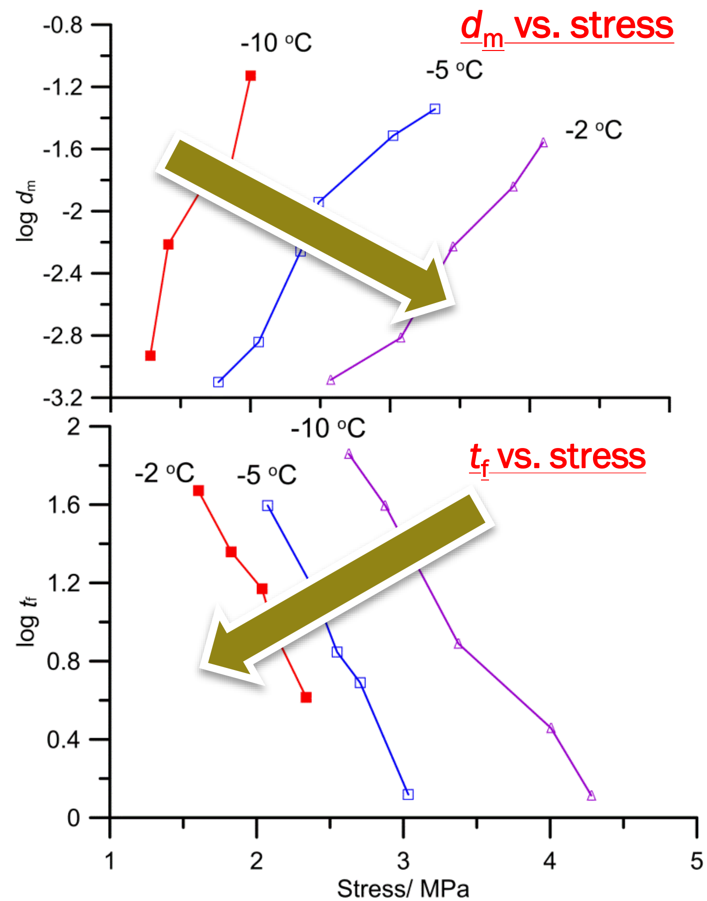
Under the same temperature, stress increases with strain rate.

From Zhu and Carbee (1984)

TEMPERATURE DEPENDENCE

From *Wu and Ma. 1994*

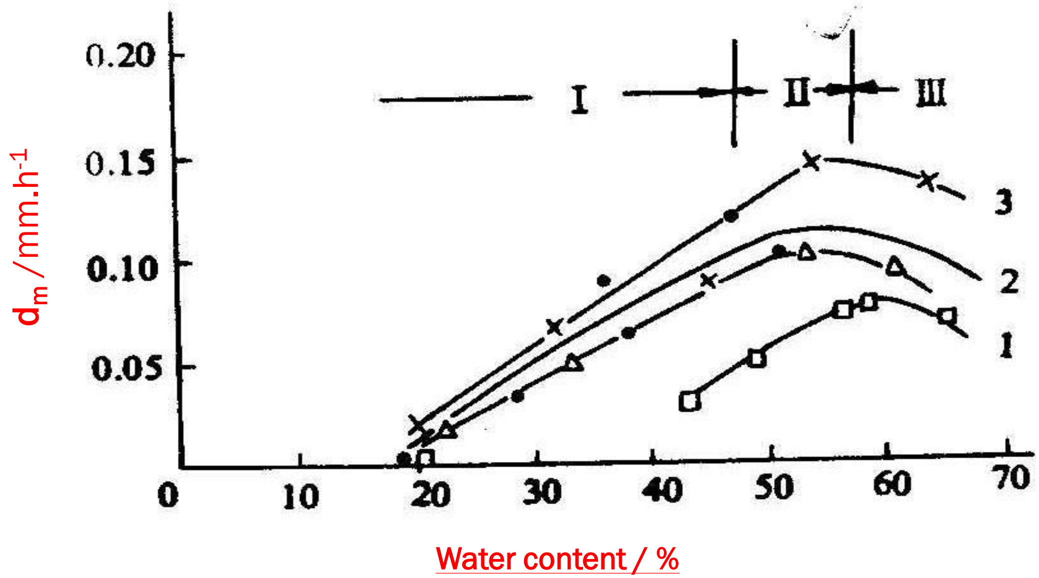




Temperature plays a role similar to load

Minimal strain rate and time to failure all dependent on temperature

DEPENDENT ON TOTAL WATER CONTENT*

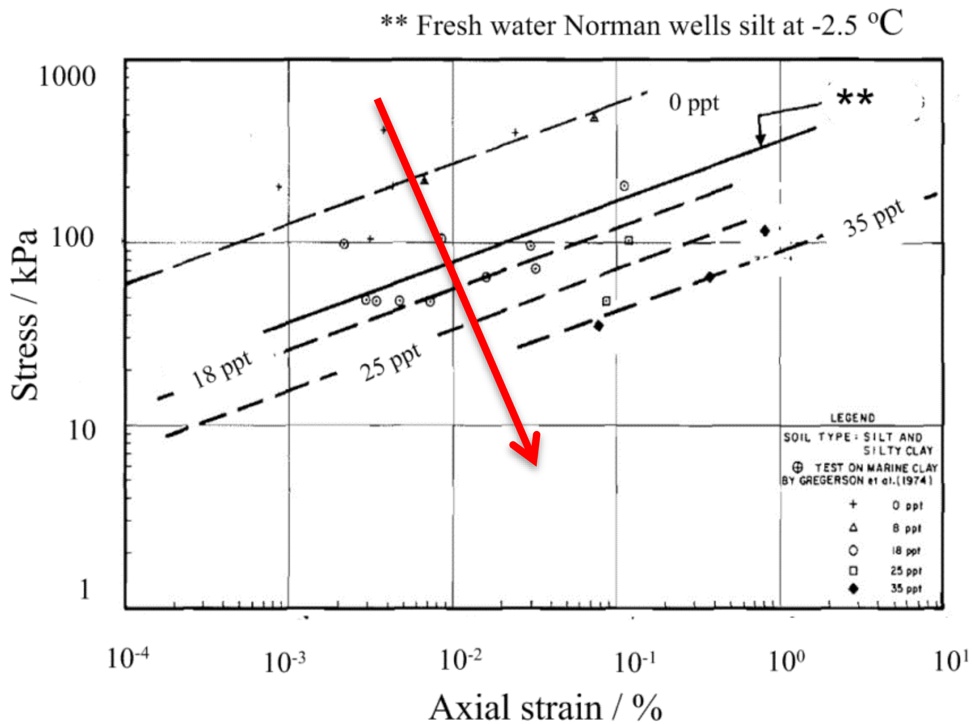


1. -0.5 °C, 1.4 MPa; 2. -1.0 °C, 2.2 MPa; 3. -2.0 °C, 3.4 MPa From *Wu and Ma. 1994*

Generally, different water content range, different minimal strain rate development for frozen silty clay (samples from Lanzhou, China)

* Total water content: ice is counted as water together with unfrozen water

SOLUTE DEPENDENCE



Solute (Nixon and Lem 1984):

The higher solute content, the larger axial strain under a certain stress level.

AGENDA

- Factors influencing creep of frozen soils
- Creep of Warm Frozen Soils： Observation data
- State of The Art: Creep Models for Frozen Soils
- Our attempts on constitutive modeling
- Concluding Remarks

DEFINE WARM FROZEN SOIL

State-of-the-art:

— Shields et al. (1985): -2.5 and -3.0 °C.
— Foster et al. (1991): -1 °C
— Tsytovich(1975): different temperature boundaries for different soils

Temperature boundary:

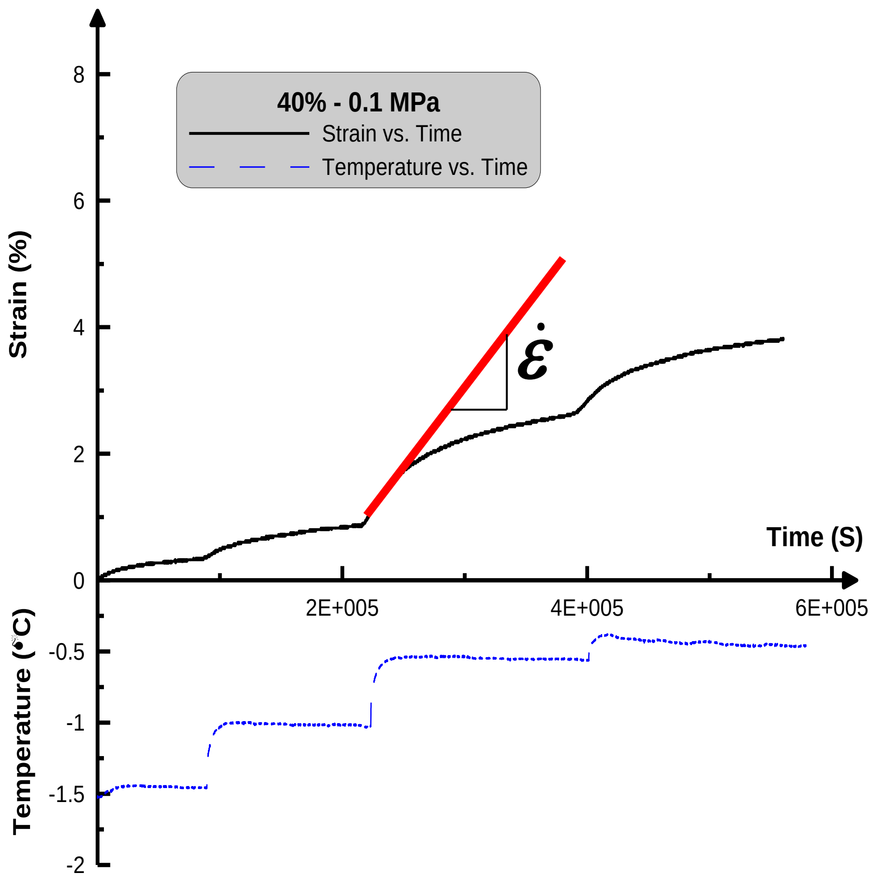
— Detecting unfrozen water content is rather difficult (NMR)
— Tend to use mechanical properties

