Computational Fluids Mechanics

Part 1 : Numerical methods
Part 2 : Turbulence models
Part 3 : Practice of numerical simulation

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Computational Fluid Mechanics
Part1：Numerical Methods

Objective：

Numerical methods for fluids mechanics and their accuracy

1. Introduction
2. Numerical methods for fluid mechanics (1) ~ Basic equations
3. Numerical methods for fluid mechanics (2) ~Discretizing schemes
4. Numerical methods for fluid mechanics (3) ~Coupling algorism
5. Numerical methods for fluid mechanics (4) ~Additional problems
6. Reliability of numerical simulation

Summary

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Computational Fluid Mechanics

Part1 : Numerical Methods (1a)

1. Introduction
   - What do we want to know ?
   - How to do numerical simulation ?
   - Basic mathematics for Fluid Mechanics

2. Numerical methods for fluid mechanics (1) ~ Basic equations
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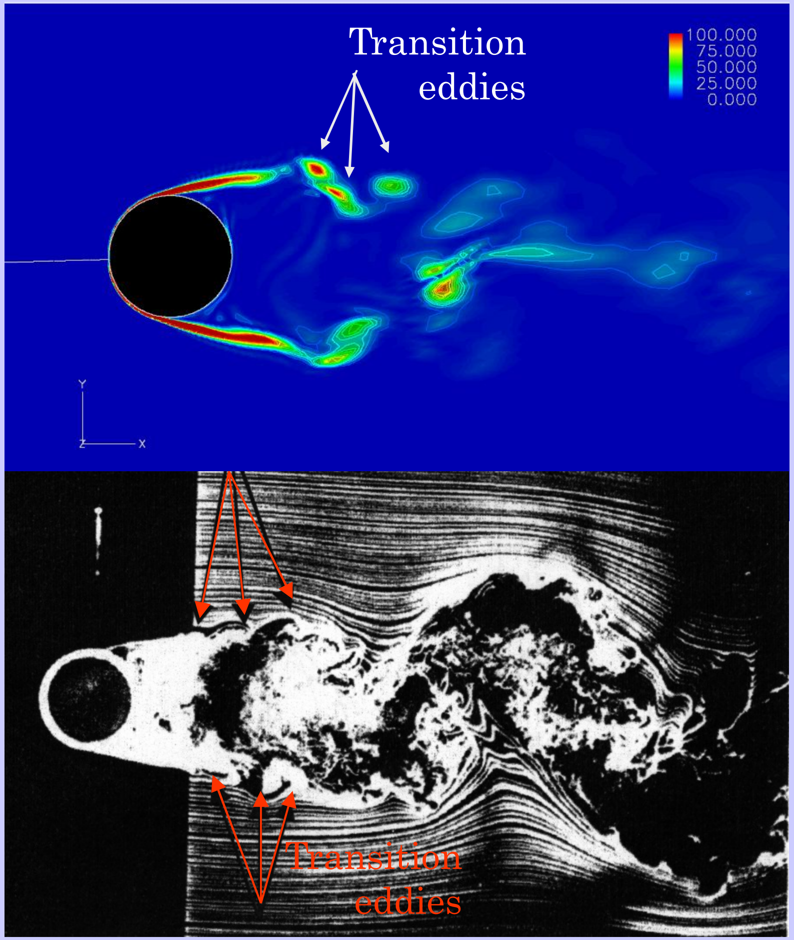
6. Reliability of numerical simulation

What we want to know ？

- ***3-D and unsteady features of fluid flow***
  - turbulence
  - vortex interactions
- ***Realistic design of fluid machine***
  - detail design (ex. parts design, flow conditions, etc.)
  - unsteady conditions (ex. cascade, sloshing, etc.)
- ***Dynamic interaction in complex flows***
  - reactive flows / multiphase flows
  - flow induced instability and acoustics problems

Instability of free shear layer



blade

LES result (Re=3900, case 2)
Instantaneous enstrophy distribution

Experiment (Corke Re=8000[13])
Streaklines visualization(smoke wire)

Vortex structures in wall share layer

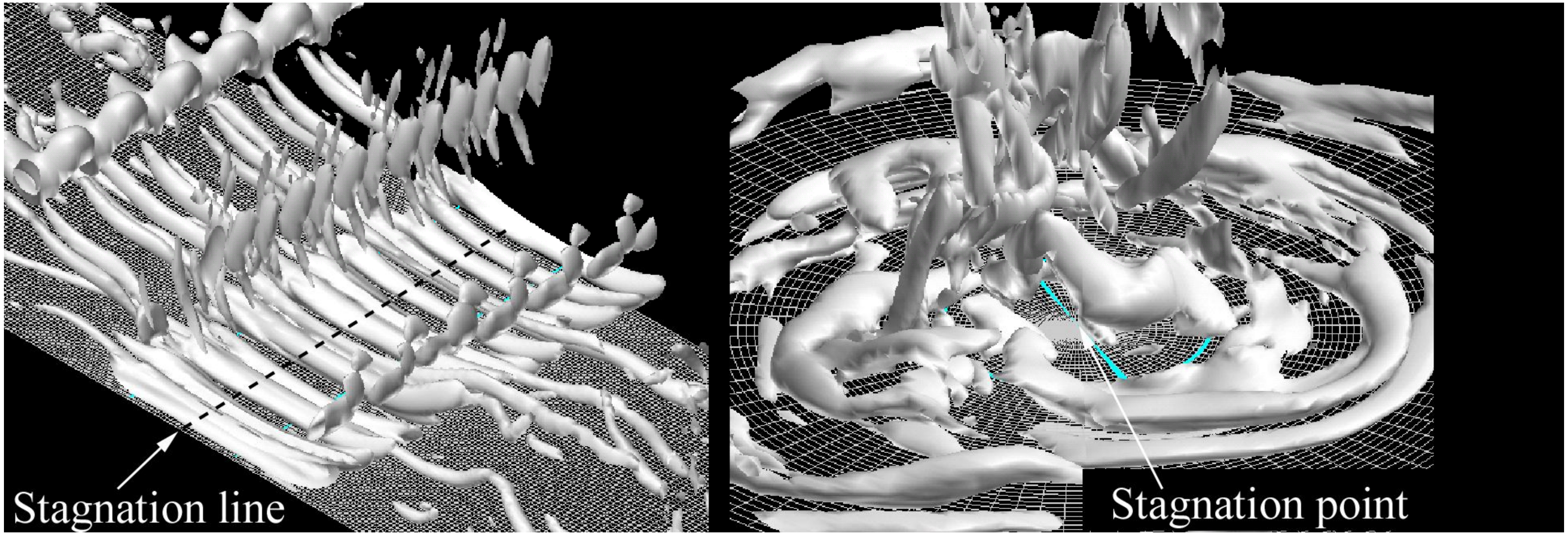


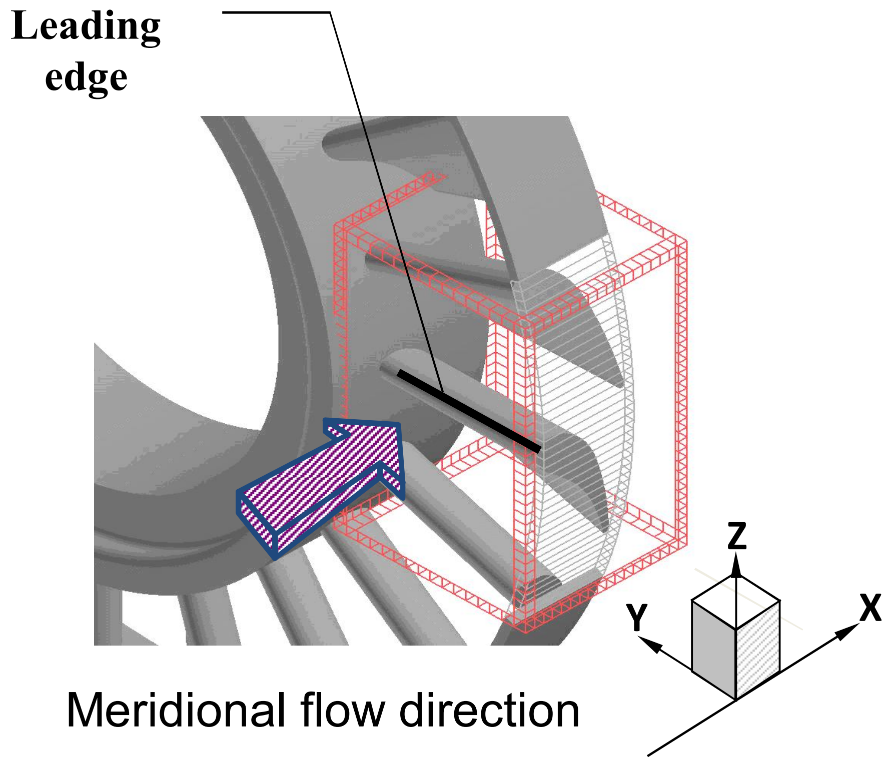
Figure 4: Instantaneous eddy structures at the stagnation region of plane (left) and round (right) impinging jets when six waves are imposed at the inlet (left: $\lambda_z/\pi D=1/6$; right: $\lambda_\theta/\pi D=1/6$, iso-surface: $\Delta p'=4.5$)

