



---

# **Failure Analysis of RC Structures using Volume Control Technique**

**-COE Intensive Course-**

---

**Feb. 3, 2005**

**Hokkaido University, Sapporo, Japan**

**Ha-Won Song**

**School of Civil and Environmental Engineering**



**Concrete Materials, Mechanics and Engineering Lab.  
YONSEI UNIVERSITY**

# Outline

## ■ Introduction

Characteristic of failure in concrete structures

Homogenized crack model

Volume control method

## ■ Modeling for cracked RC and ECC

Constitutive equations of RC/ Layered shell element

Modeling of ECC as re-strengthening material

## ■ Analysis results and comparison

RCCV subjected to internal pressure

RC tank , RC slab, and RC box culvert subjected to various loading

RC hollow column subjected to lateral loading

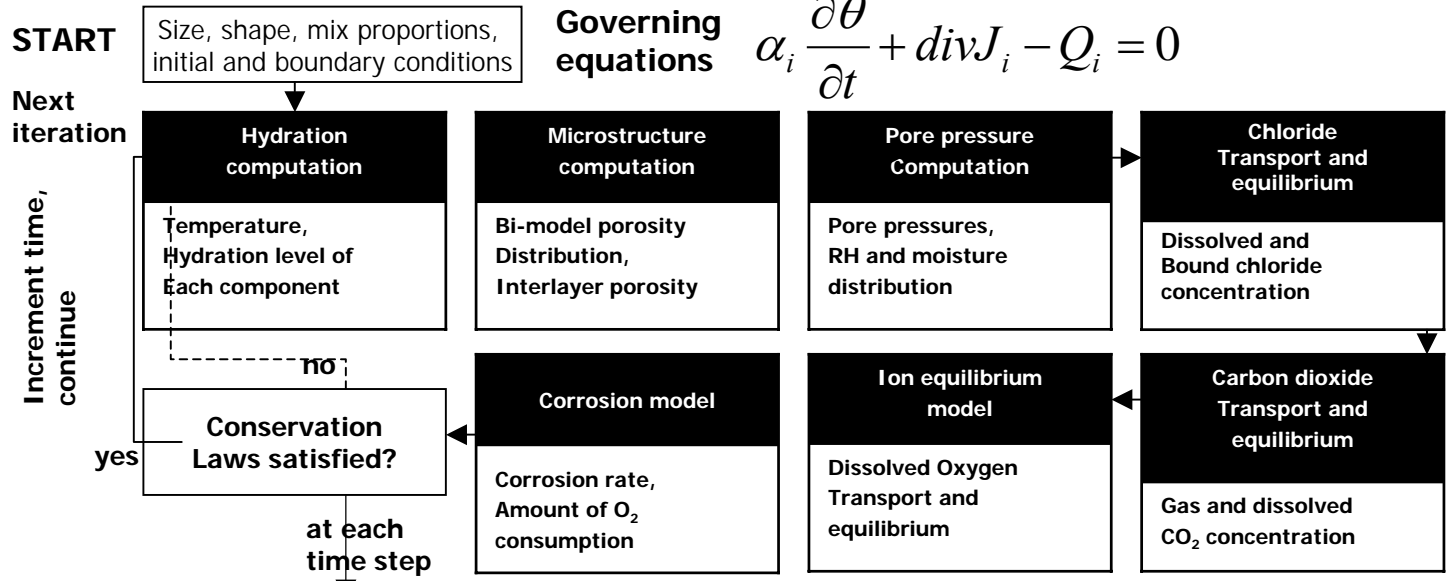
Verification with PCCV

## ■ Conclusion and future work

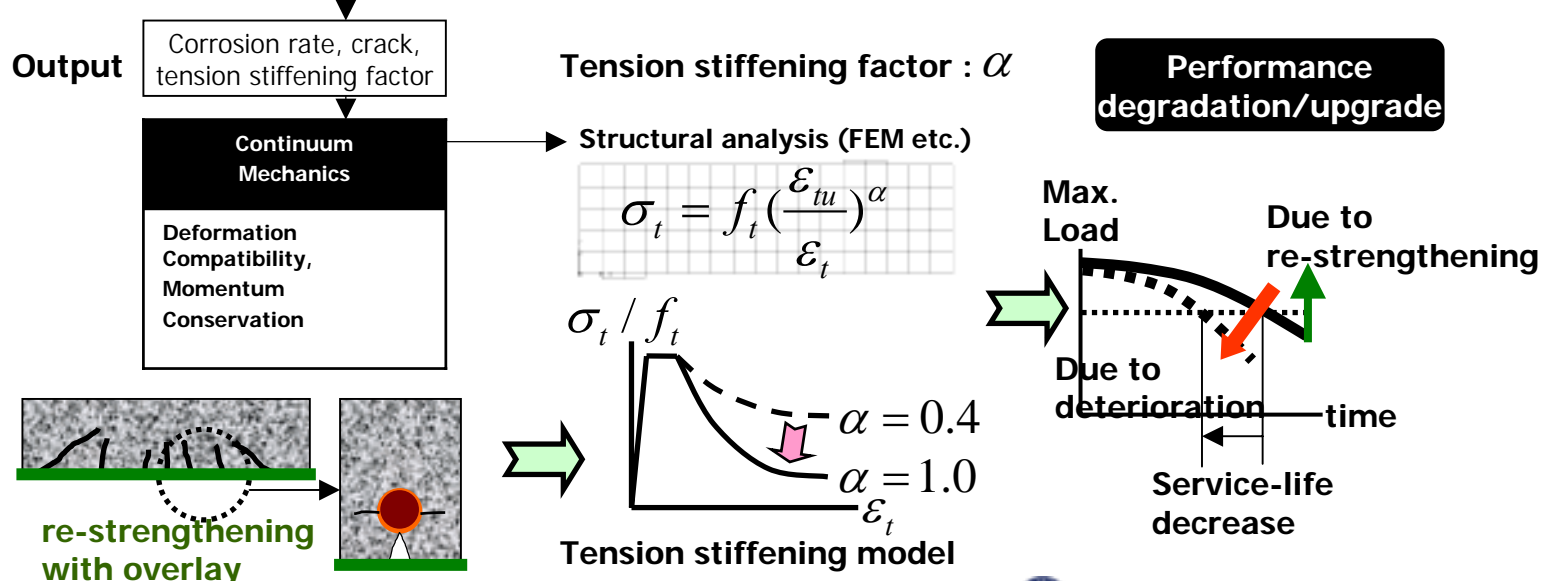


# Performance of deteriorated/repaired RC structures ?

**Durability Analysis**

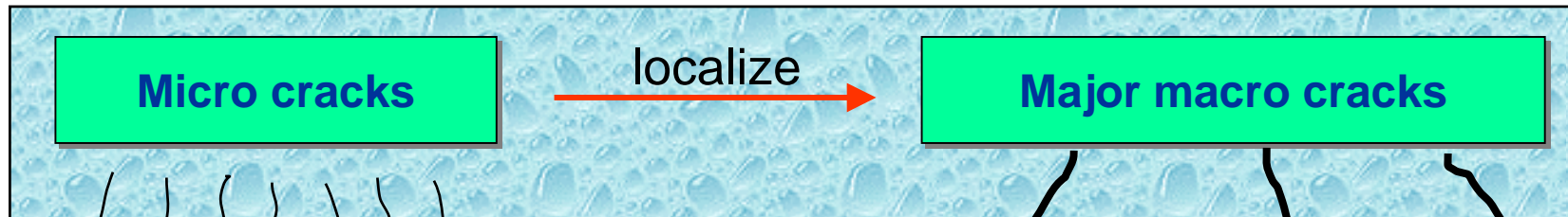


**Serviceability/Safety Analysis**



# Characteristics of concrete fracture and analysis

## ■ Material instability



### Numerical problem in concrete fracture analysis :

- Loss of ellipticity of governing equation
- Ill-posed boundary value problem

**Numerical drawback  
(Mesh sensitivity)**

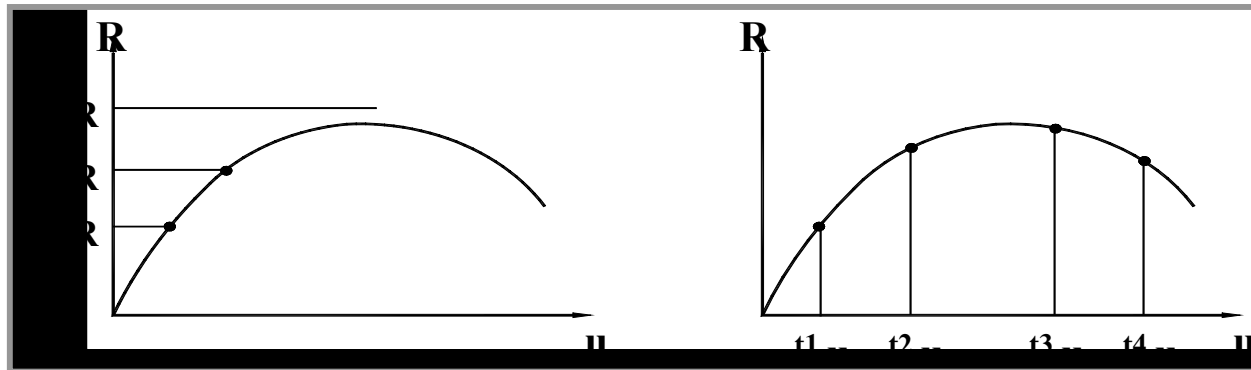
### Softening behavior

- Decreased load resistant capacity after peak
- Localized strain



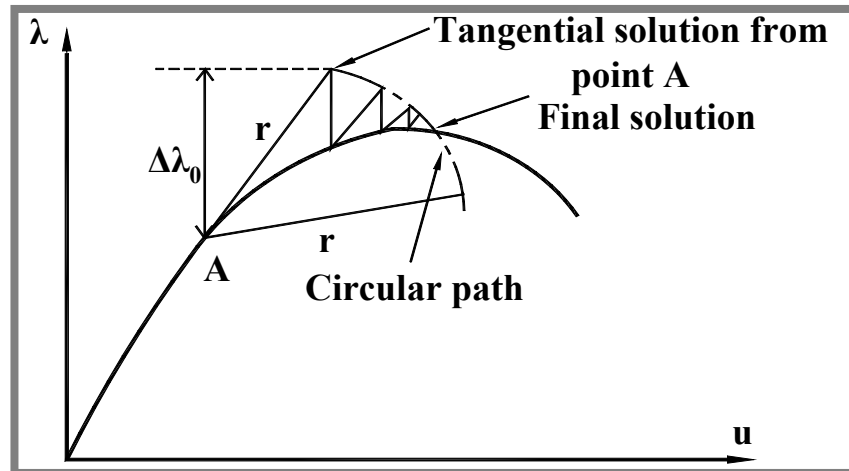
## ■ Structural instability

Sources of Non-linearity ; Material, Geometrical, Boundary and Contact



Load control

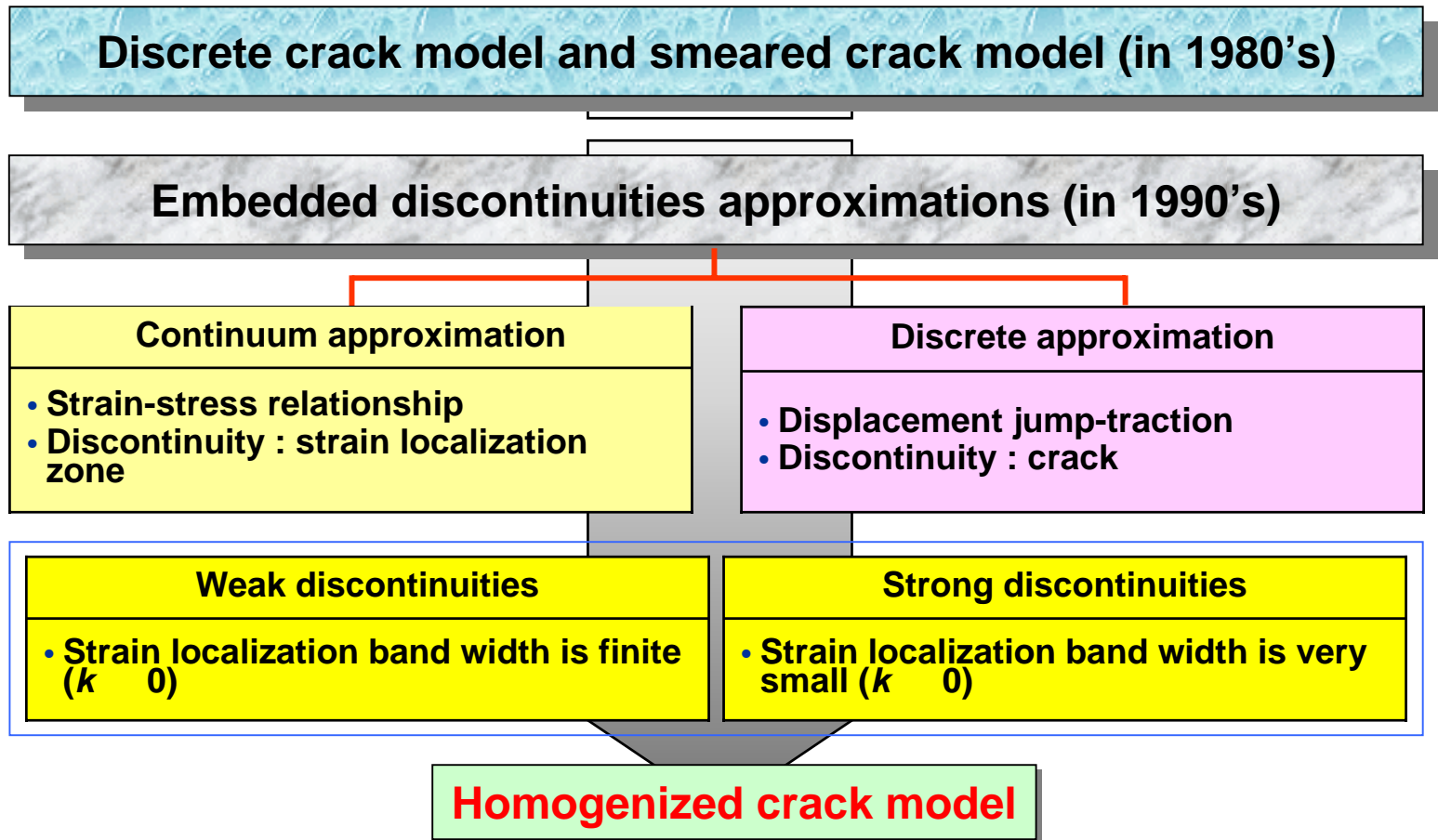
Displacement control