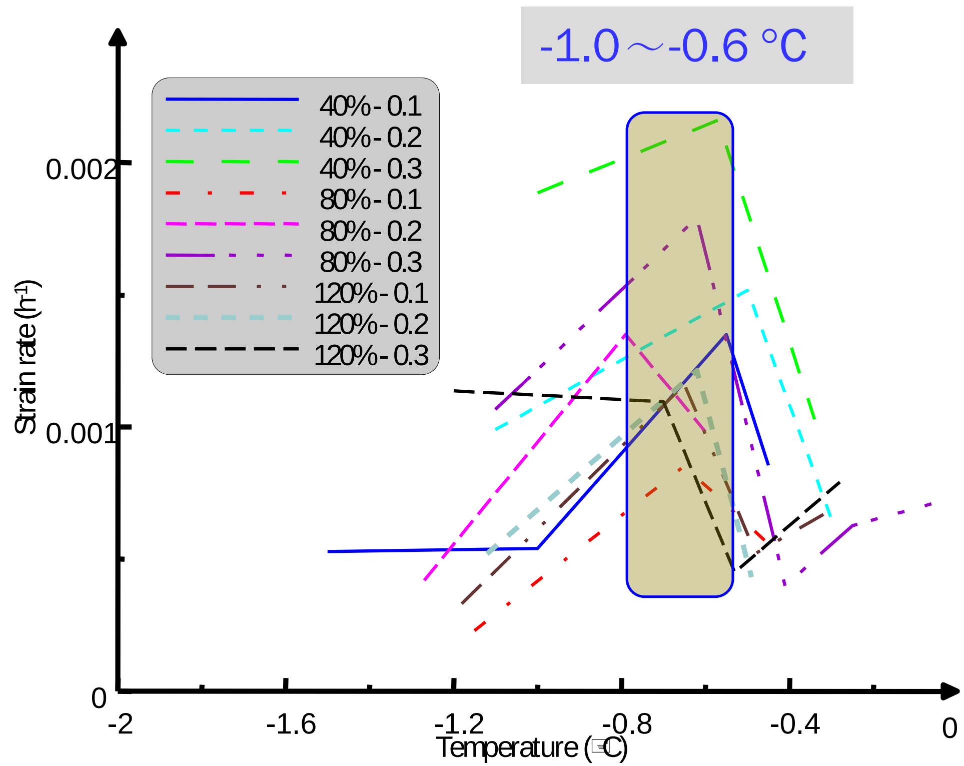
Our Work :

Constant load and stepped temperature

- Material: silty clay (CL) Qinghai-Tibet Plateau
- Different water content: 40%, 80%, 120%
- Constant Loads: 0.1, 0.2 and 0.3 MPa, respectively
- Temperature steps: -1.5, -1.0, -0.6, -0.5 and -0.3 °C

Strain rate vs. Temperature

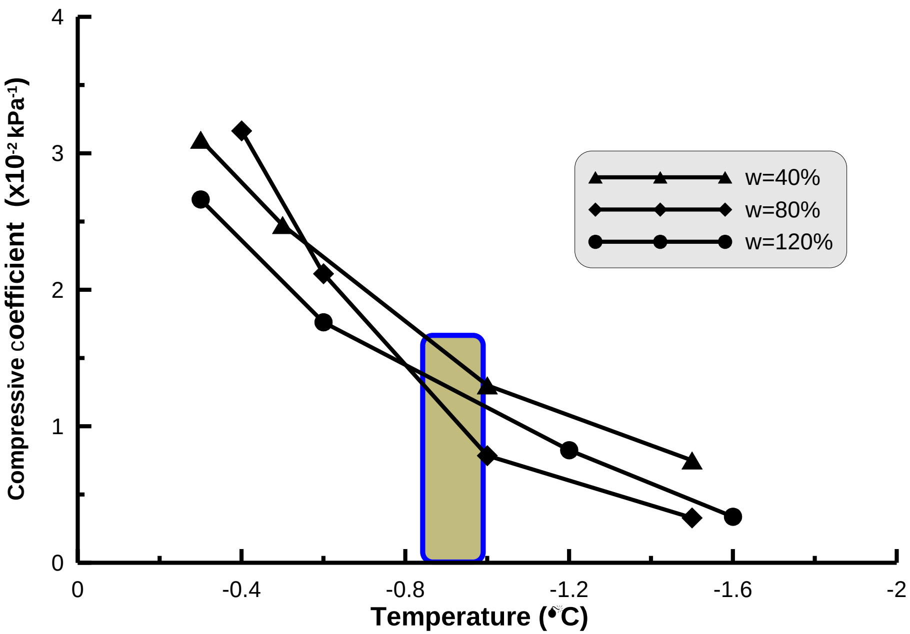




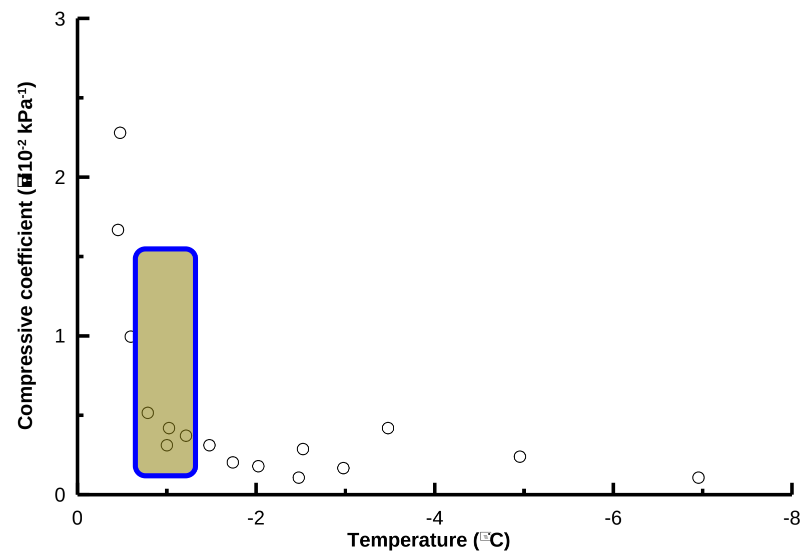
like we applied higher loads in oedometer test

Compressive Coefficient vs. Temperature

Turning point around: -1 °C



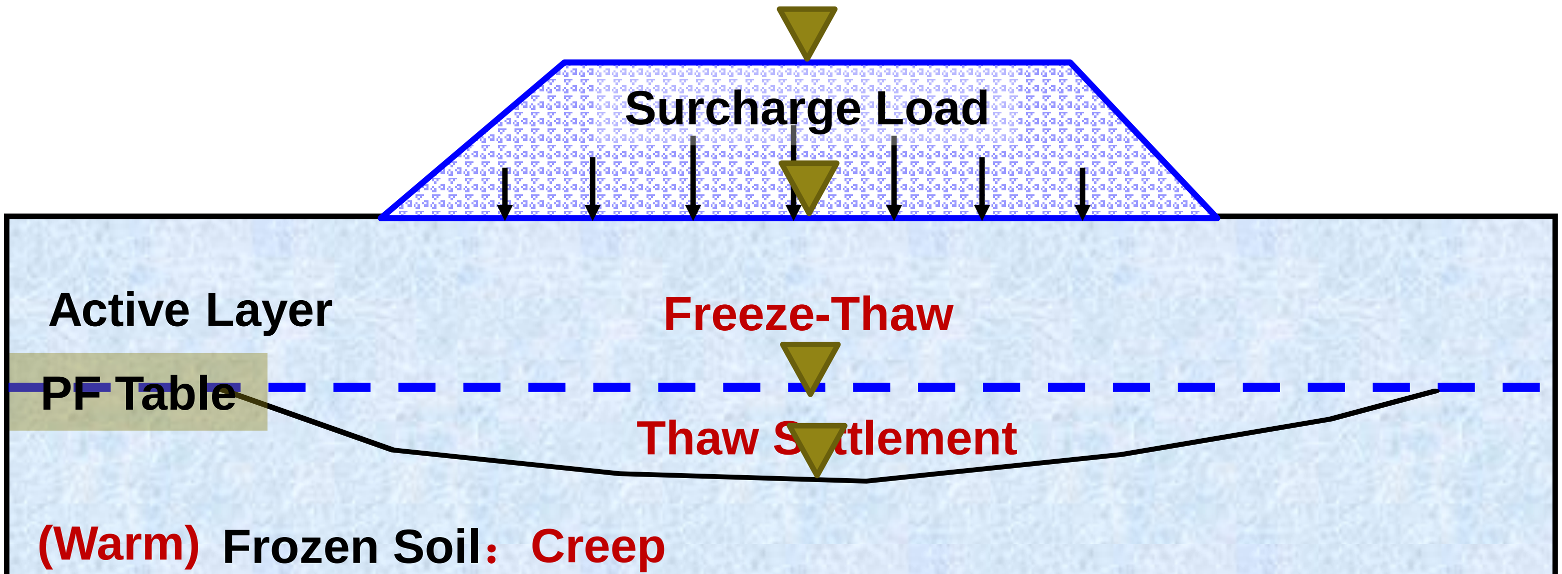
Own test



Testing results by Zhu et al. (1982)

-1 °C is defined as the temperature boundary for warm frozen soil for the silty clay we frequently encounter on the plateau.