Design of Turbomachine (1)

- 3D Measuring of Torque Converter -

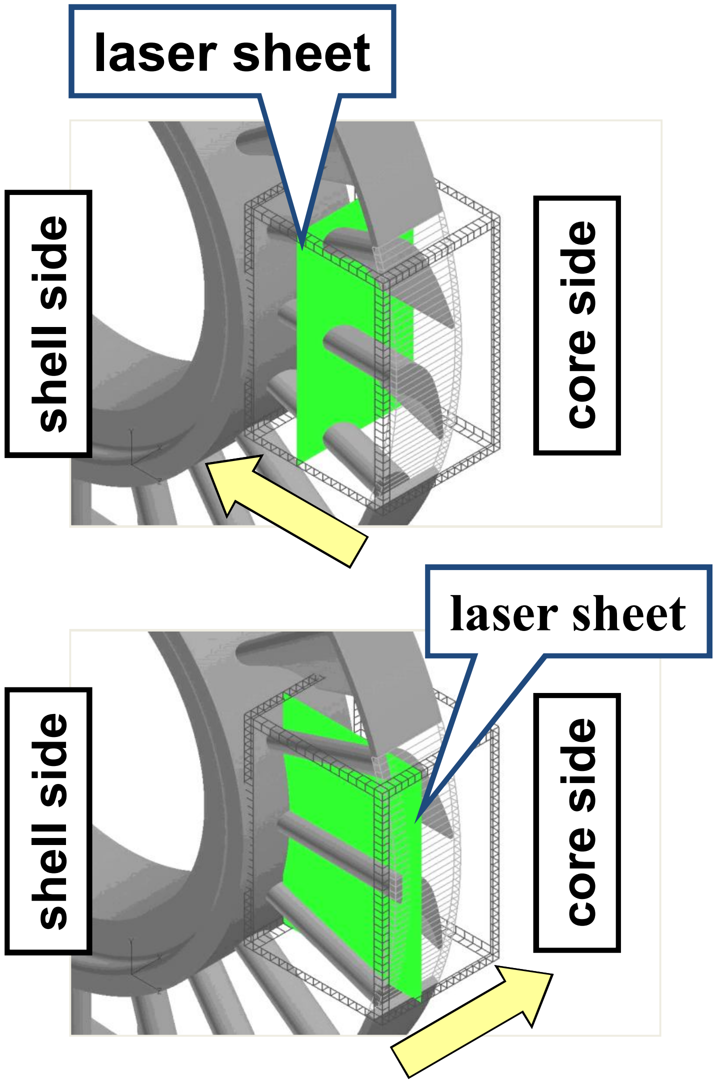
Input shaft rotating speed 400 [r/min]

Speed ratio 0.05 / 0.3 / 0.8



Meridional flow direction

(main flow direction)

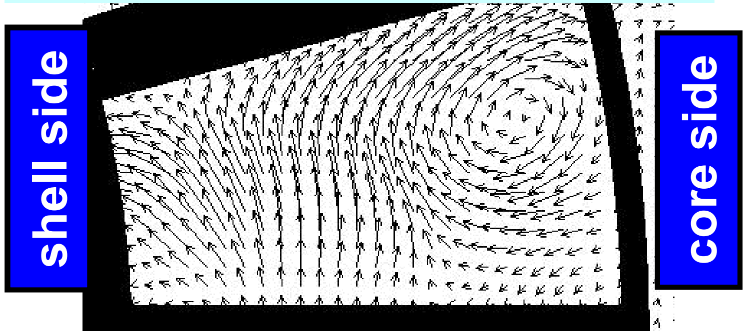


Measurement Result
Speed Ration 0.05

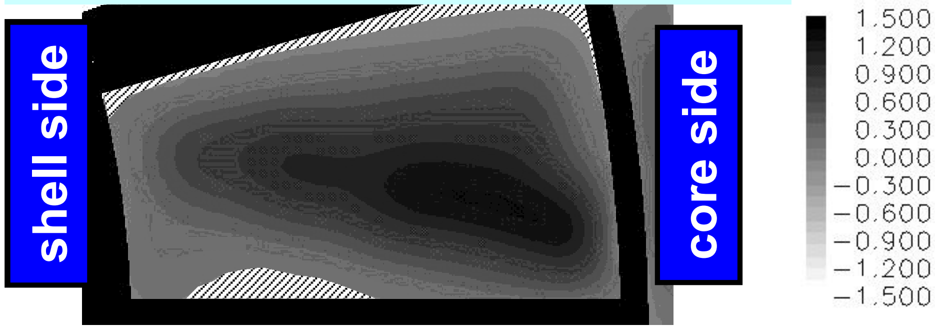
* These flow field planes are view from upstream
* 3 [mm] downstream from leading edge of stator blade

* Starting points of stream lines are located at 3 [mm] upstream from leading edge of stator blade.

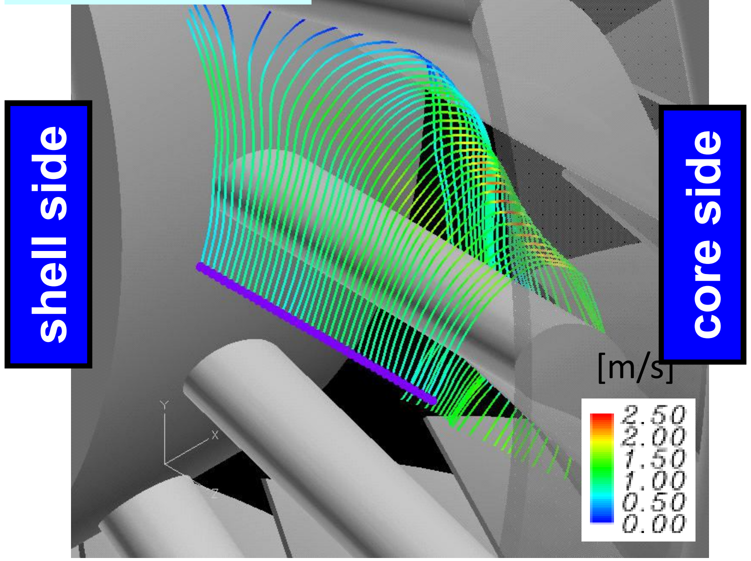
Secondary Flow Velocity Vectors



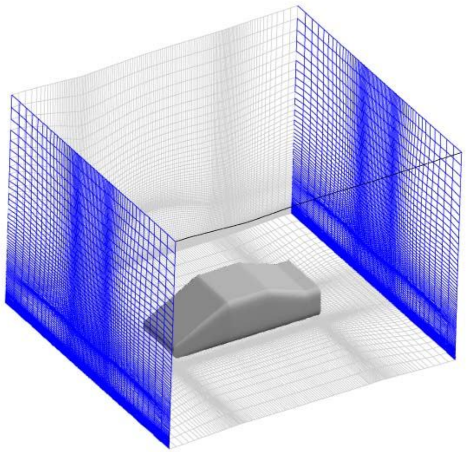
Meridional Flow Velocity Distribution



Stream Lines

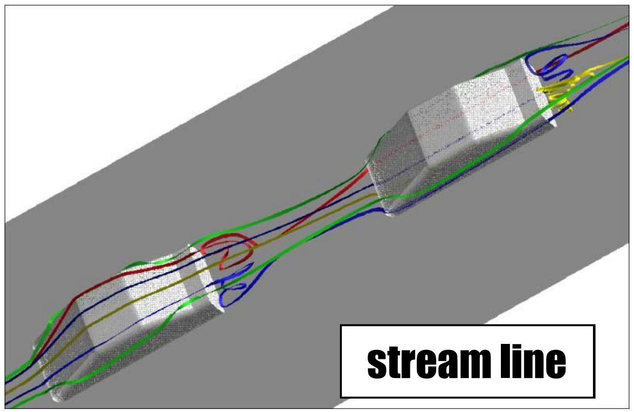


Design of Vehicle (1)