Arc-length control method



- *Effort to solve material instability in progressive fracture analysis of concrete using FEM*



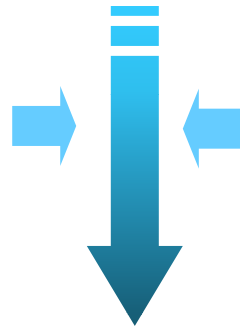
- *Effort to solve structural instability in progressive failure of concrete shell structures using FEM*

**Failure analysis of RC shell structures subjected to various loadings**

**Load Control Method** : difficulty to obtain post-peak ultimate behavior of RC structures

**Displacement Control Method** : difficulty to select a representative point for displacement control in 3D

Layered shell utilizing in-plane constitutive models of RC



- Remove the drawback of load control method
- overcome the limitation for displacement control method

**Volume Control Method**



# Volume Control Method with Pressure Node

- **Pressure Node : the uniform change of applied pressure on the shell element ( $\Delta p$ ) (Song and Tassoulas, IJNME, 1993 )**

$$\Delta V = \int_{b^e} \mathbf{n}^T \cdot \Delta \mathbf{u} \, db^e = \left( \int_{b^e} \mathbf{n}^T \cdot \mathbf{N} \, db^e \right) \Delta U$$



$$\int_{b^e} \mathbf{N}^T (\mathbf{t} + \Delta \mathbf{t}) \, db^e = -(\mathbf{p} + \Delta \mathbf{p}) \int_{b^e} \mathbf{N}^T \mathbf{n} \, db^e$$



$$\mathbf{K}_e \Delta U = -(\mathbf{p} + \Delta \mathbf{p}) \int_{b^e} \mathbf{N}^T \mathbf{n} \, db^e - \mathbf{F}_e$$

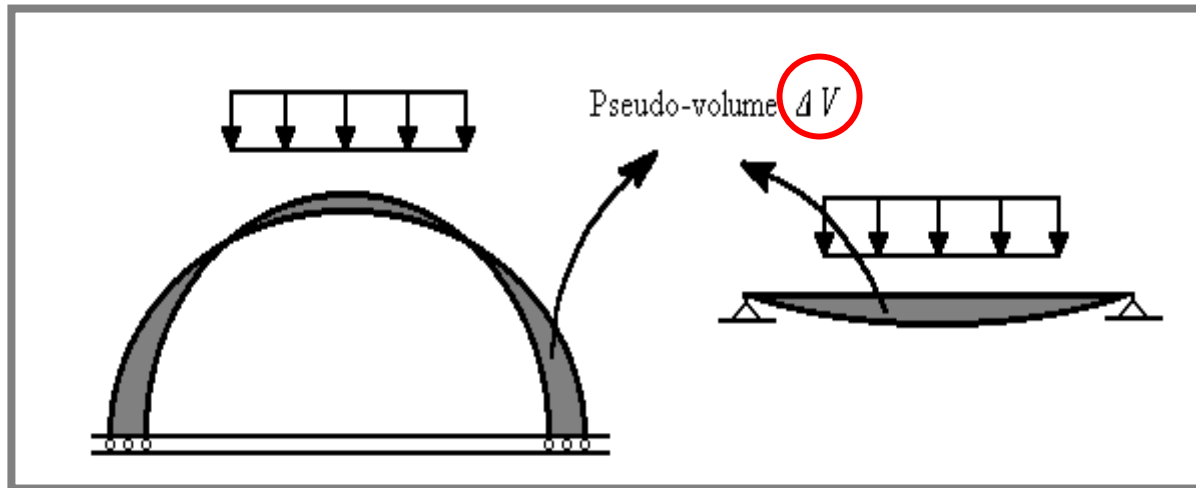


$$\begin{bmatrix} \mathbf{K}_e & \int_{b^e} \mathbf{N}^T \mathbf{n} \, db^e \\ \int_{b^e} \mathbf{N}^T \mathbf{n} \, db^e & 0 \end{bmatrix} \begin{bmatrix} \Delta U \\ \Delta p \end{bmatrix} = \begin{bmatrix} \mathbf{p} \int_{b^e} \mathbf{N}^T \mathbf{n} \, db^e - \mathbf{F}_e \\ \Delta V \end{bmatrix}$$