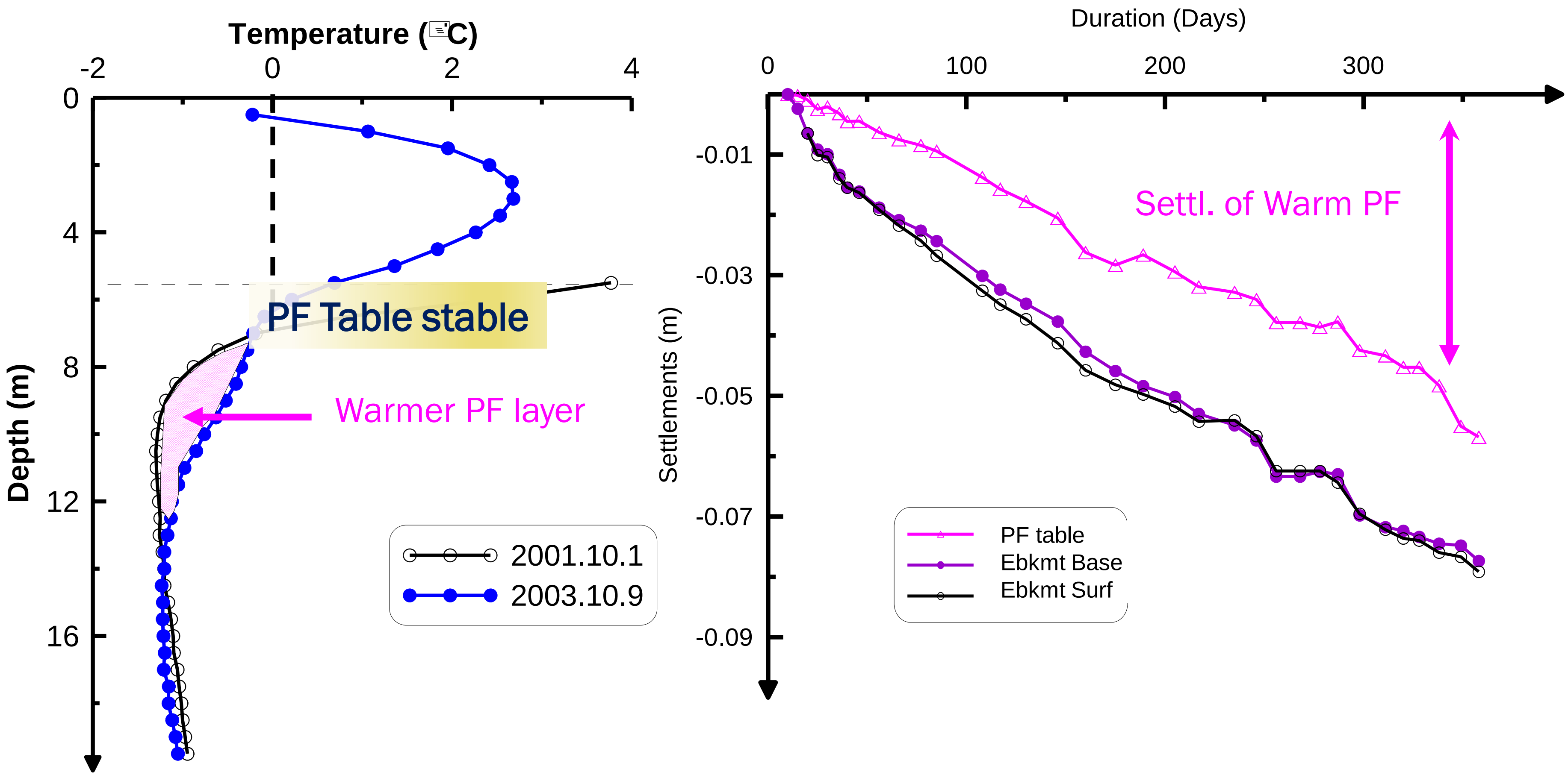
Settlement of road embankment



Different layers, different physical and mechanical processes

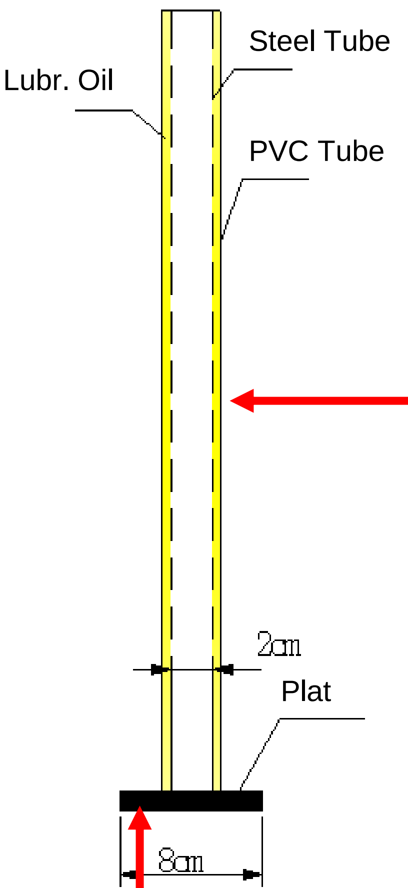
- Thaw Settlement
- Freeze-Thaw
- *Creep*

FIELD OBSERVATION QINGHAI-TIBET RAILWAY （DR. JIANMING ZHANG）



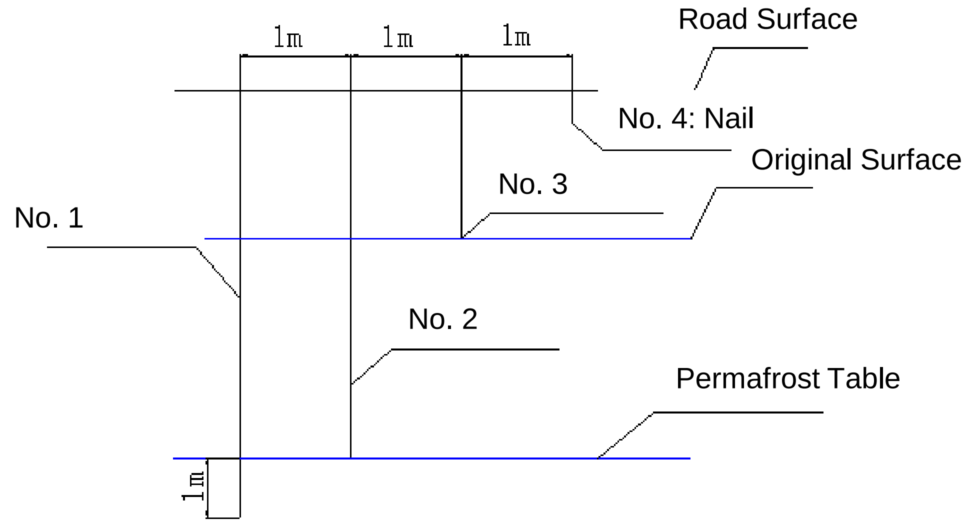
Location: DK 1136+540 (QT Railway)