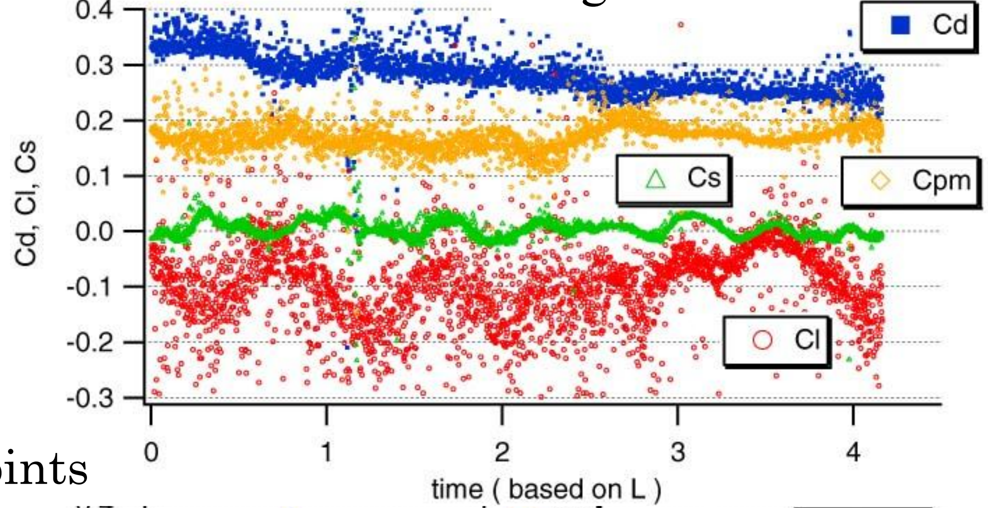


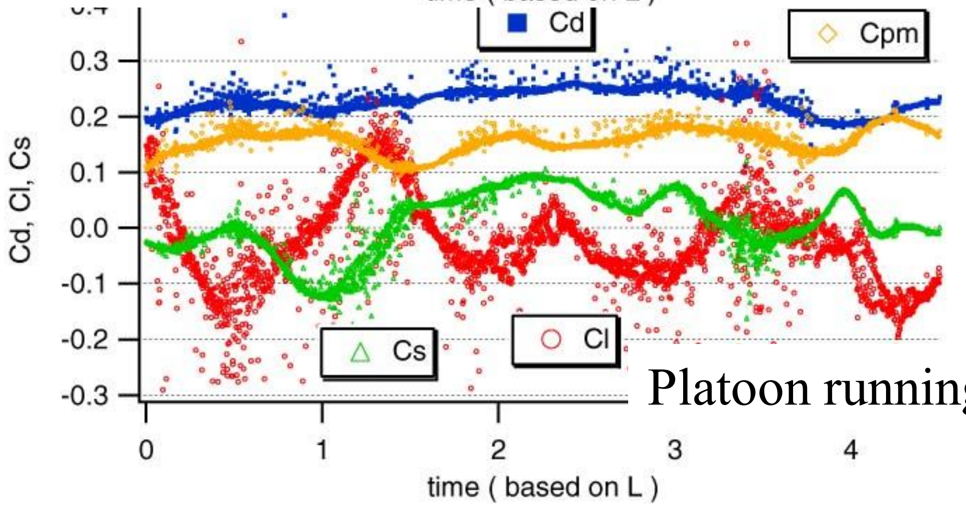
LES Grid

1,000,000 points



Single Vehicle

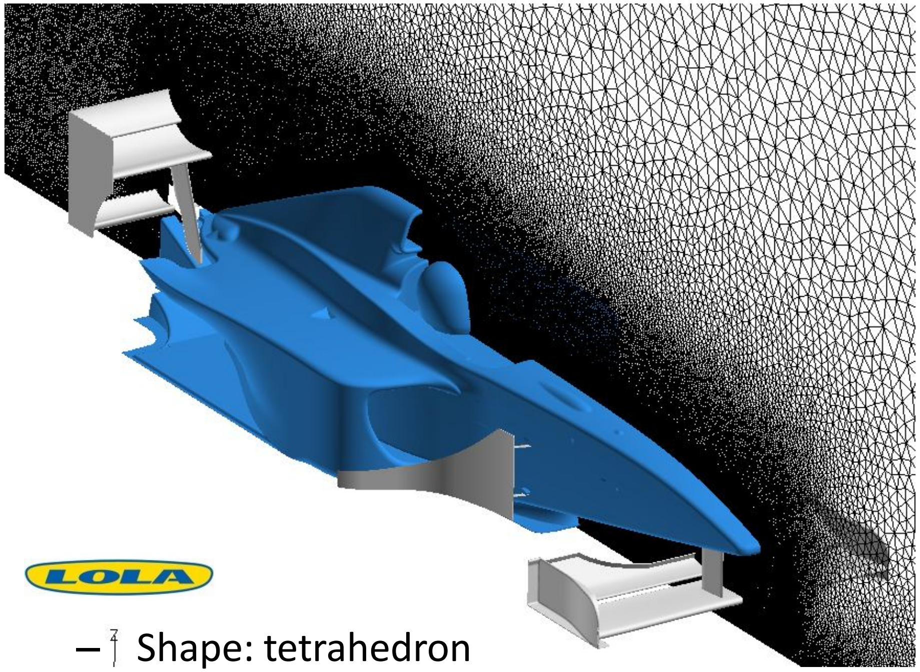




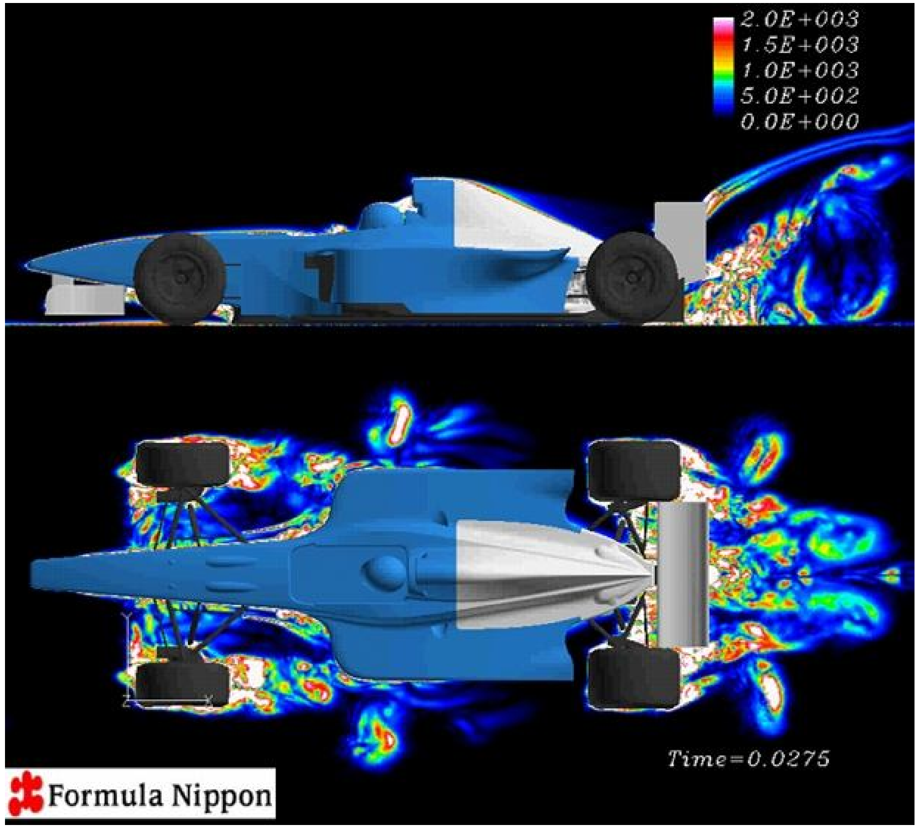
Platoon running

Unsteady flow design of Formula car

- LOLA B03/51
- Formula NIPPON (2003~05)