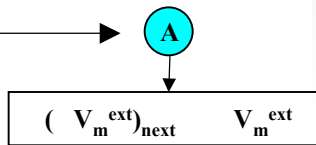
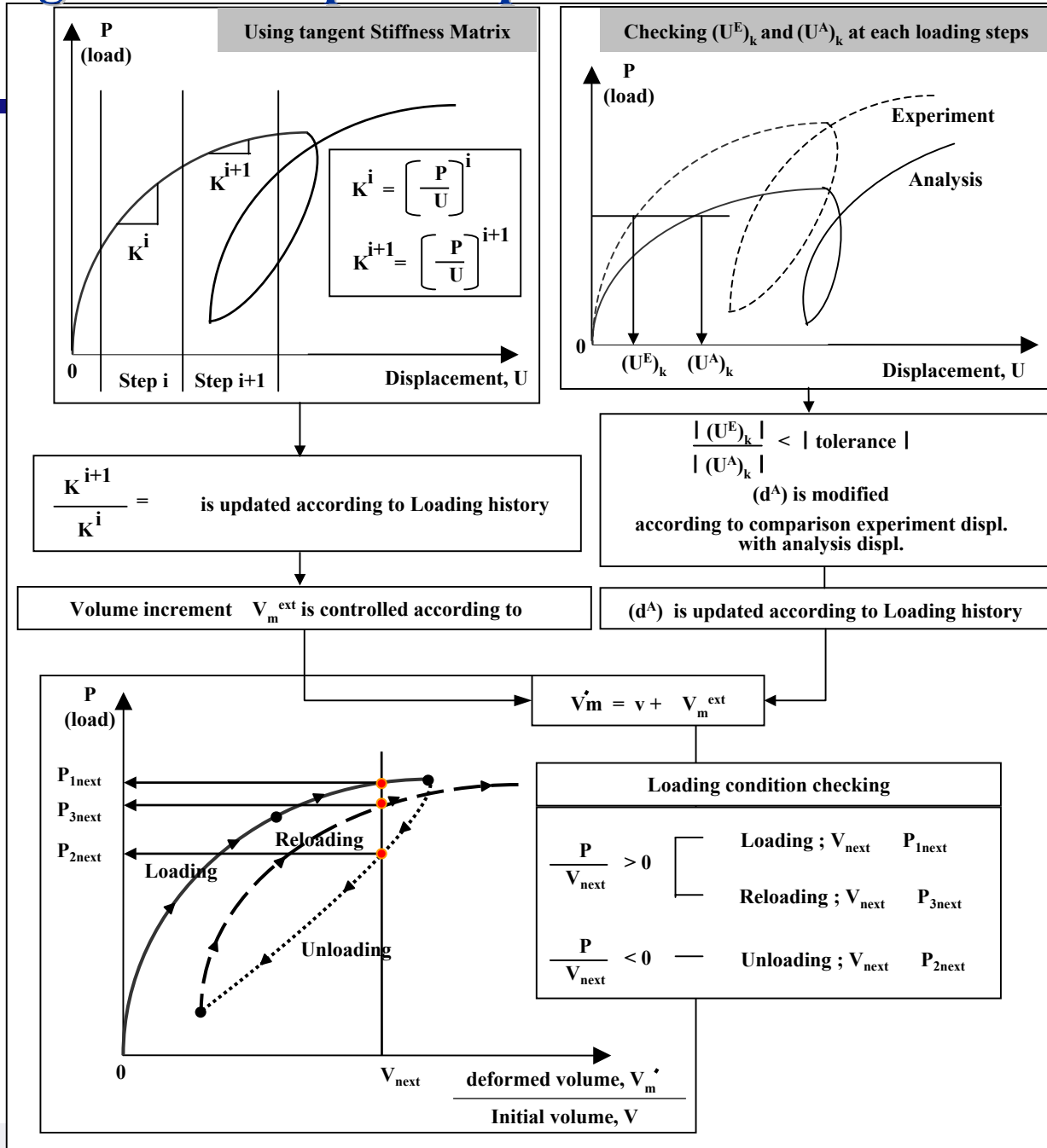
# Path dependant pseudo-volume control technique

- Pseudo Volume ( Song et. al, J. Str. Eng. 2002 )

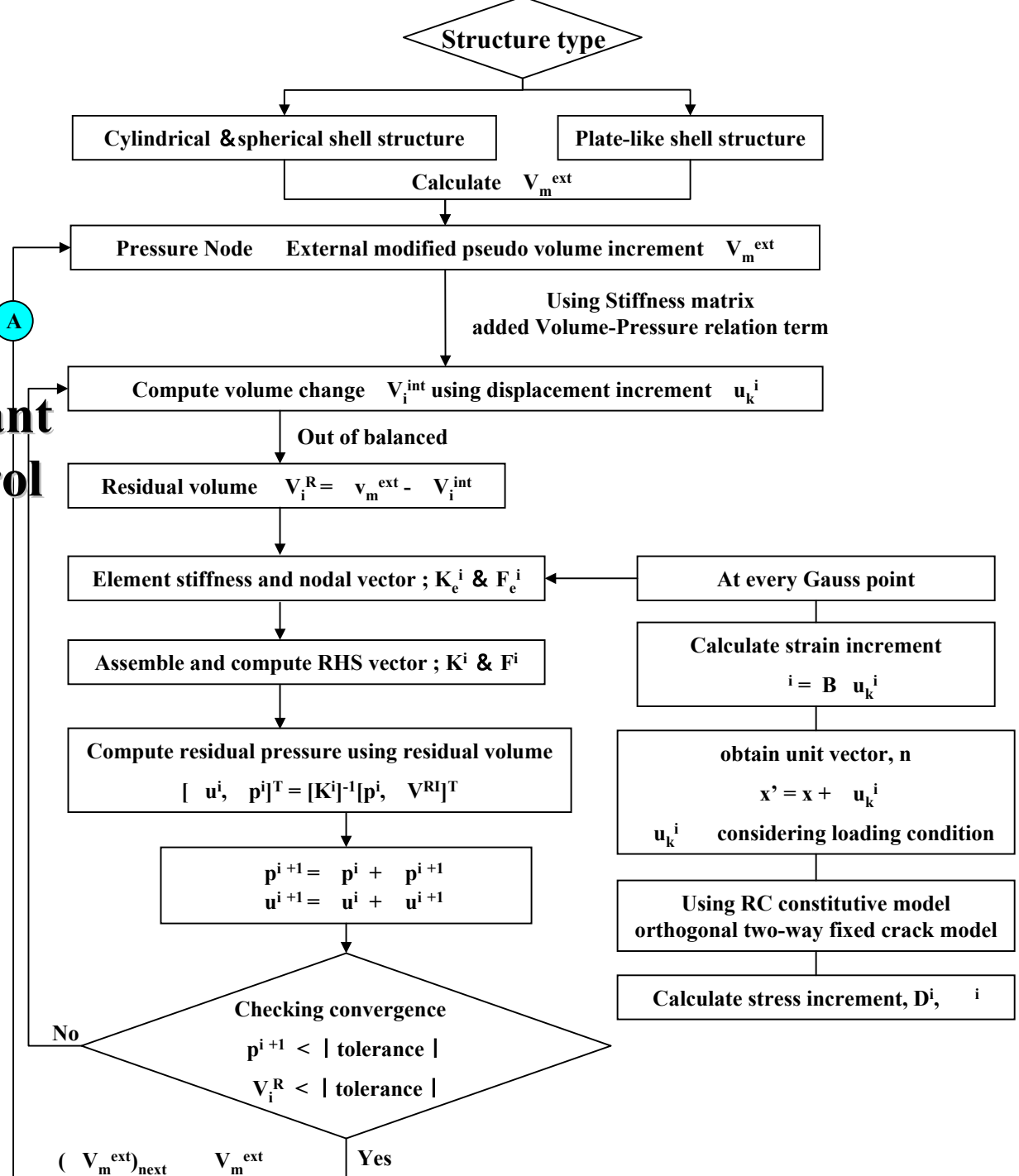


- Path-dependent Volume ( Song et. al, Nuclear Eng. and Design, 2003 )

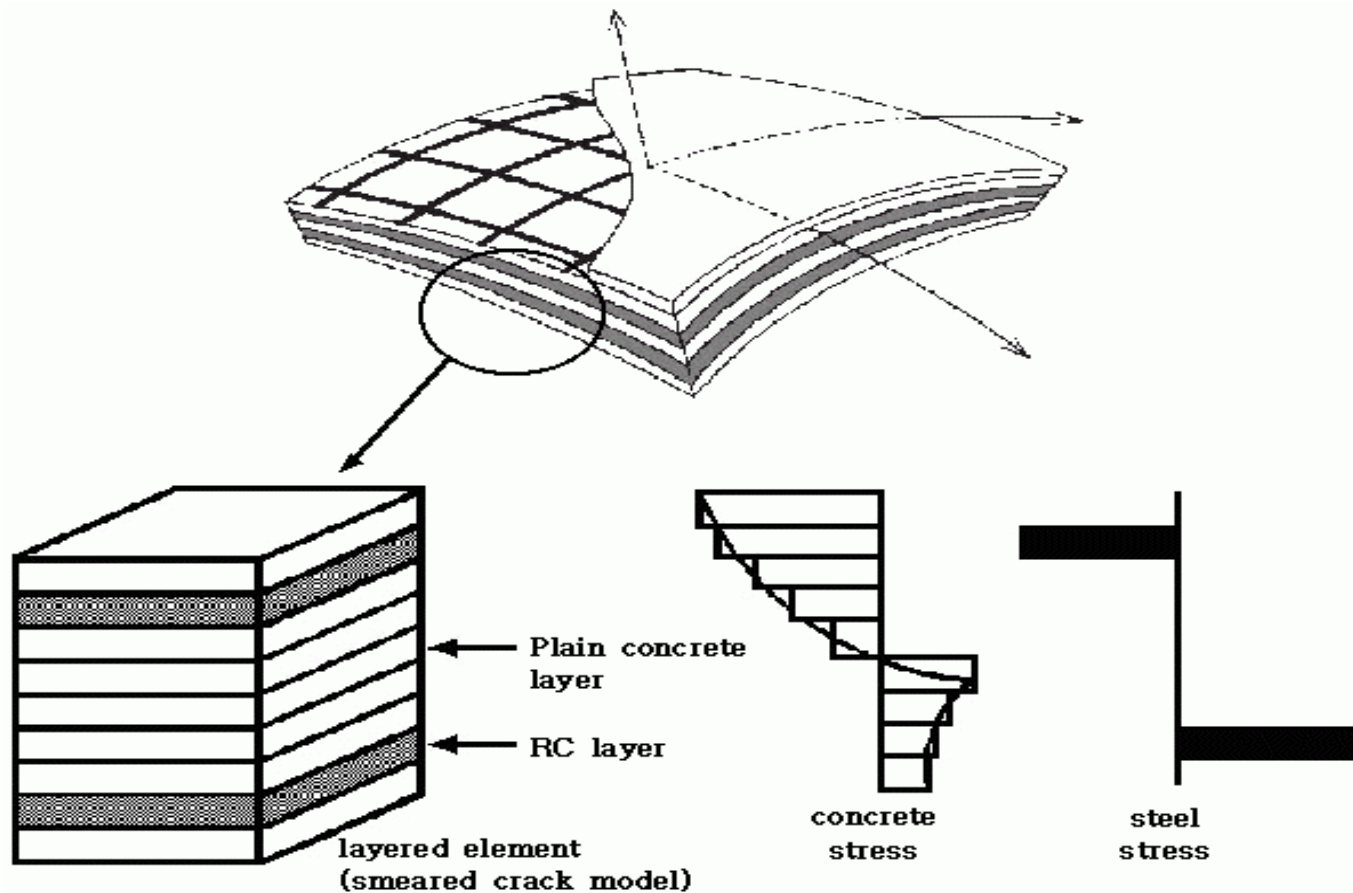
# Algorithm for path dependant volume control technique