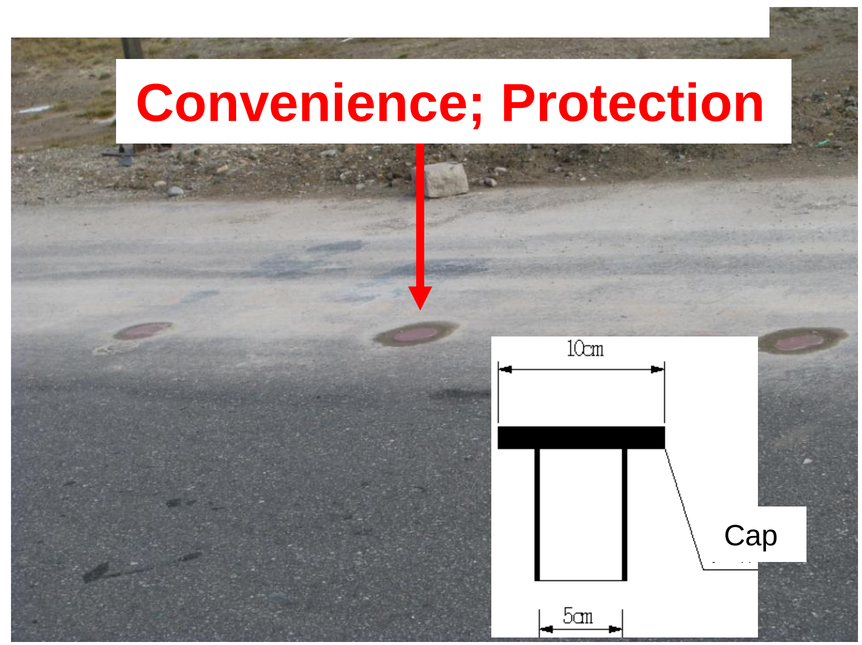
FIELD OBSERVATION QINGHAI-TIBET HIGHWAY



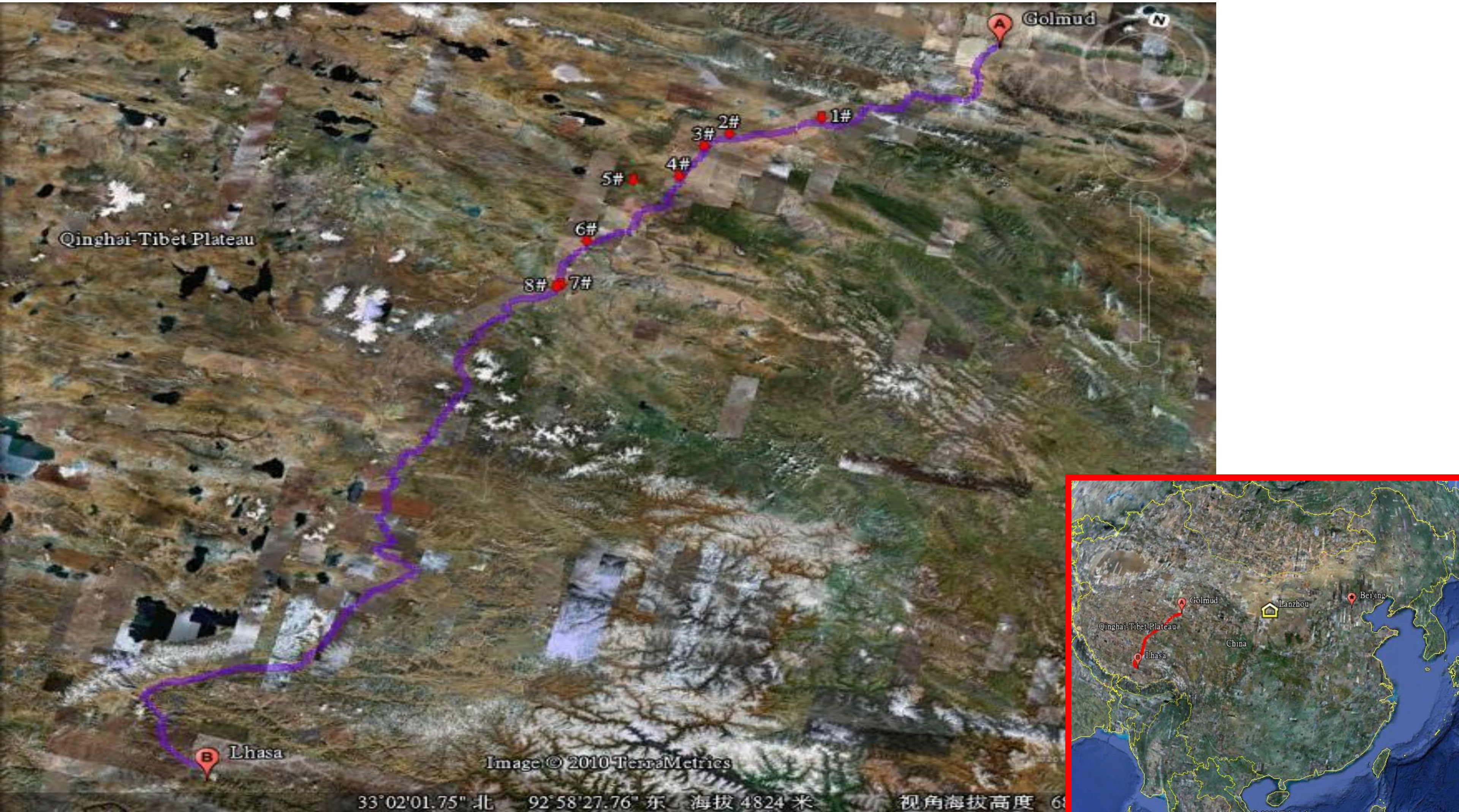
Mitigate Friction

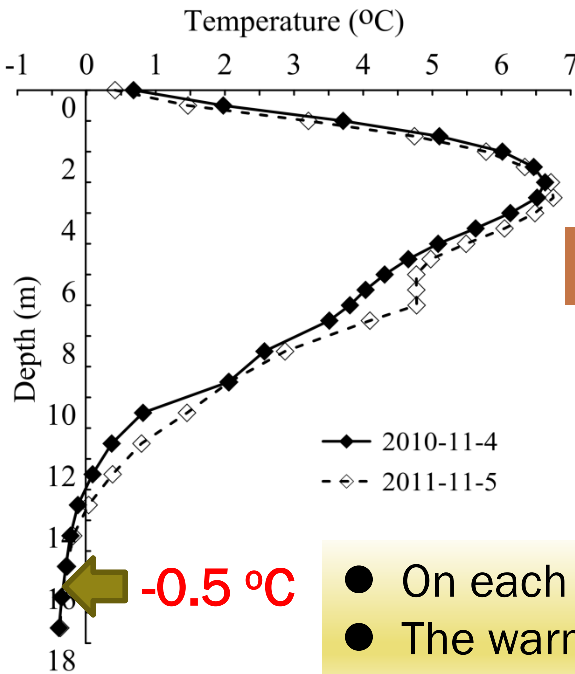
Keep Vertical





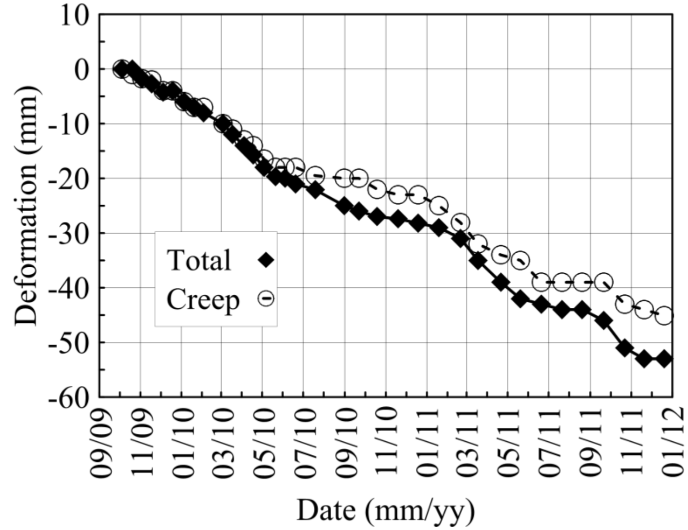
Along the highway: 300 km, 10 observation sites, 4500 m a.s.l.



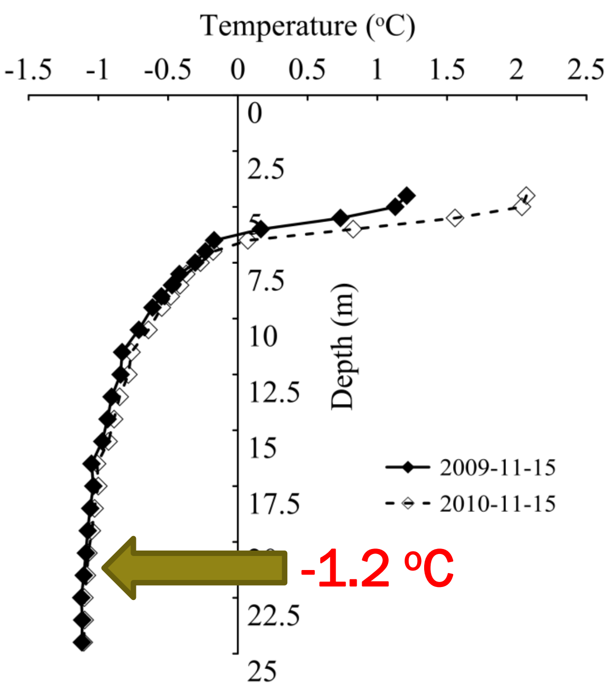


-0.5 °C

Beiluhe

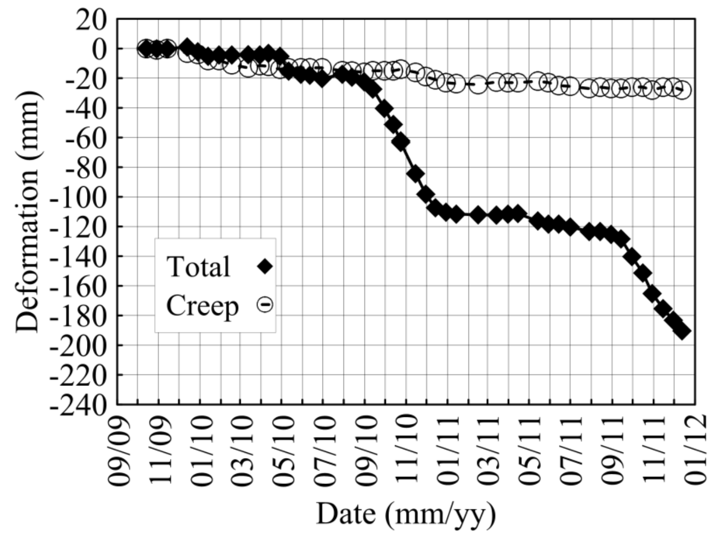


- On each site: we obtained total settlement and creep
- The warmer it is, the more creep occurred.



-1.2 °C

Test section



AGENDA

- Factors influencing creep of frozen soils
- Creep of Warm Frozen Soils
- State of The Art: Creep Models for Frozen Soils
- Our attempts on constitutive modeling
- Concluding Remarks

CREEP MODELS FOR FROZEN SOILS

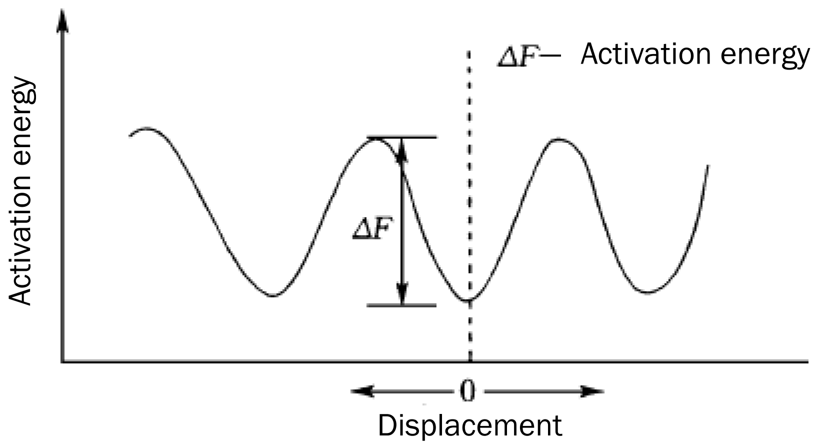
Model classification based on its scale of representation:

- **Microscopic view**
  - *The theory of rate process*
  - *Damage creep model*
- **Phenomenological view**
  - *Empirical model*
  - *Elementary element based creep model*

CREEP MODELS FROM MICROSCOPIC VIEW_1

- *The theory of rate process: Considers deformation as a thermal activation process*

Statistical thermodynamics → Activation energy → Equation for rate of deformation



$$\dot{\varepsilon}=\bar{C}\frac{kT}{h}\exp\left(-\frac{E}{RT}\right)\exp\frac{\Delta S}{k}\left(\frac{\sigma}{\sigma_0}\right)^{n+m}$$

from Fish 1983

Related studies: Andersland (1967), Assur (1980)

A reasonable physical description

Difficulty: Parameters obtained qualitatively by current testing technology

CREEP MODELS FROM MICROSCOPIC VIEW_2

***The damage creep model**: damage or recovery of soil structure*

Damage mechanics for continuum medium

↓

Damage variable

↓

Creep damage equation

CT → Damage mechanism

↓

grain orientation; damaged area + Ice content

↓

Dissipation potential

$$\dot{\varepsilon}_{kl}^{p} = \left\{\lambda\left(\frac{G}{N}\right)^{n}\left[\frac{3}{2}\left(1-3h\bar{\sigma}^{-4}J'_{3}\right)\bar{\sigma}^{-1}S_{kl}+\left(\alpha+\frac{h}{9}\bar{\sigma}^{-1}\right)\delta_{kl}\right.\right.$$

From Miao et al. 1995

Following thermodynamics; Unique internal variable in frozen soil: ice content

Difficulty: Calibration of Parameters for thermodynamics and damage mechanics

CREEP MODELS FROM PHENOMENOLOGICAL VIEW_1

- *Empirical methods：a mathematical description of creep curve*

Basic form:

1. Stress-strain-time
2. Stress-strain rate-time

Classified by creep stages:

I. Primary creep model (Vyalov, 1966)

II. Secondary creep model (Ladanyi,1972)

III. Tertiary creep model (?)

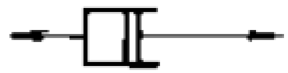
Simple structure; conveniently applied in simple engineering analysis (first estimation)
1) Poor versatility; 2) do not reflect internal mechanism

CREEP MODELS FROM PHENOMENOLOGICAL VIEW_3

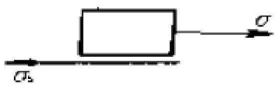
- ***Elementary creep model**: combination of mechanical elements*

Basic elements:

Elastic

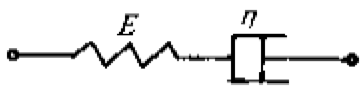


Viscous

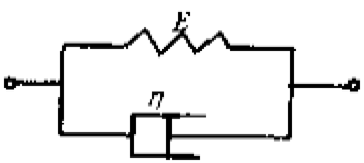


Plastic

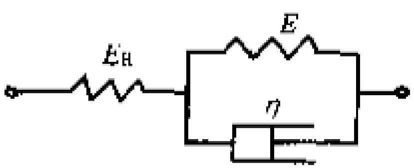
Typical Combination:



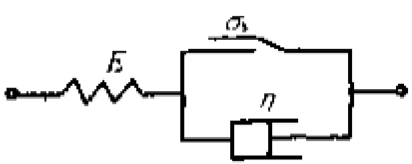
Maxwell body



Kelvin body



Standard Viscoelastic body



Generalized Bingham body

Reasonable mechanical basis; simple structure; convenient in engineering design

Reasonable parameters decide the precisions.