- Shape: tetrahedron
- Element numbers: 117,060,909
- Node numbers: 20,957,323



Design of Vehicle (2)

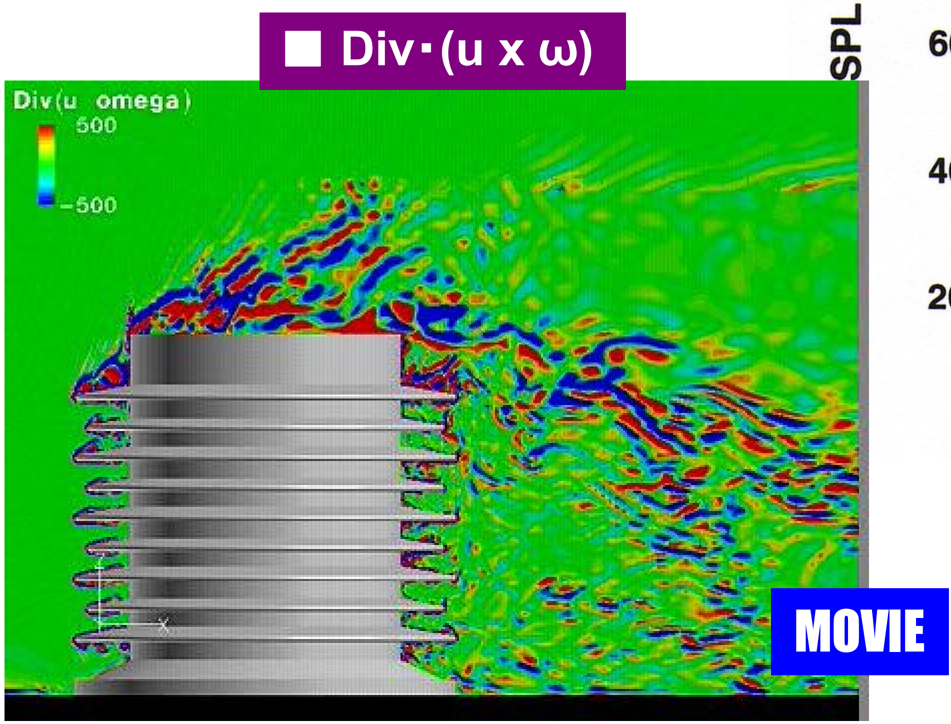
Shinkan-sen (Bullet Train)

300km/h → 360km/h

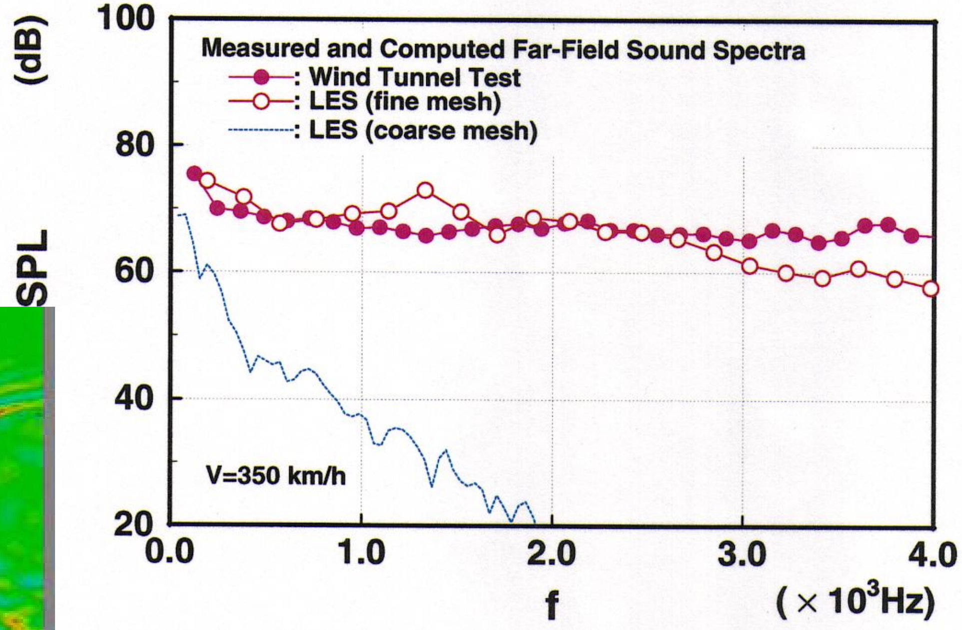
(2002) (2012)

Sources of Flow Noise
by Pantographs

Prediction of Far-field Sound Spectra & Sound Source Distribution



Identified Instantaneous Sound Sources

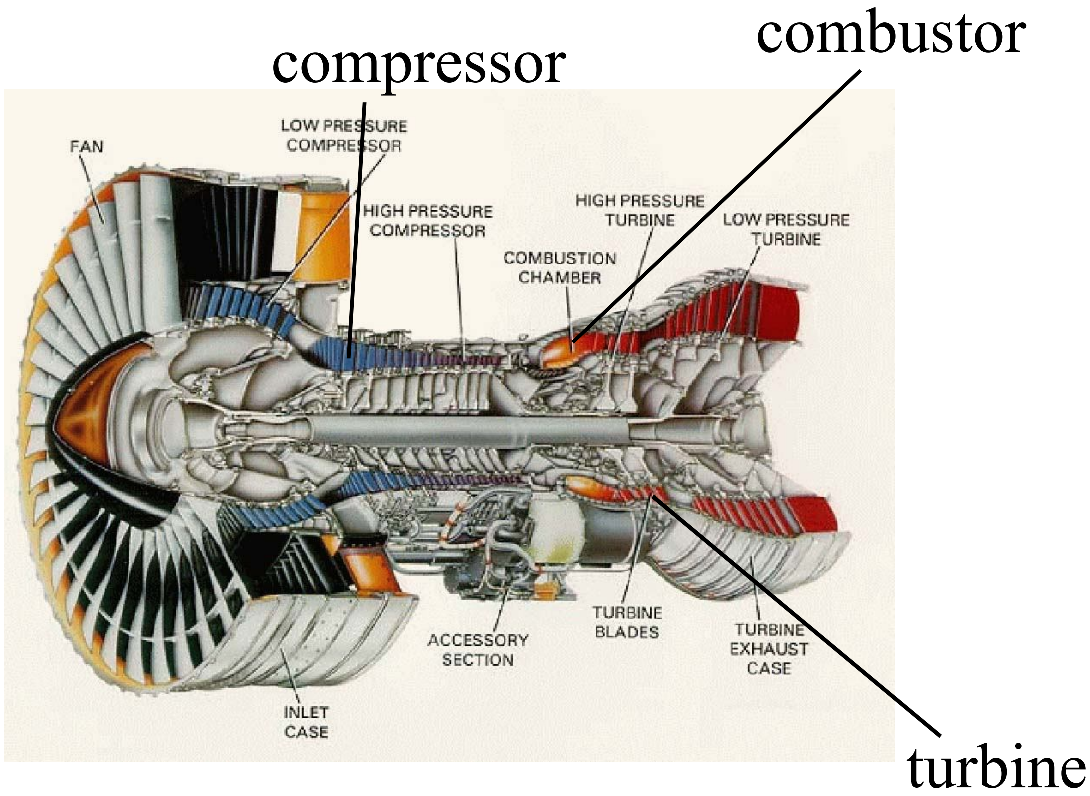


Far-Field Sound Pressure Spectra

Combustion flow in Engineering

Ex. Design of Gasturbine

- Technical Elements
  - spray
  - turbulence
  - heat transfer
  - material design
  - detail reaction
  - Ignition / extinction
  - radiation
  - noise / vibration