**Solution  
procedure  
path-dependant  
volume control  
method**

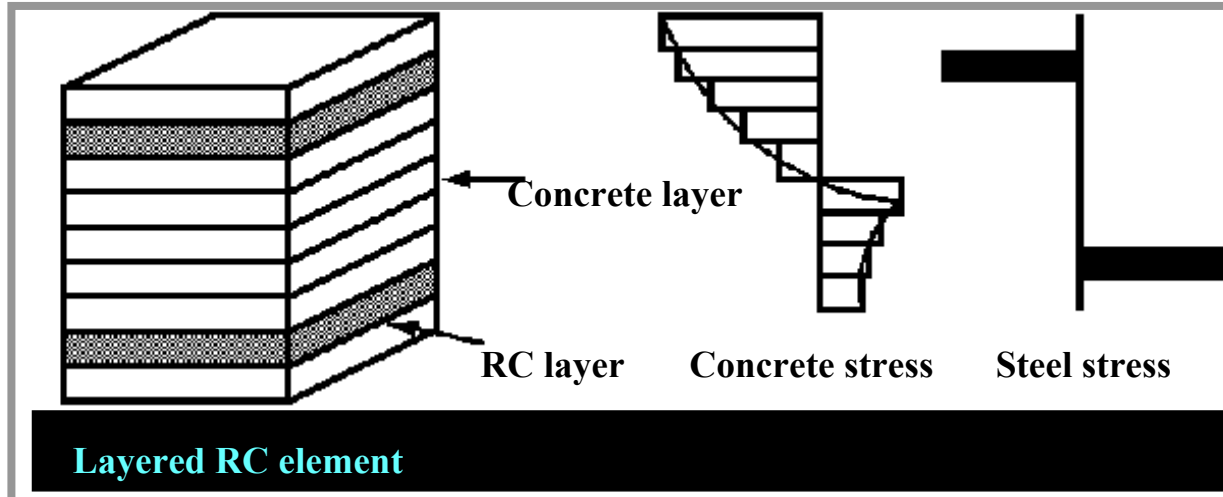


# Layered shell element



- Degenerated, isoparametric, serendipity, quadratic shell element with drilling degree of freedom
- Geometrical nonlinearity is considered by adopting total Lagrangian formulation
- In plane constitutive laws applied to each layer of element consists of RC layers and PL layers

# Constitutive law for each layer



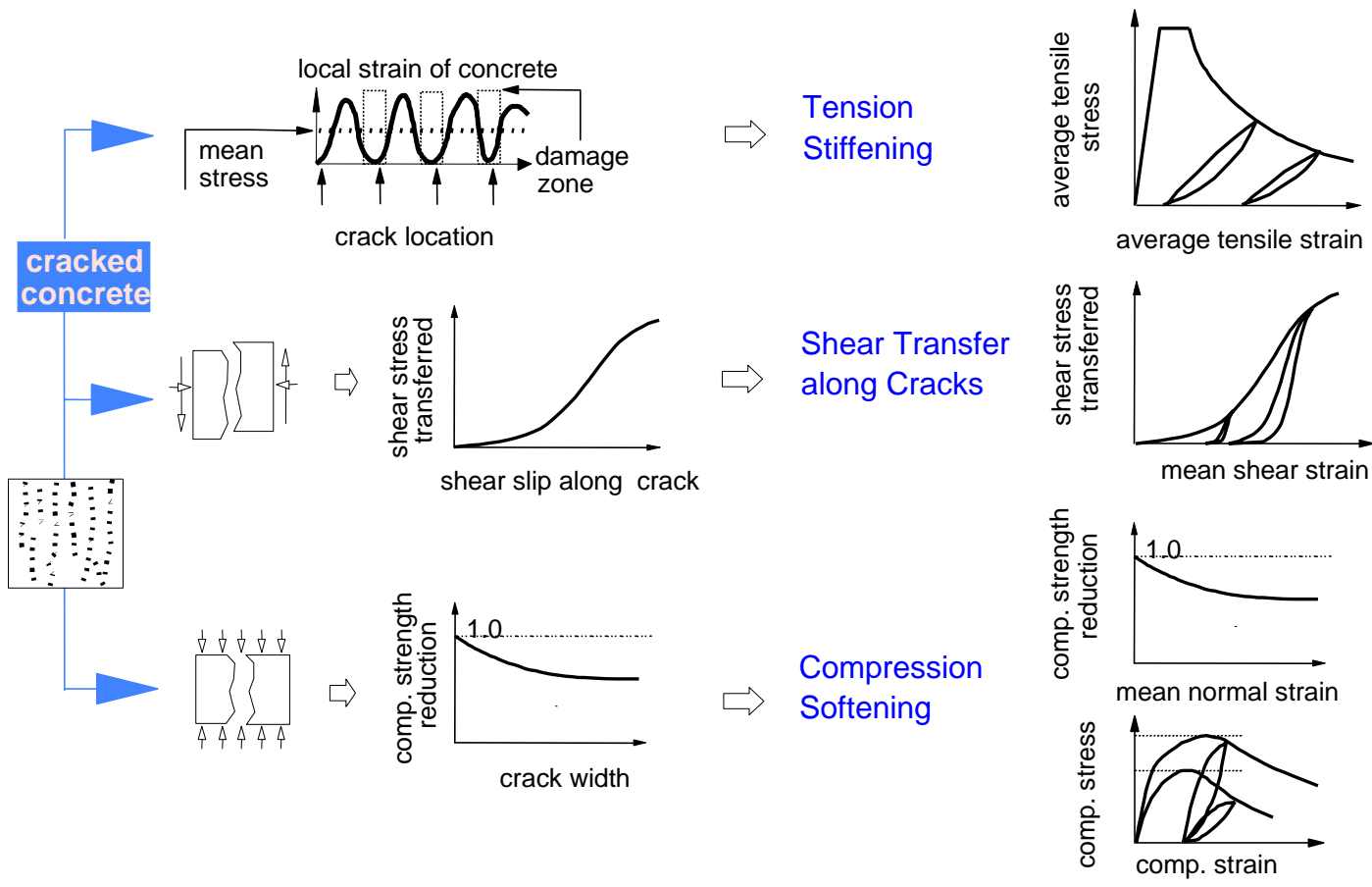
- Concrete under compression : Elasto-plastic fracture model (Maekawa et. al)
- Cracked concrete : Smearred fixed crack model
- Concrete under shear : Crack density model (Maekawa and Li)

**Shear locking + Membrane locking**



**Reduced integration ( 2 x 2 Gaussian quadrature )**

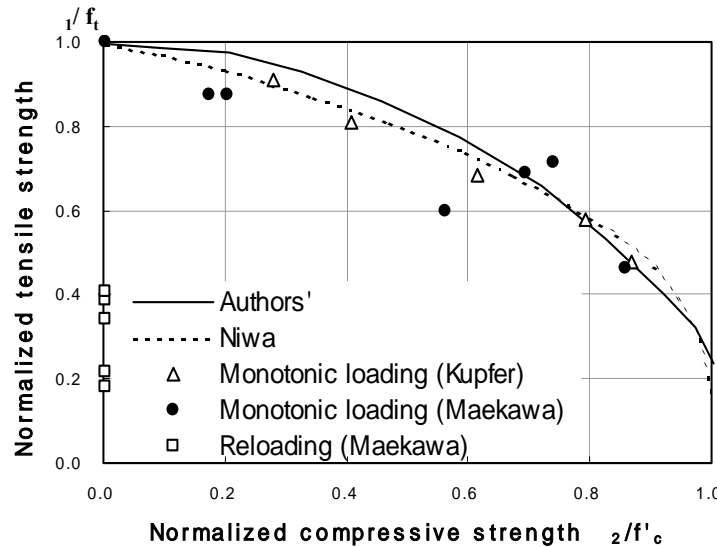
# In-plane constitutive models of cracked concrete



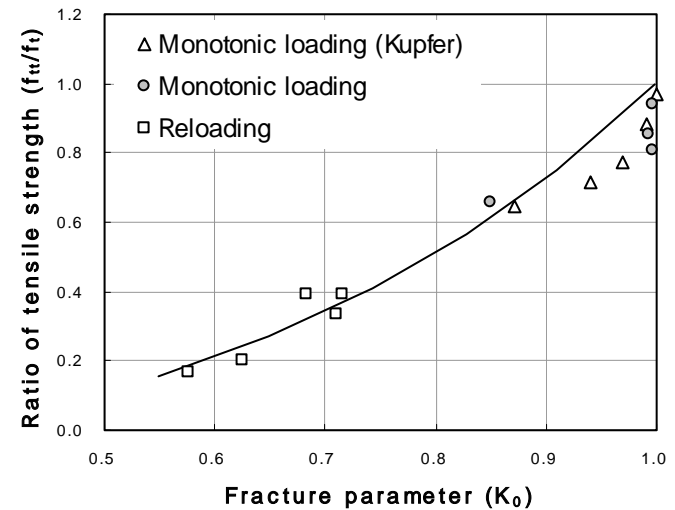
**MULTI-SYSTEM**    **LOCAL RESPONSE**    **MEAN RESPONSE**

# Cracking criteria of concrete

Cracking is affected by past loading history.