SUMMARIZATION

- Some are too complicated in form, too many parameters, even impossible to be obtained from conventional tests
- Some are lacking mechanism, just mathematical description
- Some are difficult to accommodate different thermal or load conditions

We need something new.

AGENDA

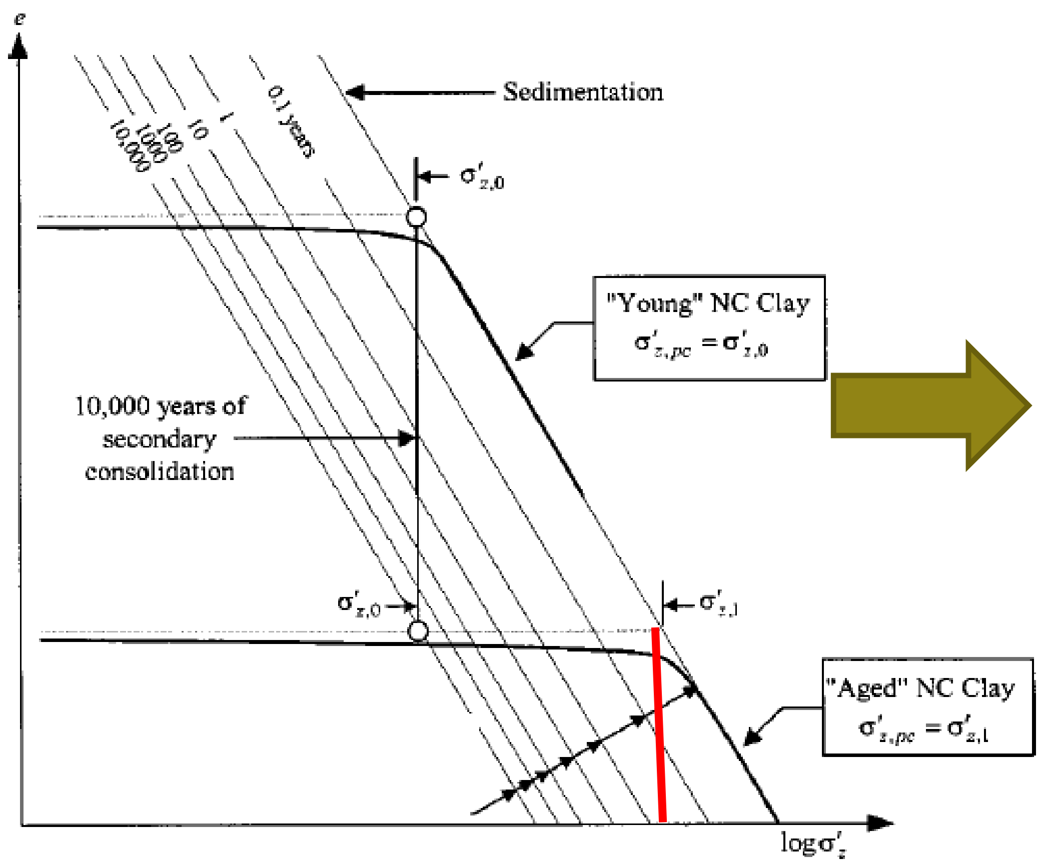
- Factors influencing creep of frozen soils
- Creep of Warm Frozen Soils
- State of The Art: Creep Models for Frozen Soils
- Our attempts on constitutive modeling
- Concluding Remarks

BORROW THEORIES FROM UNFROZEN SOILS

- Ladanyi (1999): creep of frozen soils is not very much different from that of unfrozen soils

- Warm frozen soil is between frozen and unfrozen soils, most likely closer to unfrozen soils

—Creep of unfrozen soils based on p_c



（After Bjerrum, 1973）

Yin, et al. (1989):

$$\dot{\varepsilon}_z = \frac{k/V}{\sigma_z'}\sigma_z' + \frac{\psi/V}{t_0}\exp\left[-\left(\varepsilon_z - \varepsilon_{z0}^{ep}\right)\frac{V}{\psi}\right]\left(\frac{\sigma_z'}{\sigma_{p_c}'}\right)^{\lambda/\psi}$$

Vermeer and Neher(1999):

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^c = A\frac{\dot{\sigma}'}{\sigma'} + \frac{C}{\tau}\left(\frac{\sigma'}{\sigma_p}\right)^{\frac{B}{C}}$$

$$\sigma_p = \sigma_{p_c}\exp\left(\frac{\varepsilon^c}{B}\right)$$

Questions

Is there an index in frozen soils similar to p_c?

If so

What is its relationship with creep?

How is it possible to apply creep model of unfrozen soils to frozen soils?

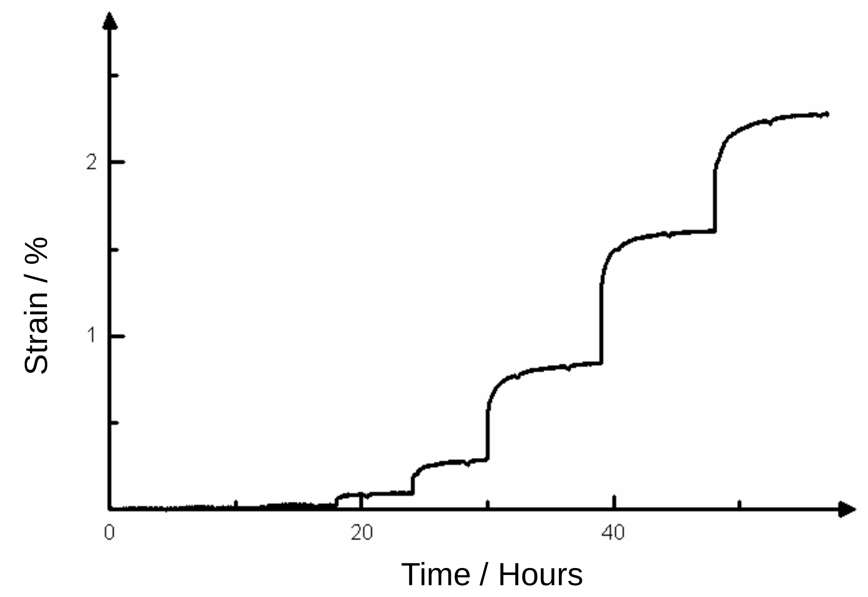
Testing

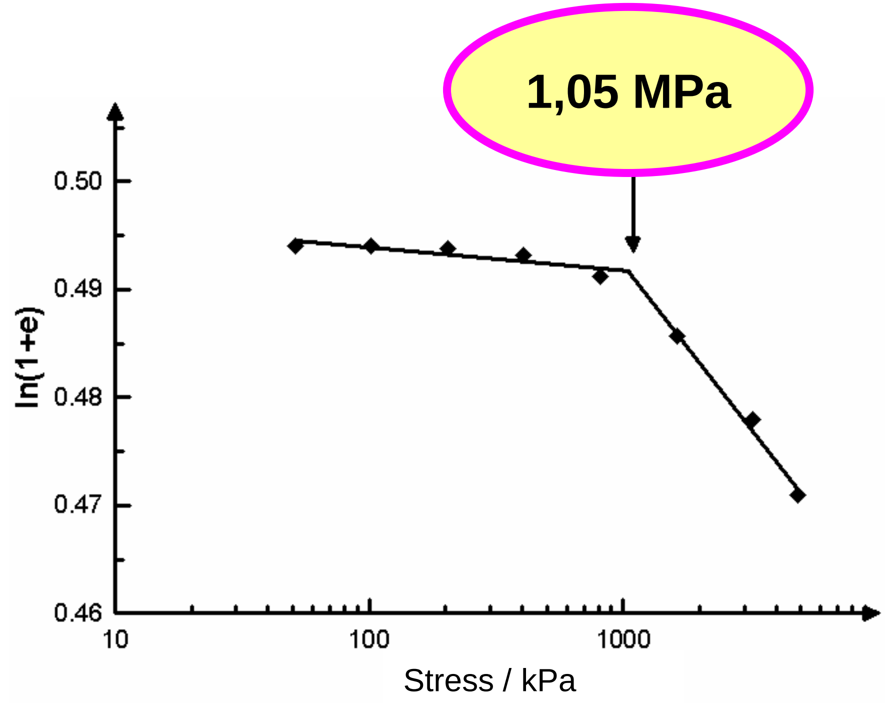
Phases	Purpose	Conditions	Samples
1	Prove the existence of an index similar to Pc	Same T Different γ_d	4
		Same γ_d Different T	4
2	Influence of creep on this index	γ_d, T, preload	30
3	Comprehensive analysis Relationship: Creep vs. Pc	Orthogonal design: γ_d, T, preload, creep time	10