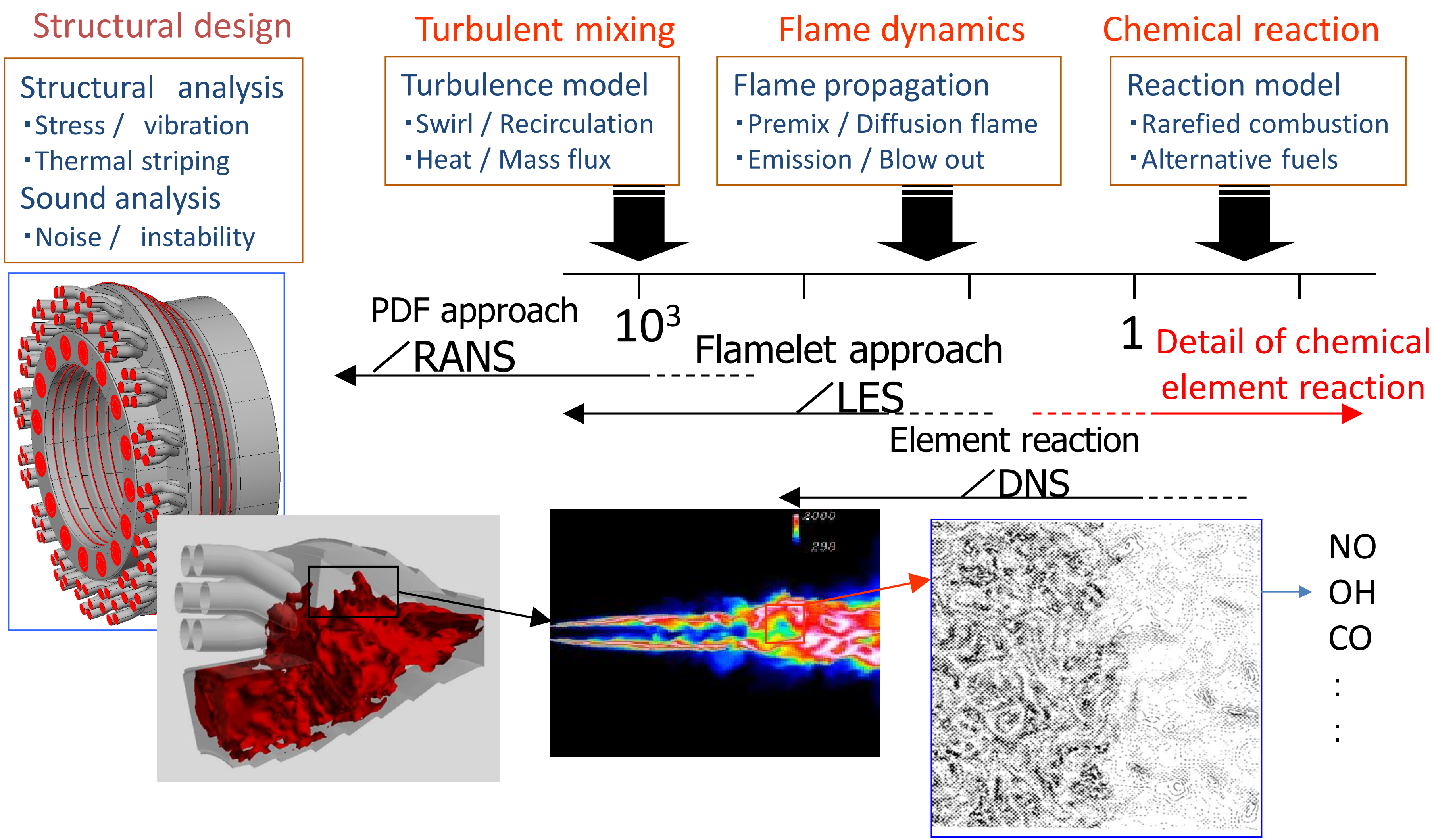


Solution for Engineering problems

- Lean premixed combustion
- Lower emission / high efficiency

Strategy for turbulent combustion design

ex. combustion flow modeling



Exercise 1a

Consider an application of computational fluid mechanics on to your interest design or research problem, and tell its targeting objective you want to know.

Computational Fluid Mechanics
Part1 : Numerical Methods (1b)

1. Introduction
   - What do we want to know ?
   - How to do numerical simulation ?
   - Basic mathematics for Fluid Mechanics

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6. Reliability of numerical simulation

How to do numerical simulation?

- Basic elements of numerical simulation

	physics	mathematics	
	Governing eq. ex. incompressible assumption turbulence model ・・・	**Numerical scheme** ex. upwind scheme MAC algorithm	global
	Boundary condition ex. inlet / outlet wall condition	**Numerical grid** ex. grid resolution unstructured mesh	local

➡ **Get a numerical solution of differential equation**

- Matrix solver ／ parallel computing
- Post processing ／ visualization

Governing eq.

Generalized convection-diffusion eq.

$$\frac{\partial \phi}{\partial t} = -\mathbf{u} \cdot \nabla \phi - \nabla \cdot \mathbf{j} + S$$

ϕ : variable＝velocity component, temperature, etc.

$-\mathbf{u} \cdot \nabla \phi$: convection term, $\mathbf{u}$: velociy vector

$-\nabla \cdot \mathbf{j}$: diffusion term, $\mathbf{j}$: diffusion flux vector

S : production (dissipation) term

$= \textit{gravity force, heat source, etc.}$

Ex. Thermal equation T(x,t)

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{Q}{\rho c_p}$$

$$\text{variable;}\ \phi \Rightarrow T, \quad \frac{\partial \phi}{\partial t} \Rightarrow \frac{\partial T}{\partial t}, \quad \text{Constant facorts;}\ u, k, Q, \rho, c_p$$

$$-\mathbf{u}\cdot\nabla T \Rightarrow -\nabla\cdot T\mathbf{u} \Rightarrow -\frac{\partial uT}{\partial x}, \quad T\mathbf{u} : \text{convection flux}$$

$$-\nabla\cdot\mathbf{j} \Rightarrow \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right), \quad \mathbf{j} = -\mathrm{k}\nabla\mathrm{T} \ \ \text{diffusion flux}$$

$$S \Rightarrow Q/\rho c_p, \quad Q : \text{heat source}, \quad \rho : \text{density}, \quad \mathrm{c_p} : \text{capacity}$$

Numerical solution of deferential eq.

Deferential eq. （+ B.C.） → Algebraic eq.

Ex.

$$\frac{df}{dx} = 2x \quad (x > 0,\ f(0) = 0)$$

$$f(x) = x^2$$

Approx. of equation

Approx. of solution

Equation system （incl. B.C.） → sequence of values

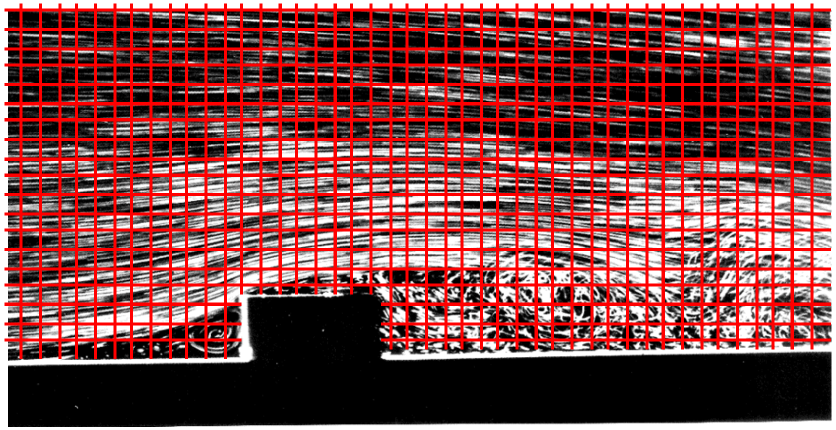
$$f(0) = 0$$
$$f(1) - f(0) = 2f(0)$$
$$f(2) - f(1) = 2f(1)$$
$$\vdots$$

$$[f(0), f(1), f(2), f(3), \cdots] = [0, 1, 4, 9, \cdots]$$

$$\mathbf{A}x = b$$

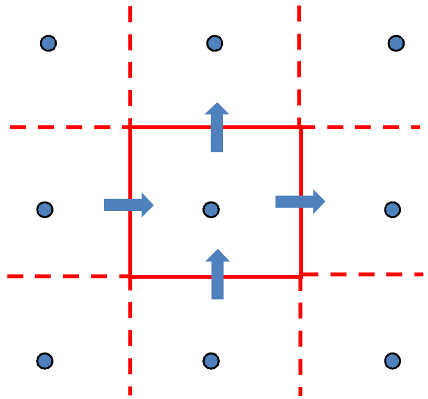
Numerical solution of 2D/3D problems

Solution



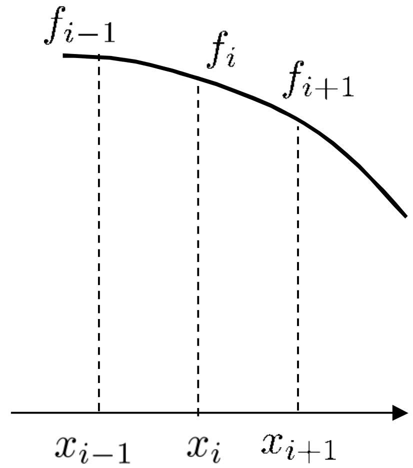
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

Numerical Grid



Conservation laws for Mass, momentum & energy

Approximation



$$\frac{\partial f}{\partial x} \sim \frac{f_{i+1} - f_{i-1}}{x_{i+1} - x_{i-1}}$$

Values on millions gird points should be solved !