< Failure envelope in tension-compression domain >



< Normalized tensile strength and fracture parameter >

By taking account influence of **continuum fracture** in past compression, cracking criterion can be defined in the space of **biaxial principal stresses**

$$\frac{\sigma_1}{(R_f \cdot f_t)} = 1.0 \quad \text{Compression-tension domain}$$

$$\left( \frac{\sigma_1}{(R_f \cdot f_t)} \right) + 0.26 \left( \frac{\sigma_2}{\sigma_1} \right) = 1.0 \quad \text{Tension-tension domain}$$

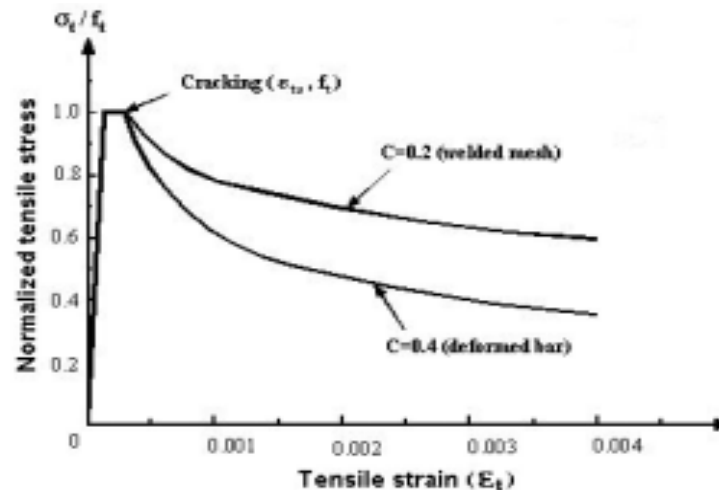
Where,  $\sigma_1, \sigma_2$ : principal stress ( $\sigma_1 > \sigma_2$ )  
 $f_t$ : uniaxial tensile strength  
 $R_f$ : tensile strength reduction factor



# Tension stiffening model

Concrete model under tensile stress is unrelated to spacing of cracks, direction of reinforcing bars and reinforcement ration.

Tension stiffening effect is known to increase overall stiffness of RC in tension compared with that of single reinforcing bar.



< Tension stiffening model for deformed bars ( $c=0.4$ ) and welded meshes ( $c=0.2$ ) >

$$\sigma_t = f_t \left( \frac{\epsilon - \epsilon_{tu}}{\epsilon_t} \right)^c$$

Where,  $\sigma_t$  : average tensile stress,  $\epsilon$  : average tensile strain

$f_t$  : uniaxial tensile strength,  $\epsilon_{tu}$  : cracking strain

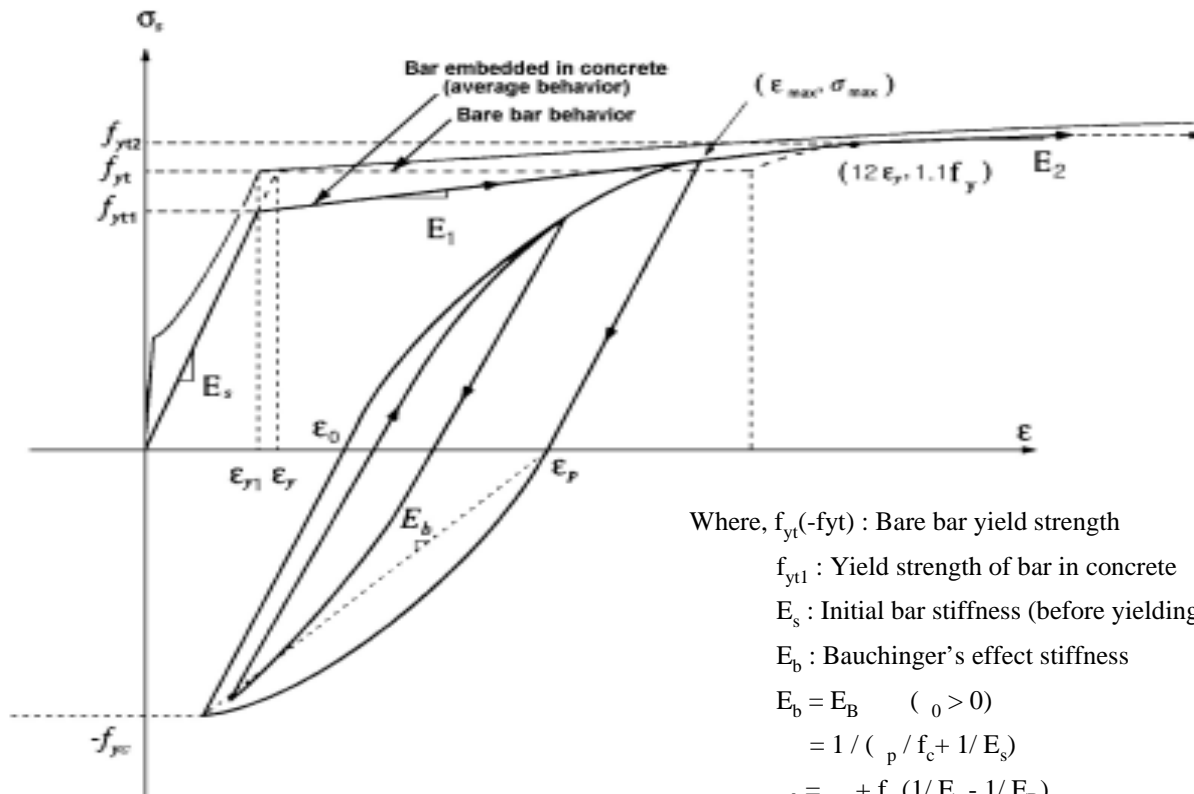
$c$  : stiffening parameter





# Steel model

Reinforcing bar model is based on the assumed cosine distribution of bar stress and concrete tension stiffening.



Where,  $f_{yt}$  (- $f_{yt}$ ) : Bare bar yield strength

$f_{yt1}$  : Yield strength of bar in concrete

$E_s$  : Initial bar stiffness (before yielding)