48 creep tests so far

K_0 Test

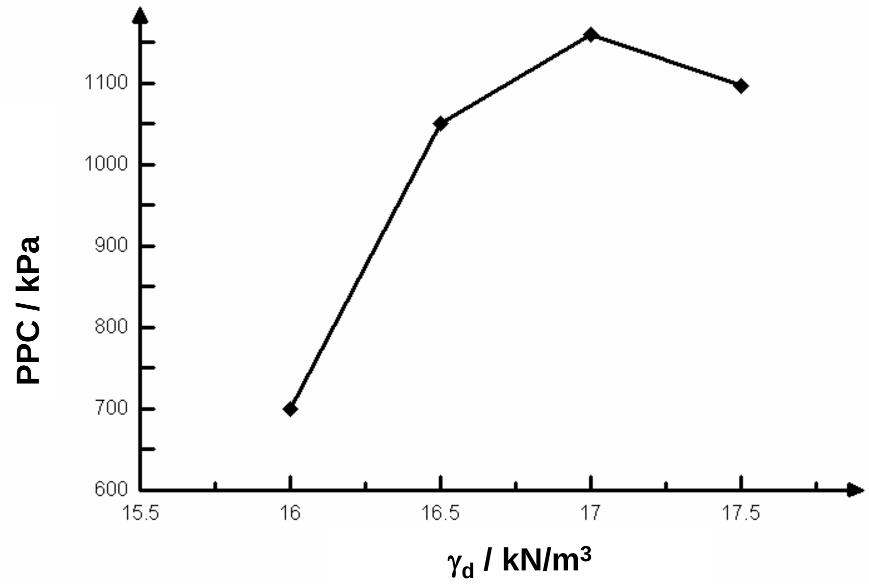
Pseudo preconsolidation pressure
PPC for frozen soils

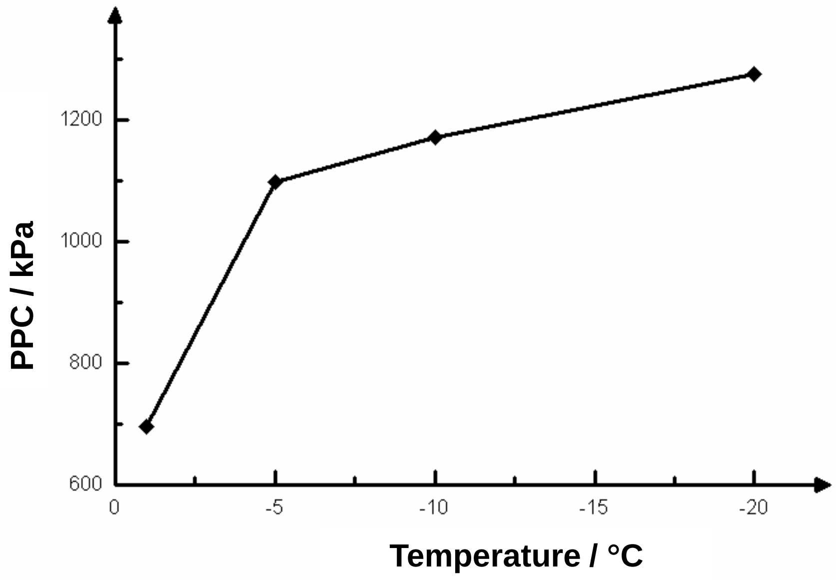




In "ln(1+e)～lnP" coordinates, there is clearly such an index

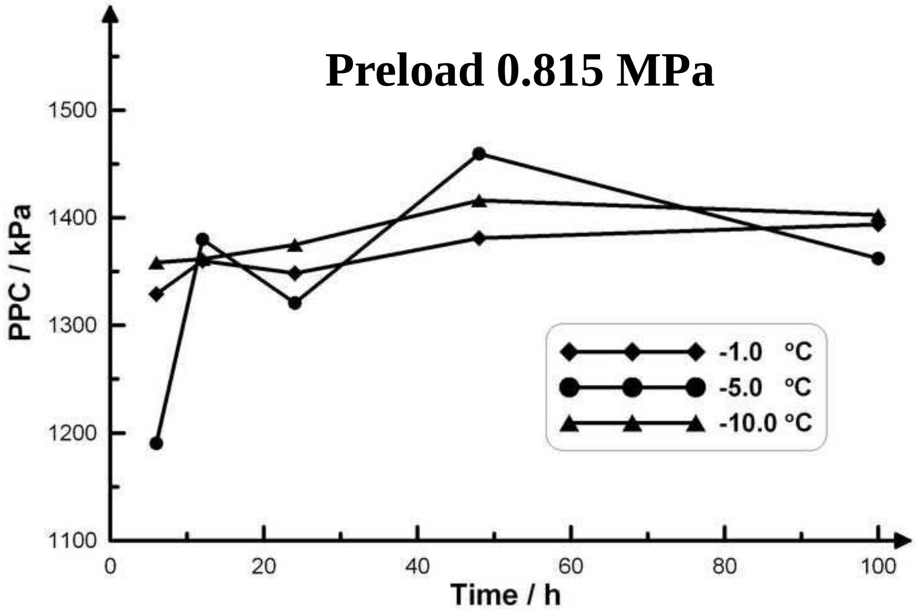
PPC: Mechanical behavior of frozen soils

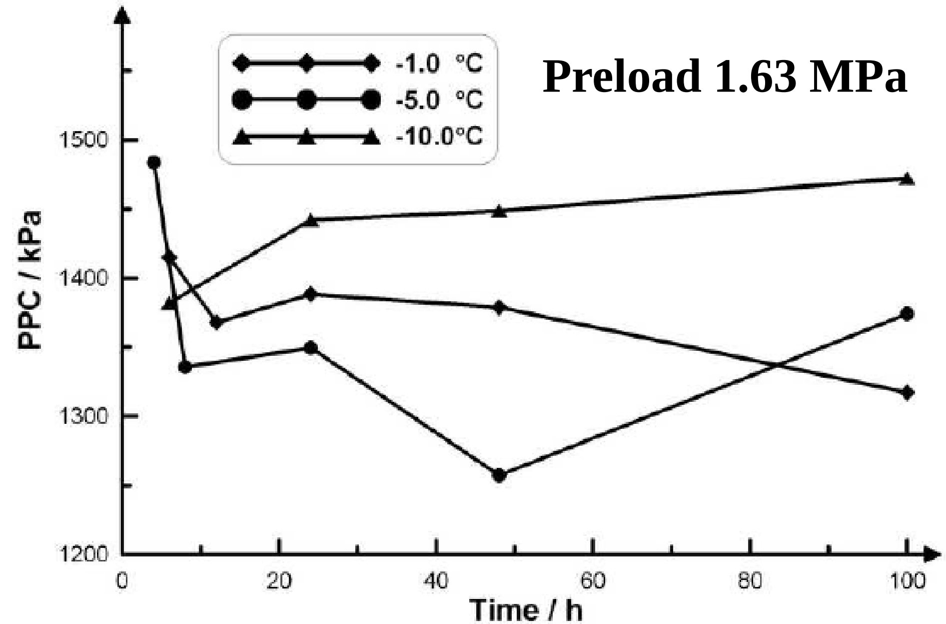




- **PPC increases with the increase in γ_d, then does not change obviously**
- **PPC increases linearly with the decrease in temperature**

Well reflects the bonding in frozen soils



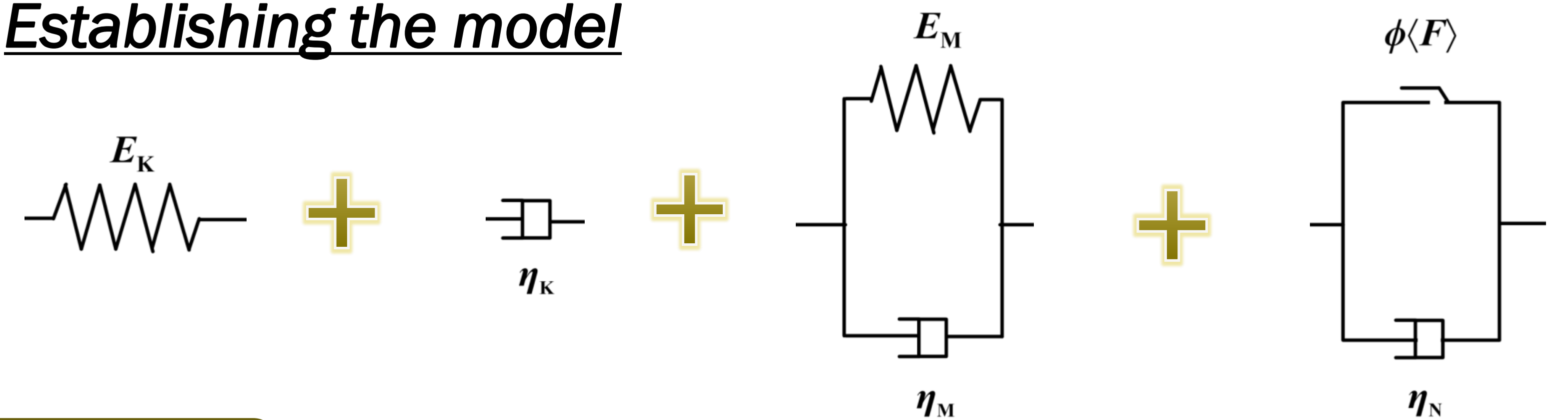


- When preload is less than original PPC, PPC increases with time
- When preload is larger than original PPC, PPC decreases with time

We are not ready to get a relationship between PPC and temperature, creep time; but we successfully proved the existence of such an index.

AN ELEMENT MODEL FOR CREEP OF FROZEN SOIL *(Dr. Songhe Wang)*

➢ *Establishing the model*



Instantaneous elastic	Viscous(strain rate approaches zero)	viscous (strain rate increases with stress)	Viscoplastic with a yield surface

The creep model is

$$e_{ij}=\frac{S_{ij}}{2G_M}+\frac{S_{ij}}{2H_M}t+\frac{S_{ij}}{2G_K}\left[1-\exp\left(-\frac{G_K}{H_K}t\right)\right]\quad(\phi(F)\le 0)$$

$$e_{ij}=\frac{S_{ij}}{2G_M}+\frac{S_{ij}}{2H_M}t+\frac{S_{ij}}{2G_K}\left[1-\exp\left(-\frac{G_K}{H_K}t\right)\right]+\frac{1}{2H_N}\langle\phi(F)\rangle\frac{\partial Q}{\partial\{\sigma\}}t\quad(\phi(F)>0)$$