Algorism for solution

Ex. 1D incompressible flow

Eq.1 $$\frac{u_{i+1}^{k+1}-u_i^{k+1}}{dx}=0$$

Eq.2 $$\frac{u_i^{k+1}-u_i^k}{dt}=-u_i^k\frac{u_i^k-u_{i-1}^k}{dx}+\mu\frac{u_{i+1}^k-2u_i^k+u_{i-1}^k}{dx^2}-\frac{1}{\rho}\frac{p_i^{k+1}-p_{i-1}^{k+1}}{dx}$$

Eq.3 $$\frac{1}{\rho}\frac{p_{i+1}^{k+1}-2p_i^{k+1}+p_{i-1}^{k+1}}{dx^2}=\frac{D_i^k}{dt}+\frac{F_{i+1}^k-F_i^k}{dx}$$

Time marching k → k+1

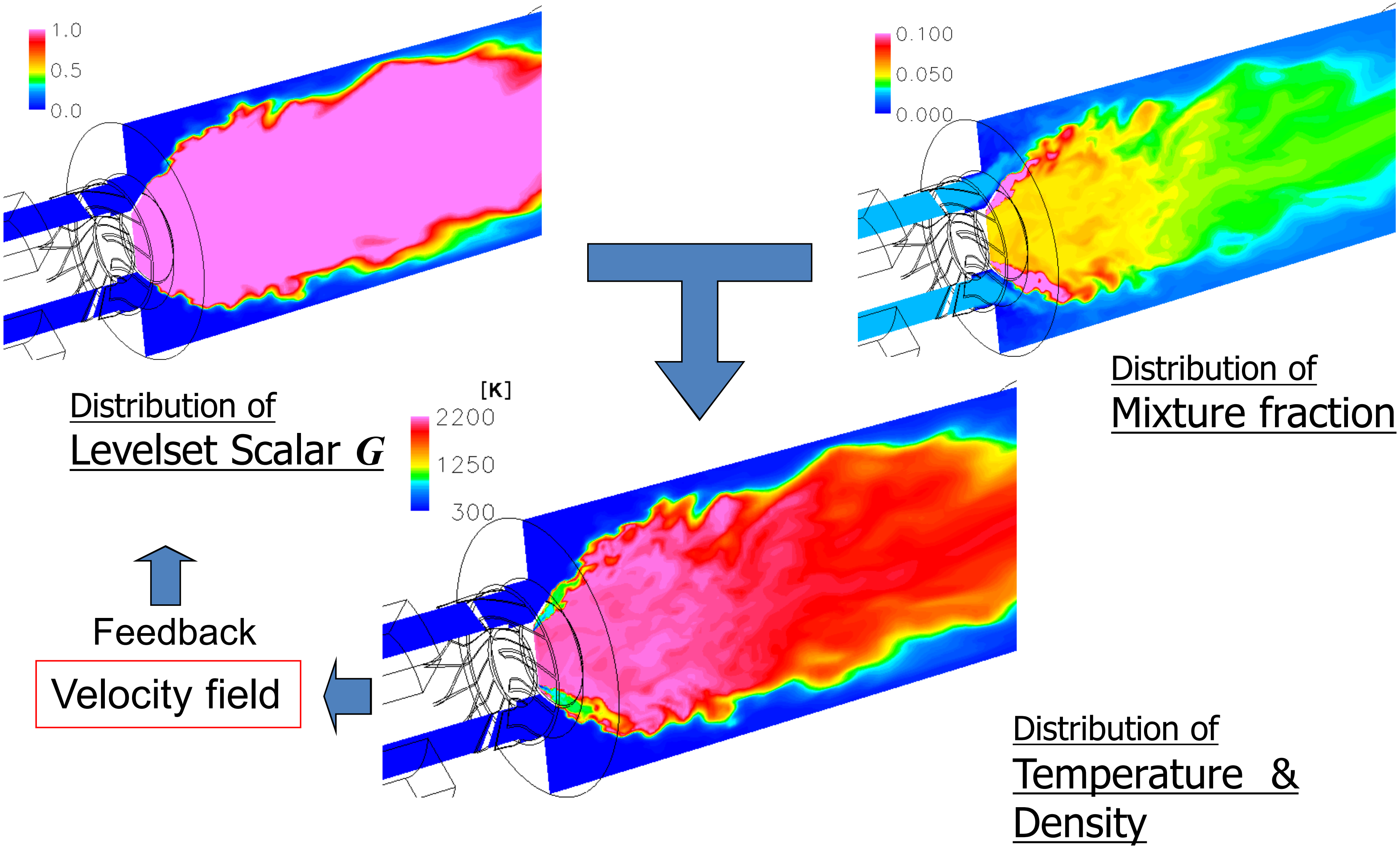
Algorism 1: Solve **u** by **eq.1** then solve **p** by **eq.2** **(successive)**

Algorism 2: Solve **p** by **eq.3** then solve **u** by **eq.2** **(successive)**

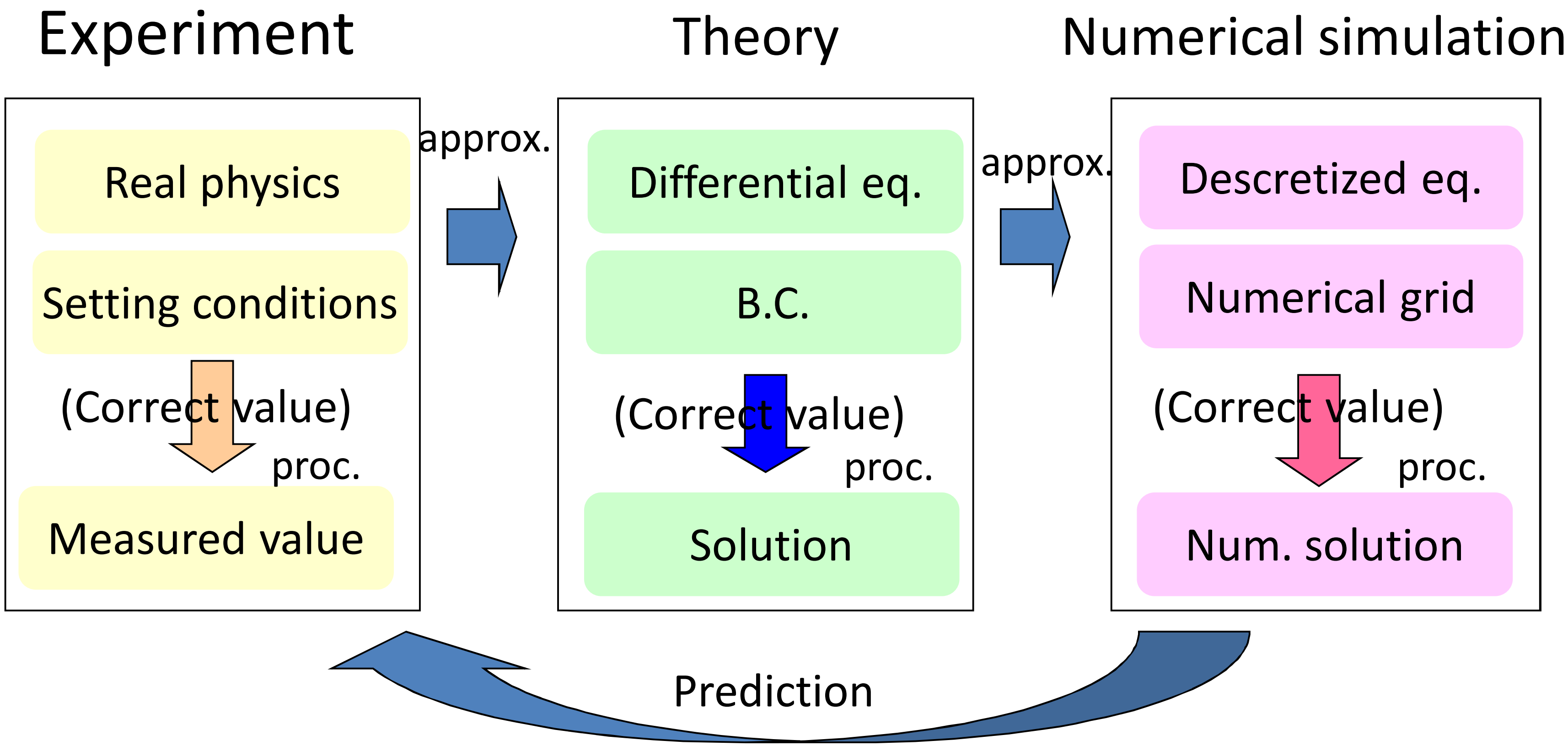
Algorism 3: Solve **u** by **eq.1** and solve **p** by **eq.3** **(parallel)**