$E_b$  : Bauchinger's effect stiffness

$E_b = E_B \quad (\rho > 0)$

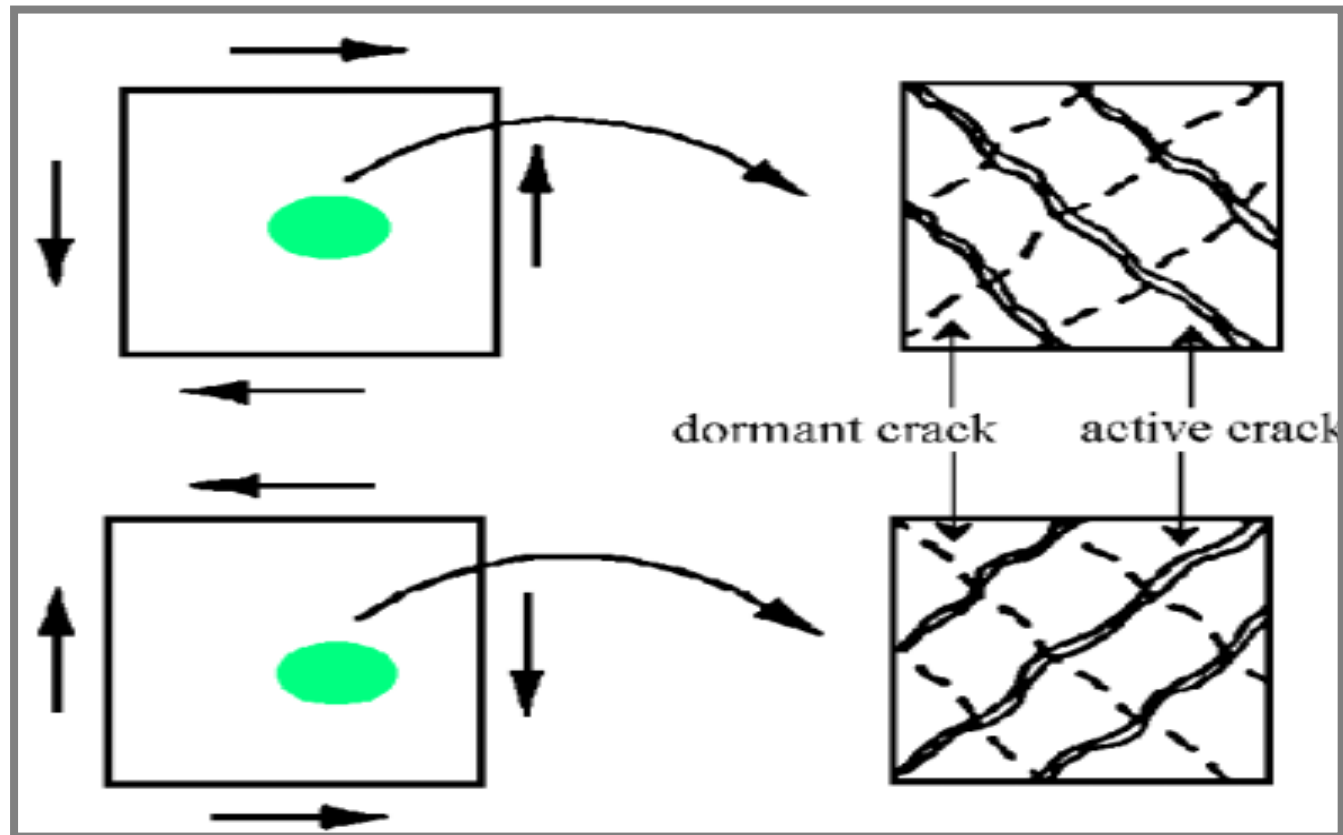
$= 1 / (\rho / f_c + 1 / E_s)$

$\rho = \rho_p + f_c (1 / E_s - 1 / E_B)$

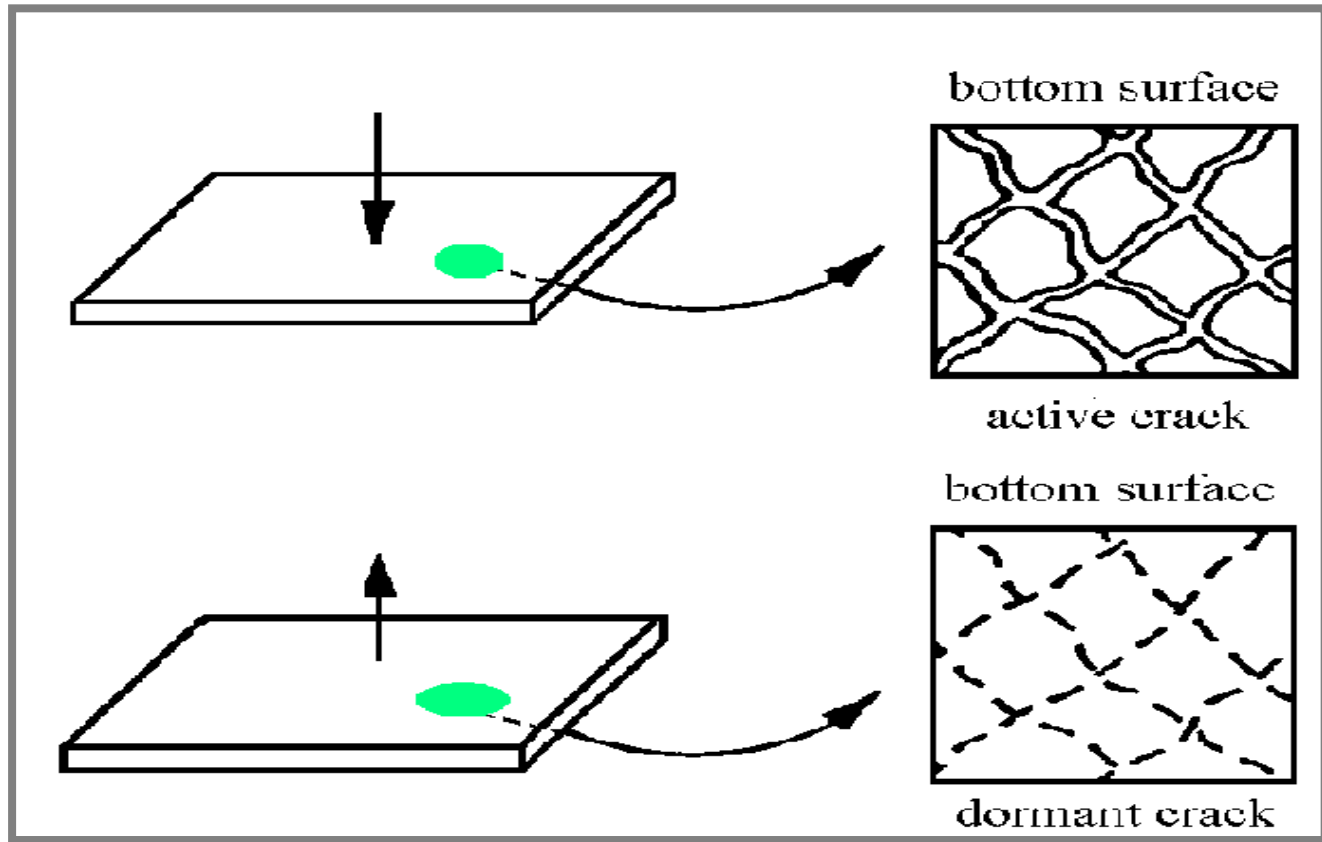
$E_B = -E_s \log_{10}(10 - \rho) / 6$



# Multi-directional smeared crack approach

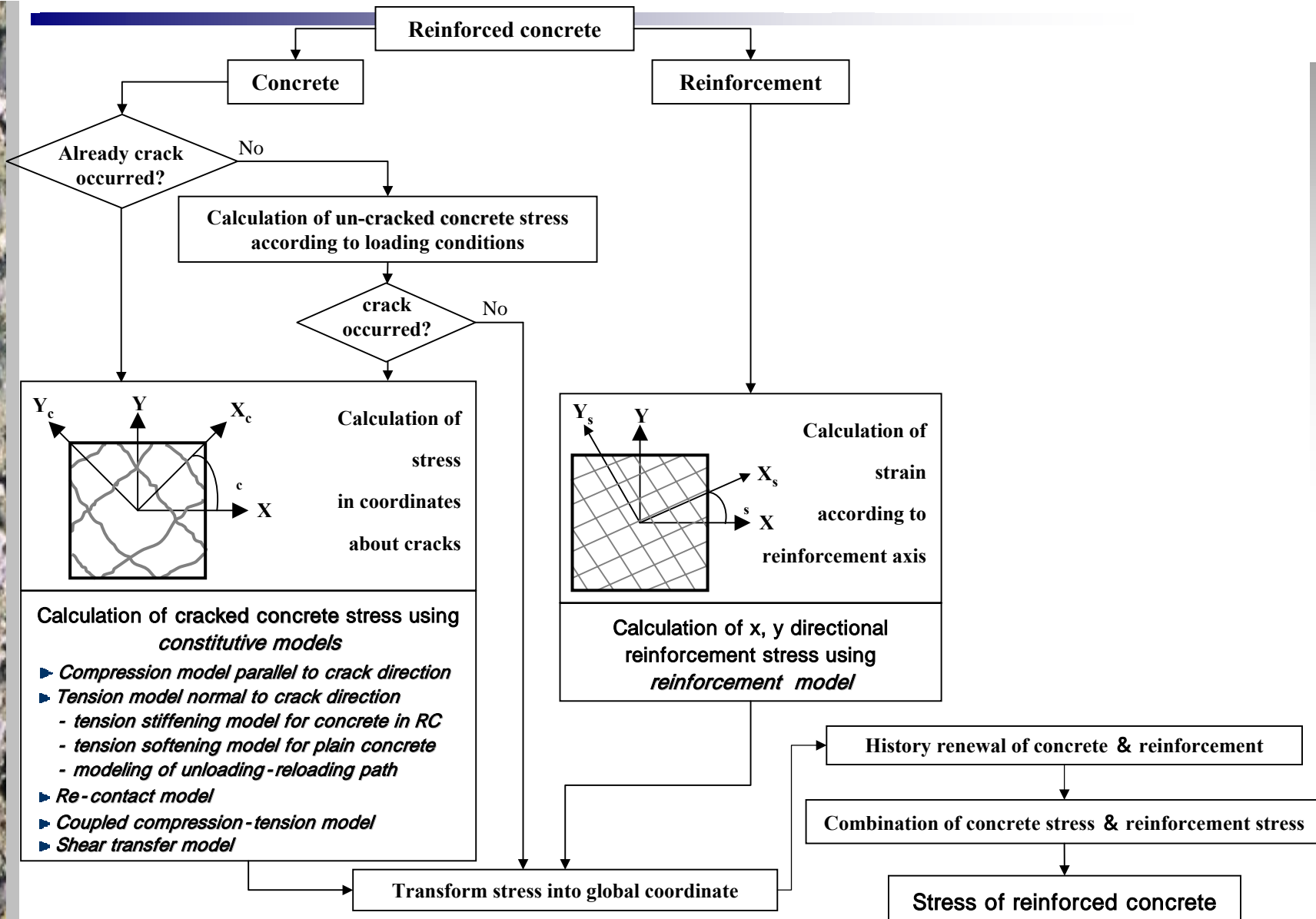


**One-way active crack governs the overall nonlinearity as for in-plane cyclic shear**



**Two-way active cracks may control the overall nonlinearity as for out of plane cyclic action**

# Orthogonal two-way fixed crack model

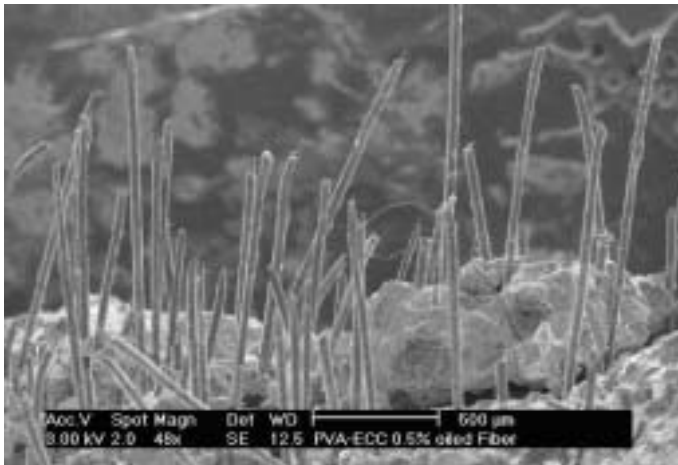


# ECC as durable overlays and repair layers

## Engineered\_Cementitious Composites (ECC)

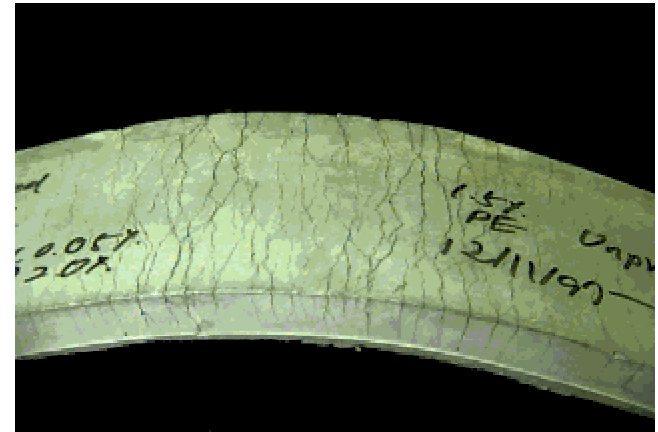
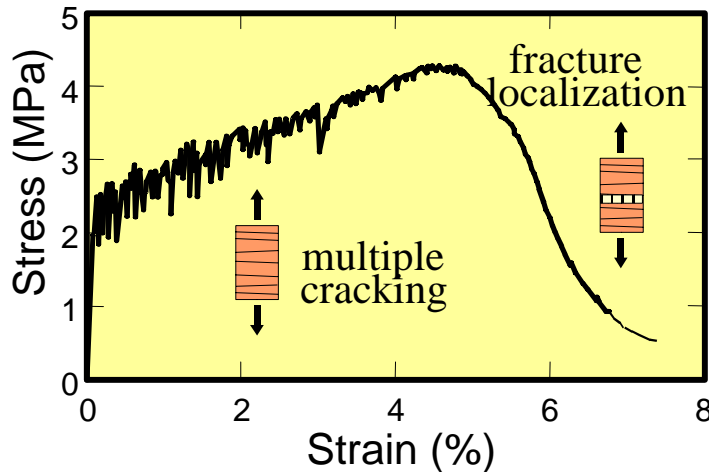
V.C. Li et al. 1992~

- cementitious matrix + short random fibers
- conscious micromechanics-based design of material composition ... **Performance Driven Design Approach**
- high performance with low fiber content (~2%)



# High performance cementitious composites

- cementitious matrix + fibers
- multiple cracking



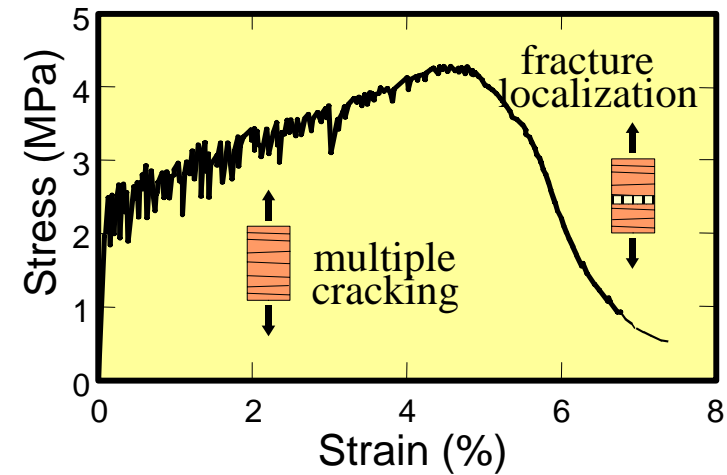
- high overall ductility in tension and shear  
with ease of processing and variability of shaping

- damage tolerance, durability, ...