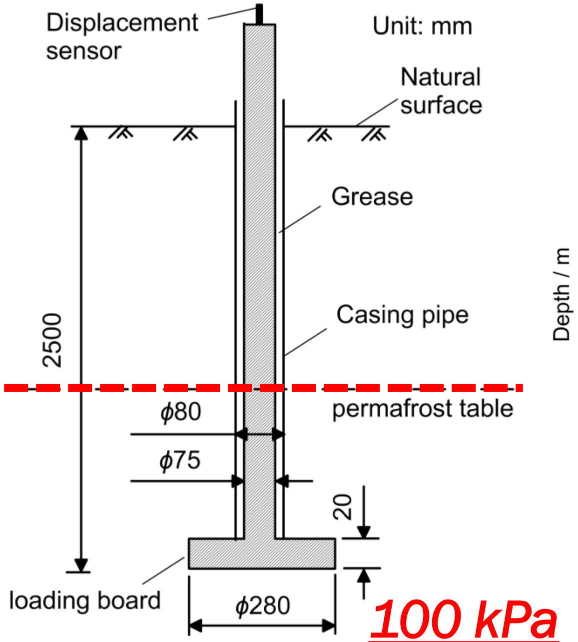
Yield criterion(Ma, et al. 1994)

$$F=\sqrt{3J_2}-c-\sigma_m\tan\varphi+\frac{\tan\varphi}{2p_m}\sigma_m^2$$

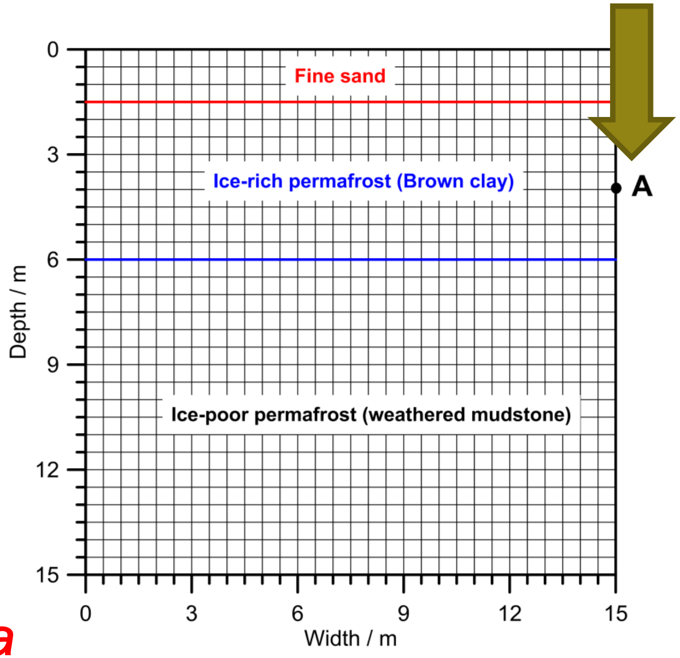
➢ Long-term load test

Field load test

Loading pile



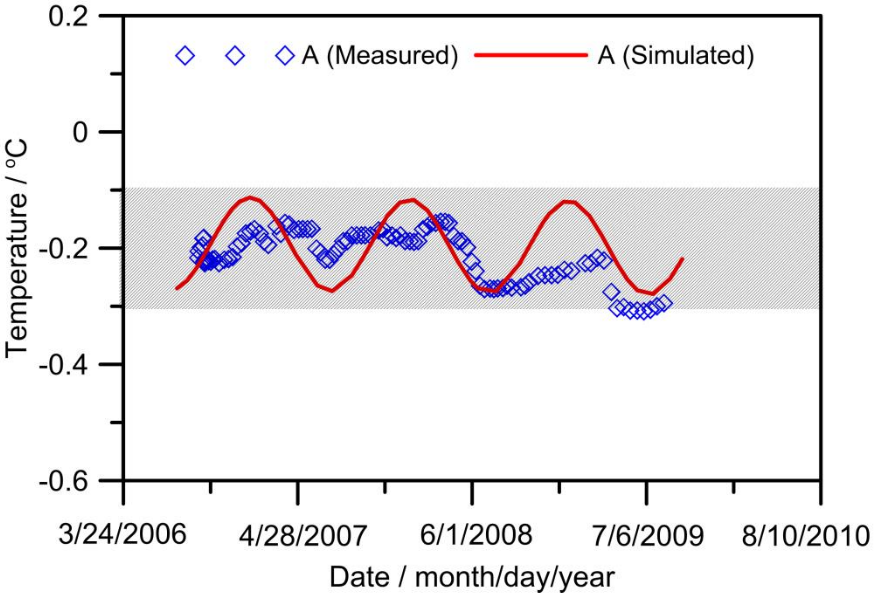
Numerical model



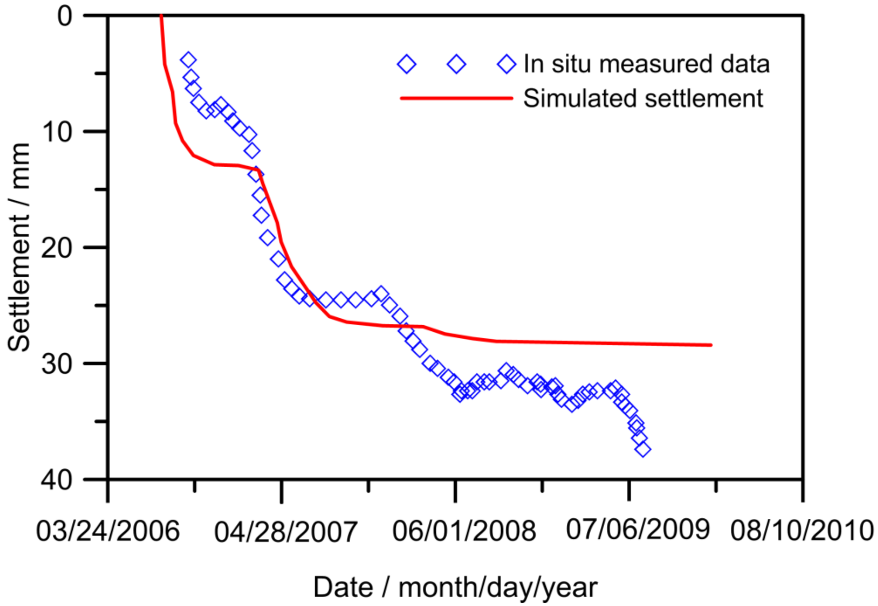
Only *creep of underlying permafrost* was considered

➢ *Numerical simulation*

After implementation of this model,



Thermal state analysis



Deformation analysis

A simple model for creep of frozen soil might provide a way in engineering analysis.

A VISCO-HYPOPLASTIC CONSTITUTIVE MODEL FOR FROZEN SOIL

(Dr. Guofang Xu, Prof. Wei Wu, Prof. Jilin Qi)

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}}_{\mathrm{s}} + \dot{\boldsymbol{\sigma}}_{\mathrm{d}}$$

$\boldsymbol{\sigma}_{\mathrm{s}}$ is static stress, $\boldsymbol{\sigma}_{\mathrm{d}}$ is dynamic stress.

Static part

$$\dot{\boldsymbol{\sigma}}_{\mathrm{s}} = c_1[\mathrm{tr}(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})]\dot{\boldsymbol{\varepsilon}} + c_2 \frac{\mathrm{tr}[(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})\dot{\boldsymbol{\varepsilon}}]}{\mathrm{tr}(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})}(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c}) + f_{\varepsilon} \cdot f_{\mathrm{cd}} \cdot \left[c_3(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})^2 + c_4(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})_{\mathrm{d}}{}^2 \right] \frac{\|\dot{\boldsymbol{\varepsilon}}\|}{\mathrm{tr}(\boldsymbol{\sigma}_{\mathrm{s}}\text{-}\boldsymbol{c})}$$

in which $\boldsymbol{c}$ is the cohesion of frozen soil, f_{ε} is a scalar function of deformation, f_{cd} is a factor of creep damage.

$$f_{\varepsilon} = 2 - \exp(\alpha \cdot l + \beta)$$

α and β are parameters, l is the accumulation of deformation.

$$l = \int_{t_0}^{t} \|\dot{\varepsilon}(\tau)\| \, \mathrm{d}\tau$$

$$f_{\mathrm{cd}} = 1 + \gamma \cdot \int_{t_1}^{t_2} \langle \ddot{\varepsilon}(\tau) \rangle \mathrm{d}\tau$$

γ is a parameter, $\langle\,\rangle$ is Macaulay brackets.

Dynamic part

$$\dot{\boldsymbol{\sigma}}_{\mathrm{d}} = \eta_1 \sqrt{\eta_2^2 + \mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^2)}\; \ddot{\boldsymbol{\varepsilon}}$$

η_1 and η_2 are parameters, $\ddot{\boldsymbol{\varepsilon}}$ is strain acceleration.

Complete constitutive model - Rate dependent

$$\dot{\boldsymbol{\sigma}} = c_1[\mathrm{tr}(\boldsymbol{\sigma}-\boldsymbol{c})]\dot{\boldsymbol{\varepsilon}} + c_2 \frac{\mathrm{tr}[(\boldsymbol{\sigma}-\boldsymbol{c})\dot{\boldsymbol{\varepsilon}}]}{\mathrm{tr}(\boldsymbol{\sigma}-\boldsymbol{c})}(\boldsymbol{\sigma}-\boldsymbol{c}) + f_{\varepsilon} \cdot f_{\mathrm{cd}} \cdot \left[c_3(\boldsymbol{\sigma}-\boldsymbol{c})^2 + c_4(\boldsymbol{\sigma}-\boldsymbol{c})_{\mathrm{d}}^{\;2} \right] \frac{\|\dot{\boldsymbol{\varepsilon}}\|}{\mathrm{tr}(\boldsymbol{\sigma}-\boldsymbol{c})} + \eta_1 \sqrt{\eta_2^2 + \mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^2)}\; \ddot{\boldsymbol{\varepsilon}}$$

➢ CALIBRATION OF THE CONSTITUTIVE MODEL

Parameters in the static part

When the two linear and nonlinear terms in the static part of the model are abbreviated as $\mathbf{L}_1$, $\mathbf{L}_2$, $\mathbf{N}_1$ and $\mathbf{N}_2$, this part can be rewritten as:

$$\dot{\boldsymbol{\sigma}}_s = c_1 \mathbf{L}_1(\boldsymbol{\sigma}_s):\dot{\boldsymbol{\varepsilon}} + c_2 \mathbf{L}_2(\boldsymbol{\sigma}_s):\dot{\boldsymbol{\varepsilon}} + c_3 \mathbf{N}_1(\boldsymbol{\sigma}_s)\left\|\dot{\boldsymbol{\varepsilon}}\right\| + c_4 \mathbf{N}_2(\boldsymbol{\sigma}_s)\left\|\dot{\boldsymbol{\varepsilon}}\right\|$$