Numerical solution of complex problems

Ex. Combustion flow by flamelet model



Approximation & Error

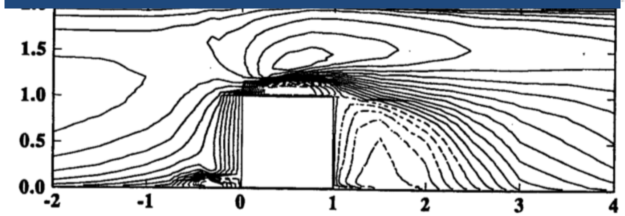


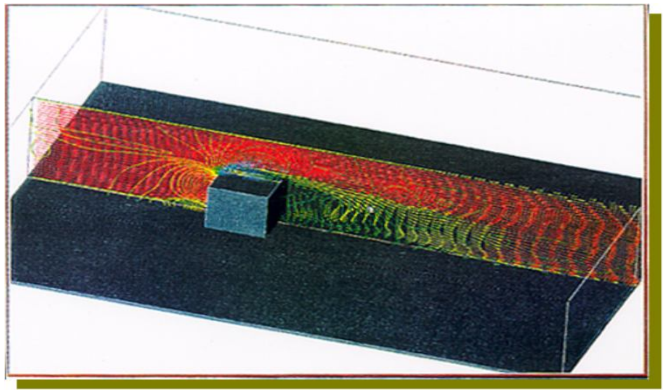
Error ＝ Analyzable discrepancy from correct answer
due to *approximation* and *procedures*

Reliability of numerical solution

Problem: How to remove numerical errors from a correct solution ?

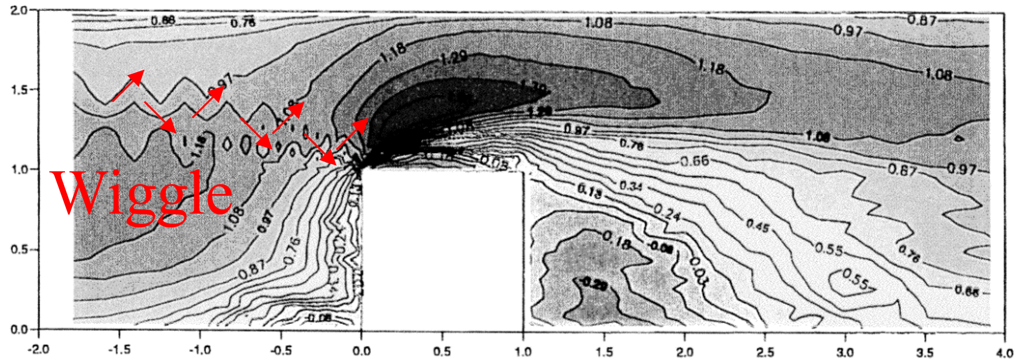
Mean velo.<U> experiment



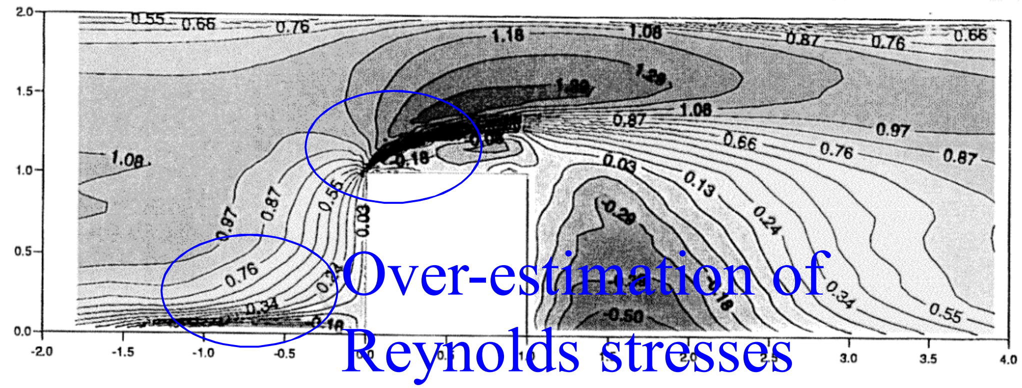


- Cubic obstacle in channel
  - Park & Taniguchi (1995)
  - LES-WS (1995, Tegernsee)

Case1 2nd Central Scheme

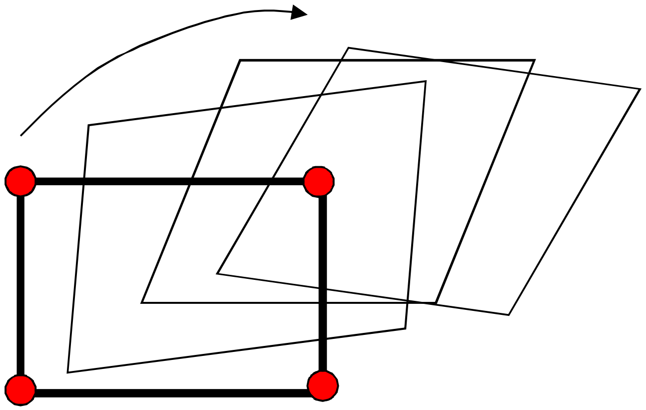


Case2 QUICK with numerical diffusion



Fluid vs. Solid

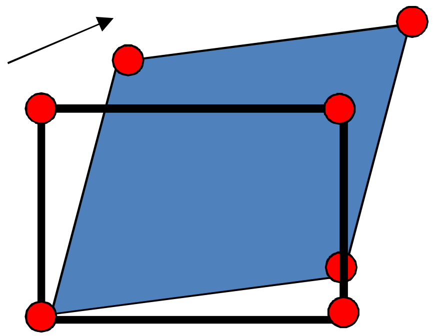
Fluid flow analysis



- Analyze shape deformation rate
- Arrangement of elements is not fixed or unobservable

- Unequilibrium physics
- Internal mechanism is statistical, isotropic and homogeneous

Solid structural analysis



- Analyze shape deformation
- Arrangement of elements is fixed and observable

- Equilibrium physics
- Internal mechanism is structural, anisotropic and inhomogeneous

Exercise 1b

Solve the three algorisms for 1D incompressible flow under the following condition

i.c. $D_i^k = \dfrac{u_{i+1}^k - u_i^k}{dx} = 0$ at $t = t^k$

b.c. $u_0 = U, \quad \dfrac{p_1 - p_0}{dx} = 0$ at $x = x_0$

$\dfrac{u_N - u_{N-1}}{dx} = 0, \quad p_N = 0$ at $x = x_N$

Computational Fluid Mechanics

Part1 : Numerical Methods (1c)

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Equations of flow phenomena

Mass

$$\frac{D\rho}{Dt}\left(\equiv\frac{\partial\rho}{\partial t}+\mathbf{u}\cdot\nabla\rho\right)=-\rho(\nabla\cdot\mathbf{u})$$