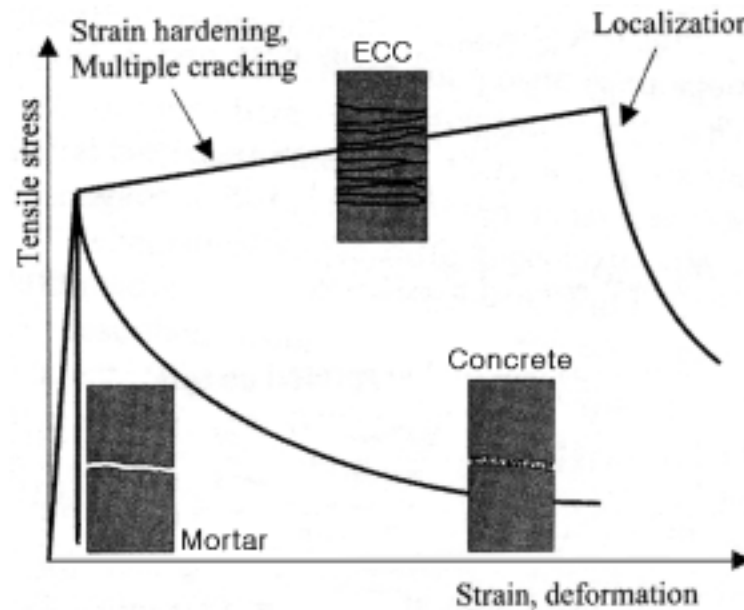
# Characteristic of ECC behavior

## mechanical properties:

- high tensile strain capacity (~5%)
- small crack width  $O(10\sim 100\ \mu\text{m})$

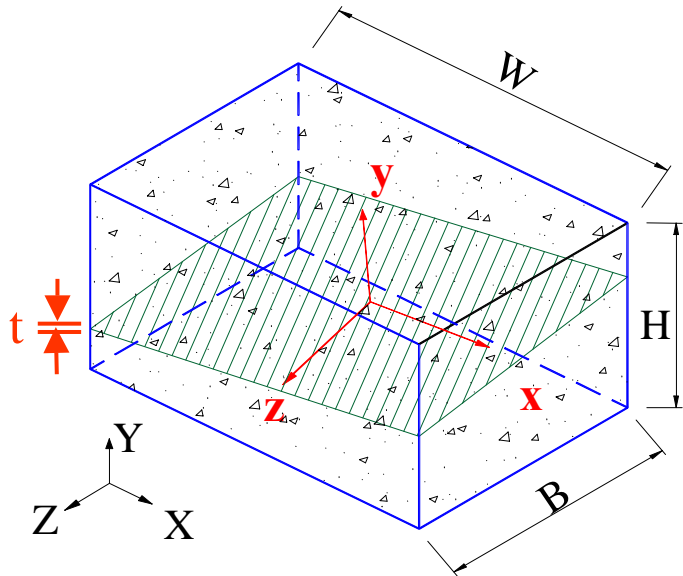


- Strain hardening
- Multiple cracking
- Localized failure



# 3-d homogenized crack model

- 3-D formulation of homogenized crack model



Representative elementary volume (REV)

$\sigma_i, \varepsilon_i$ : concrete

$\sigma_j, \varepsilon_j$ : crack

$\sigma, \varepsilon$ : concrete with crack

- Mixture rule (R.E.V)

$$\sigma = \mu_i \sigma_i + \mu_j \sigma_j \quad \varepsilon = \mu_i \varepsilon_i + \mu_j \varepsilon_j$$

$$\mu_i + \mu_j = 1$$

- Equilibrium & compatibility

$$\begin{aligned} \sigma &= \sigma_i = \sigma_j & \varepsilon &= \varepsilon_i = \varepsilon_j \\ \tau_{xy} &= \tau_{xy}^i = \tau_{xy}^j & \varepsilon_{xx} &= \varepsilon_{xx}^i = \varepsilon_{xx}^j \\ \tau_{yz} &= \tau_{yz}^i = \tau_{yz}^j & \gamma_{zx} &= \gamma_{zx}^i = \gamma_{zx}^j \end{aligned}$$

- Velocity discontinuity at crack surface

$$\mathbf{g} = \{g_y, g_x, g_z\}^T \quad [\delta] \sigma^i = [K] \mathbf{g}$$

$$[\delta] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \quad [K] = [K^e] = \begin{bmatrix} K_N & 0 & 0 \\ 0 & K_{S1} & 0 \\ 0 & 0 & K_{S2} \end{bmatrix}$$







If  $t \ll H$ ,

$$\mu_i = \frac{BW(H - t)}{HBW} \cong 1$$

$$\mu_j = \frac{BWt}{HBW} \cong \frac{t}{H}$$

$$\underbrace{\frac{1}{t} \langle \mathbf{g} \rangle [\delta] \langle \mathbf{i} \rangle}_{\text{averaged crack strain}}$$

averaged crack strain

Let  $\mu$  be the ratio of the crack area and REV., i.e. (  $\mu := \frac{1}{H}$  )

Then,  $\langle \mathbf{\varepsilon} \rangle = \mu_i \langle \mathbf{\varepsilon}_i \rangle + \mu_j \langle \mathbf{\varepsilon}_j \rangle$  can be written as

$$[\delta] \langle \mathbf{\varepsilon} \rangle \mu_i [\delta] \langle \mathbf{\varepsilon}_i \rangle + \mu_j [\delta] \langle \mathbf{\varepsilon}_j \rangle$$

$$= 1 \cdot [\delta] \langle \mathbf{\varepsilon} \rangle + \frac{t}{H} \cdot \frac{1}{t} \langle \mathbf{g} \rangle$$

$$= [\delta] \langle \mathbf{\varepsilon} \rangle + \mu \langle \mathbf{g} \rangle$$



• Structural relationship

$$\sigma_i = [D] \varepsilon_i \quad \sigma_i = \mu_i \sigma_i + \mu_j \sigma_j$$

$$[\delta] \varepsilon_i \approx [\delta] \varepsilon_i + \mu g$$

$$[\delta] \varepsilon_i = [A] \varepsilon_i + [B] g$$

$$[A] = \begin{bmatrix} \frac{-D_{21}}{C_1} & \frac{-K_{11}}{\mu C_1} & \frac{-D_{23}}{C_1} & \frac{-D_{24}}{C_1} & \frac{-D_{25}}{C_1} & \frac{-D_{26}}{C_1} \\ \frac{-D_{41}}{C_2} & \frac{-D_{42}}{C_2} & \frac{-D_{43}}{C_2} & \frac{-K_{22}}{\mu C_2} & \frac{-D_{45}}{C_2} & \frac{-D_{46}}{C_2} \\ \frac{-D_{51}}{C_3} & \frac{-D_{52}}{C_3} & \frac{-D_{53}}{C_3} & \frac{-D_{54}}{C_3} & \frac{-K_{33}}{\mu C_3} & \frac{-D_{56}}{C_3} \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & \frac{K_{12} + \mu D_{24}}{C_1} & \frac{K_{13} + \mu D_{25}}{C_1} \\ \frac{K_{21} + \mu D_{42}}{C_2} & 0 & \frac{K_{23} + \mu D_{45}}{C_2} \\ \frac{K_{31} + \mu D_{52}}{C_3} & \frac{K_{32} + \mu D_{54}}{C_3} & 0 \end{bmatrix} \quad \begin{aligned} C_1 &= D_{22} + \frac{K_{11}}{\mu} \\ C_2 &= D_{44} + \frac{K_{22}}{\mu} \\ C_3 &= D_{55} + \frac{K_{33}}{\mu} \end{aligned}$$



- Structural relationship of (tensile) crack

$$[\delta] \varepsilon = [\delta] \varepsilon + \mu g$$

$$\varepsilon = [S_1] \varepsilon \quad \text{or} \quad [\delta] \varepsilon = [S] \varepsilon$$

i.e.,  $[\delta] \varepsilon = [S] \varepsilon + \mu g$

$$\therefore \mu g = ([\delta] - [S]) \varepsilon$$

$$g = \frac{1}{\mu} ([\delta] - [S]) \varepsilon$$

Let  $[S_2] = \frac{1}{\mu} ([\delta] - [S])$

then  $g = [S_2] \varepsilon$



- Total strain relationship

$$[\delta] \varepsilon^i = [S] \varepsilon^e$$

$$[S] = \left( [I] + \frac{1}{\mu} [B] \right)^{-1} \left( [A] + \frac{1}{\mu} [B][\delta] \right)$$

$$\varepsilon^i = [S_1] \varepsilon^e$$

$$[S_1] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Homogenized constitutive equation

$$\sigma = \mu_i \sigma_i + \mu_i \sigma_i$$

$$\approx \sigma_i$$

$$= [D][S_1] \varepsilon$$

$$\sigma = [D^{eq}] \varepsilon$$

$$[D^{eq}] = [D][S_1]$$

### Remark

Crack width,  $t$ , is removed in the final constitutive equation only expressed with  $\mu$ . This is a solution for the mesh sensitivity problem without the introduction of additional length scale such as a characteristic length.