In a conventional triaxial test, owing to $\dot{\sigma}_2 = \dot{\sigma}_3 = 0$,the above equation can be divided into two scalar equations as follows:

$$\dot{\sigma}_1 = c_1 L_{11}\dot{\varepsilon}_1 + c_2 L_{12}\dot{\varepsilon}_3 + c_3 N_{11}\sqrt{\dot{\varepsilon}_1^2 + 2\dot{\varepsilon}_3^2} + c_4 N_{12}\sqrt{\dot{\varepsilon}_1^2 + 2\dot{\varepsilon}_3^2}$$

$$\dot{\sigma}_3 = c_1 L_{21}\dot{\varepsilon}_1 + c_2 L_{22}\dot{\varepsilon}_3 + c_3 N_{21}\sqrt{\dot{\varepsilon}_1^2 + 2\dot{\varepsilon}_3^2} + c_4 N_{22}\sqrt{\dot{\varepsilon}_1^2 + 2\dot{\varepsilon}_3^2}$$

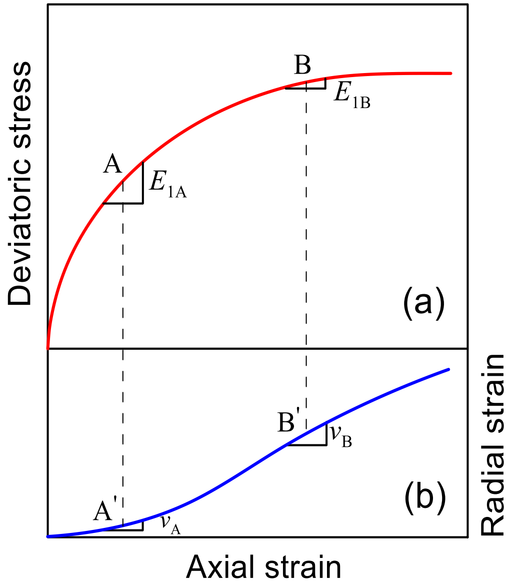
Owing to the radial stiffness $E_{A3} = E_{B3} = 0$, we have

$$E_{A1} = c_1 L_{11} + c_2 L_{12} v_A + c_3 N_{11}\sqrt{1+2v_A^2} + c_4 N_{12}\sqrt{1+2v_A^2}$$

$$0 = c_1 L_{21} + c_2 L_{22} v_A + c_3 N_{21}\sqrt{1+2v_A^2} + c_4 N_{22}\sqrt{1+2v_A^2}$$

$$E_{B1} = c_1 L_{11} + c_2 L_{12} v_B + c_3 N_{11}\sqrt{1+2v_B^2} + c_4 N_{12}\sqrt{1+2v_B^2}$$

$$0 = c_1 L_{21} + c_2 L_{22} v_B + c_3 N_{21}\sqrt{1+2v_B^2} + c_4 N_{22}\sqrt{1+2v_B^2}$$



The parameters c_i (i = 1, …, 4) can be obtained by solving the above equation system with respect to the variables c_i.

Parameters α and β

Parameters α and β can be obtained from the following expressions:

$$\alpha = 1 - (T/T_{\mathrm{ref}})^{\eta_1}$$

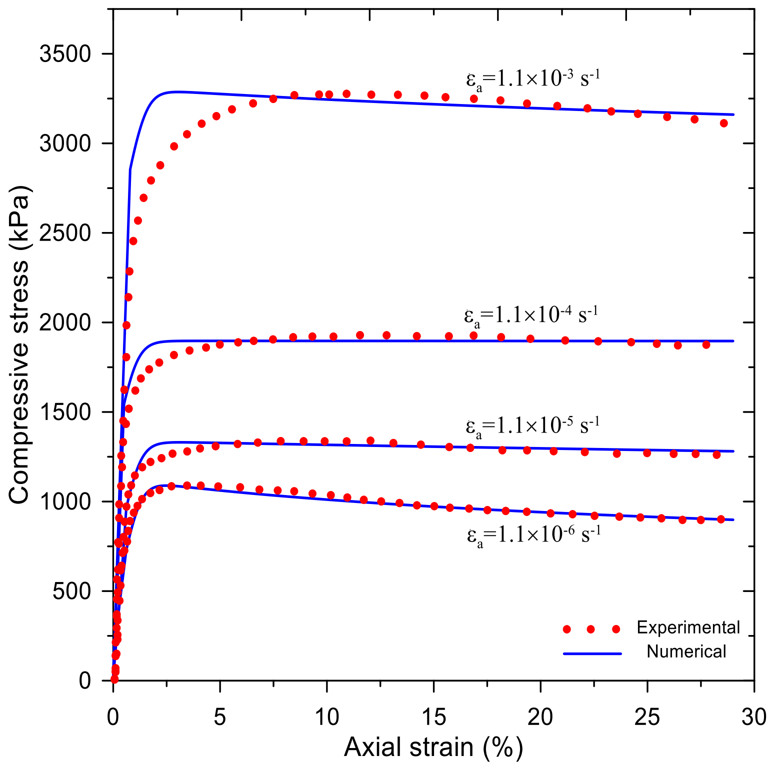
$$\beta = -(T/T_{\mathrm{ref}})^{\eta_2}$$

in which T_{ref} is a reference temperature and can be regarded as -1°C (Zhu and Carbee, 1984).

Parameters η_1 and η_2 in the dynamical part can only be obtained by fitting the experimental data, as done by Hanes and Inman (1985).

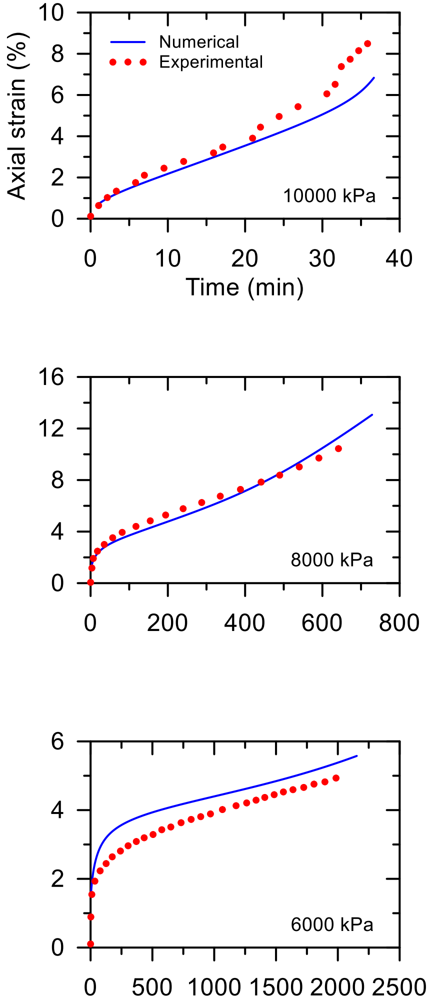
➢ VERIFICATION OF THE CONSTITUTIVE MODEL

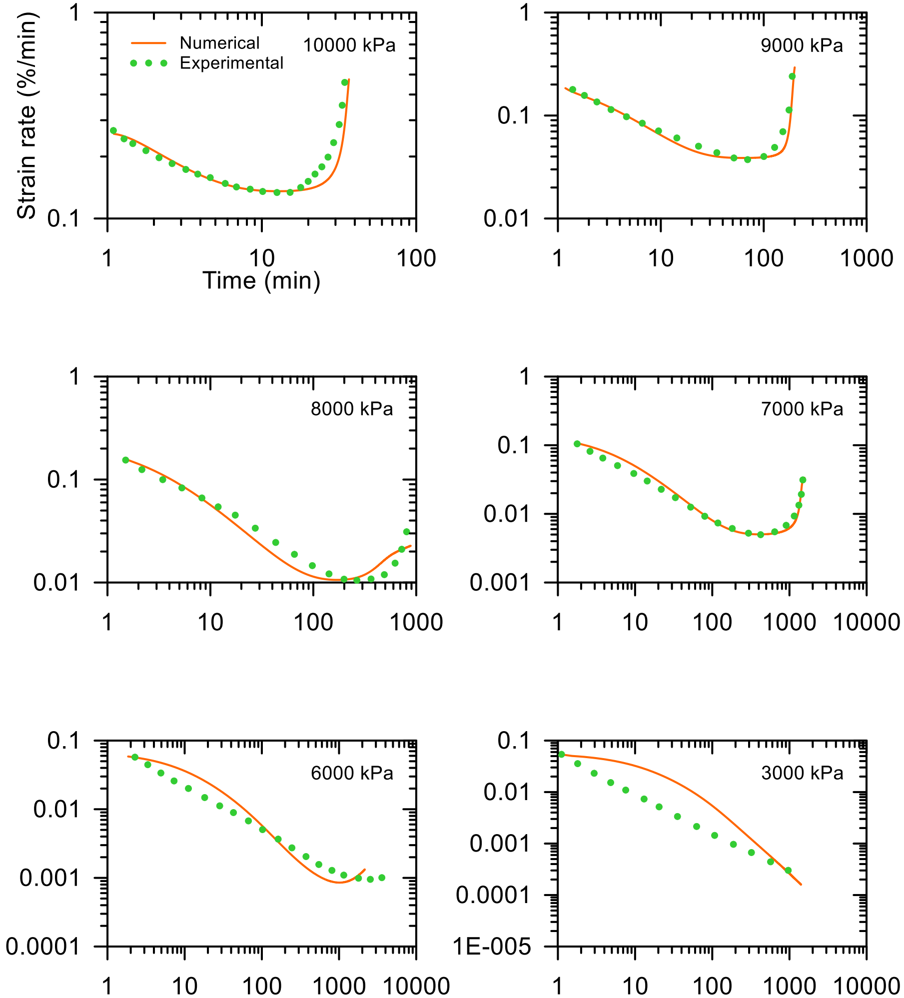
Uniaxial compression tests at different loading rates



Stress-strain relationship at different strain rates (Data from Zhu and Carbee, 1984)

Uniaxial creep tests at different stress levels





Creep strain vs. time
(Test data from Orth (1986))

Creep strain rate vs. time
(Test data from Orth (1986))

CONCLUDING REMARKS

- General features in stress-strain-time curves for frozen are similar to that of unfrozen soils. Warm frozen soil is closer to unfrozen soils.
- A warm frozen soil is defined according to mechanical properties. Its creep was successfully observed in situ.
- Our own attempts: one is too simple, the other has too many parameters.
- No generally recognized constitutive models are found for creep of frozen soils. We have tried in different ways.


Thank you for your attention!