Momentum

$$\frac{D\mathbf{u}^t}{Dt}\left(\equiv\frac{\partial\mathbf{u}^t}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}^t\right)=-\frac{(\nabla p)^t}{\rho}-\nabla\cdot\mathbf{\Sigma}+\mathbf{f}^t$$

$$\mathbf{u}^t=[u\quad v\quad v],\qquad \mathbf{\Sigma}=\nu\left(\frac{2}{3}(\nabla\cdot\mathbf{u})\mathbf{I}-2\mathbf{S}\right),\qquad \mathbf{I}\equiv\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Energy

$$\frac{De}{Dt}\left(\equiv\frac{\partial e}{\partial t}+\mathbf{u}\cdot\nabla e\right)+\nabla\cdot q=\quad T\frac{Ds}{Dt}\quad-p\frac{Dv}{Dt}$$

diffusion — heat source — pressure work

$$-p\frac{D\mathrm{v}}{Dt}=\frac{p}{\rho^2}\frac{D\rho}{Dt}=-\frac{p}{\rho}(\nabla\cdot\mathbf{u})\ ,\qquad v=\frac{1}{\rho}$$

Basic mathematics for fluid mechanics

Operations of vector and tensor variables

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad \mathbf{a}^t = \begin{bmatrix} a_1 & a_2 \end{bmatrix}, \quad \mathbf{b}^t = \begin{bmatrix} b_1 & b_2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad \mathbf{A}^t = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^t \mathbf{b} = \begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 b_1 + a_2 b_2 \end{bmatrix} \qquad |\mathbf{a}|^2 = \mathbf{a} \cdot \mathbf{a} = \mathbf{a}^t \mathbf{a} \qquad \mathbf{A}\mathbf{b} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_{11} b_1 + a_{12} b_2 \\ a_{21} b_1 + a_{22} b_2 \end{bmatrix}$$

$$\mathbf{b} \cdot \mathbf{A} = \mathbf{b}^t \mathbf{A} = \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} b_1 a_{11} + b_2 a_{21} & b_1 a_{12} + b_2 a_{22} \end{bmatrix}$$

$$\mathbf{a}\mathbf{b}^t = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \begin{bmatrix} b_1 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{bmatrix} \qquad \mathbf{b}\mathbf{a}^t = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \begin{bmatrix} a_1 & a_2 \end{bmatrix} = \begin{bmatrix} b_1 a_1 & b_1 a_2 \\ b_2 a_1 & b_2 a_2 \end{bmatrix} = \left(\mathbf{a}\mathbf{b}^t\right)^t$$

$$(\mathbf{c} \cdot \mathbf{a})\mathbf{b}^t = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \begin{bmatrix} b_1 & b_2 \end{bmatrix} = \mathbf{c} \cdot \left(\mathbf{a}\mathbf{b}^t\right)$$

Derivative operators for vector variables

$$\nabla = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} \quad \Rightarrow \quad \nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \qquad \nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix}$$

$$\nabla \mathbf{u}^t = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix} \begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} \partial u/\partial x & \partial v/\partial x \\ \partial u/\partial y & \partial v/\partial y \end{bmatrix}$$

$$\nabla \cdot \mathbf{A} = \begin{bmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial a_{11}}{\partial x} + \dfrac{\partial a_{21}}{\partial y} & \dfrac{\partial a_{12}}{\partial x} + \dfrac{\partial a_{22}}{\partial y} \end{bmatrix}$$

$$(\mathbf{u} \cdot \nabla)\mathbf{u} = \mathbf{u} \cdot \nabla \mathbf{u}^t = \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \mathbf{u}, \qquad \nabla \cdot \mathbf{u} f = f\, \nabla \cdot \mathbf{u} + \mathbf{u} \cdot \nabla f$$

$$\therefore \ \nabla \cdot (\rho \mathbf{u} \phi) = (\rho \mathbf{u}) \cdot \nabla \phi + \phi (\nabla \cdot \rho \mathbf{u})$$ Convection of scalar ϕ

$$\frac{\partial}{\partial x_j}(\rho u_j \phi) = \rho u_j \frac{\partial \phi}{\partial x_j} + \phi \frac{\partial}{\partial x_j}(\rho u_j)$$

$$\therefore \ \nabla \cdot (\rho \mathbf{u} \mathbf{u}^t) = \mathbf{u}^t (\nabla \cdot \rho \mathbf{u}) + (\rho \mathbf{u}) \cdot \nabla \mathbf{u}^t$$ Convection of vector $\mathbf{u}$

$$\frac{\partial}{\partial x_j}(\rho u_j u_i) = \rho u_j \frac{\partial u_i}{\partial x_j} + u_i \frac{\partial}{\partial x_j}(\rho u_j)$$

Derivative operators for vector variables (continued)

$$\nabla = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} \quad \Rightarrow \quad \nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}, \qquad \nabla f = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix}$$

$$\nabla \mathbf{u}^t = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix} \begin{bmatrix} u & v \end{bmatrix} = \begin{bmatrix} \partial u/\partial x & \partial v/\partial x \\ \partial u/\partial y & \partial v/\partial y \end{bmatrix}$$

$$\nabla \cdot \nabla f = \begin{bmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \nabla^2 f$$

Scalar Laplacian operator

$$\nabla \cdot \nabla \mathbf{u}^t = \begin{bmatrix} \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \end{bmatrix} \begin{bmatrix} \partial u/\partial x & \partial v/\partial x \\ \partial u/\partial y & \partial v/\partial y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} & \dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} \end{bmatrix} = \left(\nabla^2\right)\mathbf{u}^t$$

Vector Laplacian operator

Derivative operators for vector variables (continued)

$$\nabla \mathbf{u}^t = \mathbf{S} + \mathbf{\Omega}$$

$$\mathbf{S} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] = \frac{1}{2}\begin{bmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} & 2\frac{\partial w}{\partial z} \end{bmatrix}$$

Strain tensor

(symmetric component)

$$\mathbf{\Omega} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right] = \frac{1}{2}\begin{bmatrix} 0 & \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} & 0 & \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} & 0 \end{bmatrix}$$

Voricity tensor

(anti-symmetric component)

$$= \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}, \qquad \boldsymbol{\omega} = \nabla \times \mathbf{u} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

Other Eqs. of flow phenomena

Kinetic energy $K \equiv \frac{1}{2}|\mathbf{u}|^2 = \frac{1}{2}u_k u_k = \frac{1}{2}\left(u^2 + v^2 + w^2\right)$

$\mathbf{u}\cdot$ （Momentum eq.）^{t}

$$\Rightarrow \quad \mathbf{u}\cdot\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot(\mathbf{u}\cdot\nabla)\mathbf{u} = -\mathbf{u}\cdot\frac{\nabla p}{\rho} - \mathbf{u}\cdot(\nabla\cdot\Sigma)^t + \mathbf{u}\cdot\mathbf{f}$$

$$\therefore \frac{\partial K}{\partial t} + (\mathbf{u}\cdot\nabla)K = -\frac{\nabla\cdot(p\mathbf{u})}{\rho} + \frac{p(\nabla\cdot\mathbf{u})}{\rho} + \nabla\cdot(u\cdot\mathbf{\Sigma})^t - \Phi + \mathbf{u}\cdot\mathbf{f}$$

$$\nabla\cdot(\mathbf{u}\cdot\mathbf{\Sigma})^t = \frac{\partial}{\partial x_j}\left(u_i \Sigma_{ij}\right), \qquad \Phi = \frac{\partial u_i}{\partial x_j}\Sigma_{ij}$$

Vorticity $\boldsymbol{\omega} \equiv \nabla\times\mathbf{u}$

$$\nabla\times \boxed{\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \left\{\nabla\cdot(2\nu\mathbf{S})\right\}^t + \mathbf{f}}$$

Exercise 1c

Complete the component formulation of vorticity equation

$$\frac{\partial \omega_i}{\partial t} + w\frac{\partial \omega_i}{\partial x} + v\frac{\partial \omega_i}{\partial y} + w\frac{\partial \omega_i}{\partial z} = ?$$

Confirm the following relation between Stress tensor formulations

$$-\nabla \cdot \mathbf{\Sigma} = \nabla \cdot \nu\left(-\frac{2}{3}(\nabla \cdot \mathbf{u})\mathbf{I} + 2\mathbf{S} \right)$$

$$= \frac{\nu}{3}\{\nabla(\nabla \cdot \mathbf{u})\}^t + \nu\nabla^2\mathbf{u}^t$$