**Regularization of the continuum model**

## ■ Constitutive equation for crack

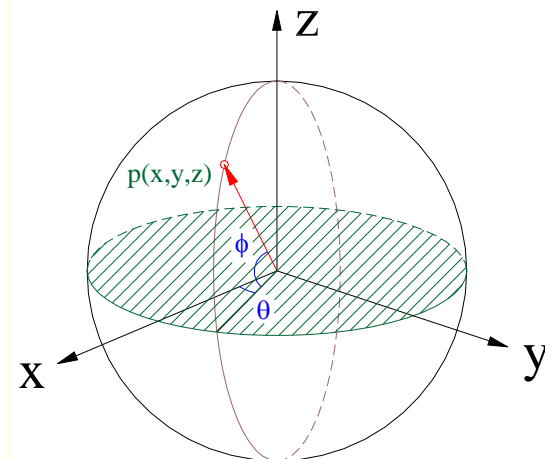
$$\begin{aligned}
 \mathbf{t} &= \{ t_n^{cr}, t_{S_1}^{cr}, t_{S_2}^{cr} \}^T \\
 &\quad \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \sigma_{yy} & \tau_{xy} & \tau_{yz} \end{array} \\
 &= [\delta] \boldsymbol{\sigma} = [\mathbf{K}] \mathbf{g}
 \end{aligned}$$

### ➤ Compression

- Bifurcation analysis for crack initiation

$$F(\mathbf{n}) = \det(n_i D^{eq}_{ijkl} n_l)$$

$$[\mathbf{K}] = \begin{bmatrix} K_N^{ep} & 0 & 0 \\ 0 & K_{S_1}^{ep} & 0 \\ 0 & 0 & K_{S_2}^{ep} \end{bmatrix}$$



# Failure criteria and softening curve (compression)

## ➤ Drucker-Prager type

$$F = \alpha I_1 + \sqrt{J_2} - k(\bar{\sigma}, \bar{\varepsilon}^p) = 0$$

## ➤ Hardening and softening function

### 1) Song and Na (1997)

$$k(\bar{\varepsilon}^p, \bar{\sigma}) = \sigma_0 + \bar{\sigma} \bar{\varepsilon}^p + (\sigma_\infty - \sigma_0)[1 - e^{-\beta \bar{\varepsilon}}] - \sqrt{\frac{3}{2}} \alpha p$$

### 2) Song et al (2003), Farahat et al.(1995)

$$k(W^p) = k_0 e^{-[(\beta W^p)^\gamma - \xi]^2}$$

### 3) Barcelona model

#### ■ $k(\varepsilon^p)$ (J. Lubliner, 1996)

$$k(\varepsilon^p) = f_{N0} [(1 + a_N) \exp(-b_N \varepsilon^p) - a_N \exp(-2b_N \varepsilon^p)]$$

#### ■ $k(W^p)$ (Modified Barcelona Model, MBM)

$$k(W^p) = f_{N0} [(1 + a_N) \exp(-b_N W^p) - a_N \exp(-2b_N W^p)]$$

# Failure criteria and softening function (tension)

## ➤ Failure criterial (Gopalaratnam and Shah, 1985)

$$F = \sigma_1 - k(\cdot)$$

## ➤ Hardening and softening function

### 1) Gopalaratnam and Shah (1985)

$$k(t) = f_t (e^{-\kappa t^\lambda})$$

### 2) Song et al. (2003)

$$k(\underbrace{\varepsilon^p}_y) = f_t (e^{-\kappa (\eta \underbrace{\varepsilon^p}_y)^\lambda})$$

### 3) Barcelona model

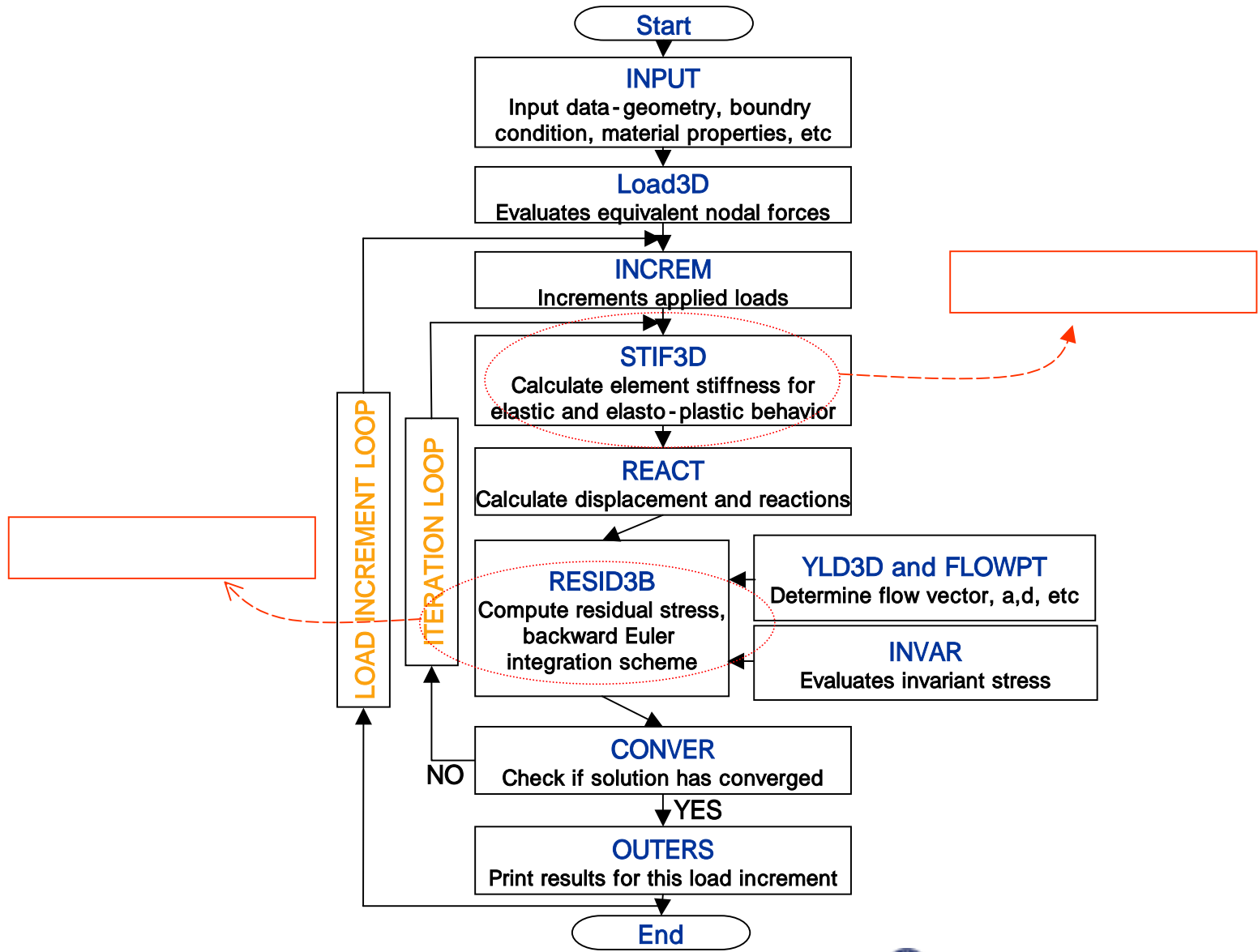
- $k(\varepsilon^p)$  (J. Lubliner, 1996)

$$k(\varepsilon^p) = f_{t0} [(1 + a_t) \exp(-b_t \varepsilon^p) - a_t \exp(-2b_t \varepsilon^p)]$$

- $k(\underbrace{\varepsilon^p}_y)$  (MBM)

$$k(\underbrace{\varepsilon^p}_y) = f_{t0} [(1 + a_t) \exp(-b'_t \underbrace{\varepsilon^p}_y) - a_t \exp(-2b'_t \underbrace{\varepsilon^p}_y)]$$





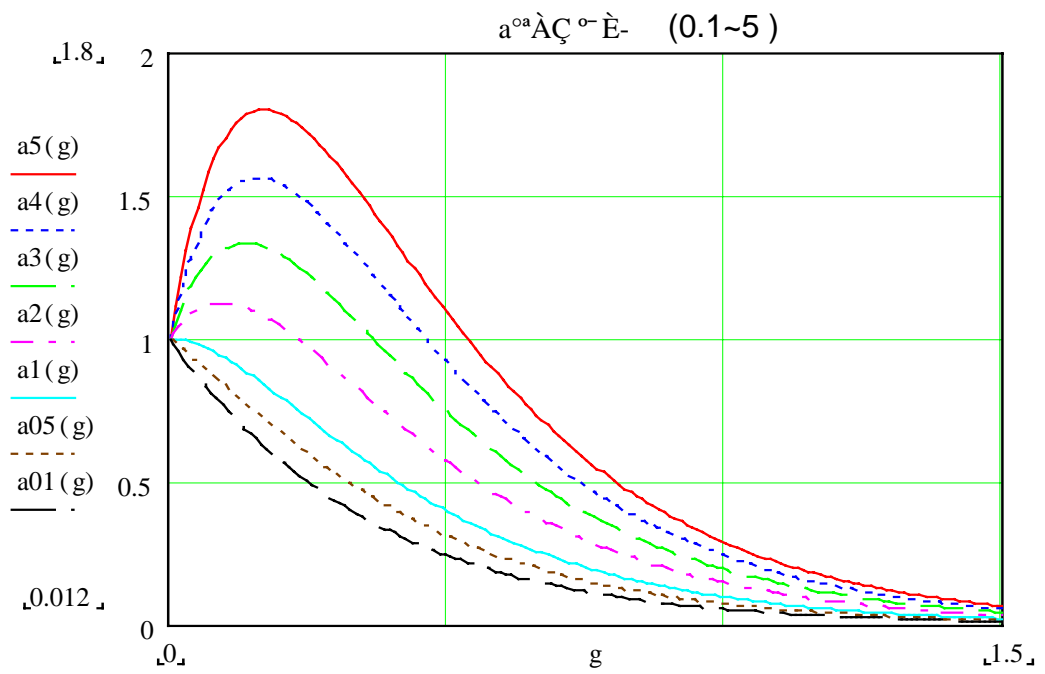
# Hardening and softening curve for concrete and ECC (tension)

## ➤ J. Lubliner (1996)

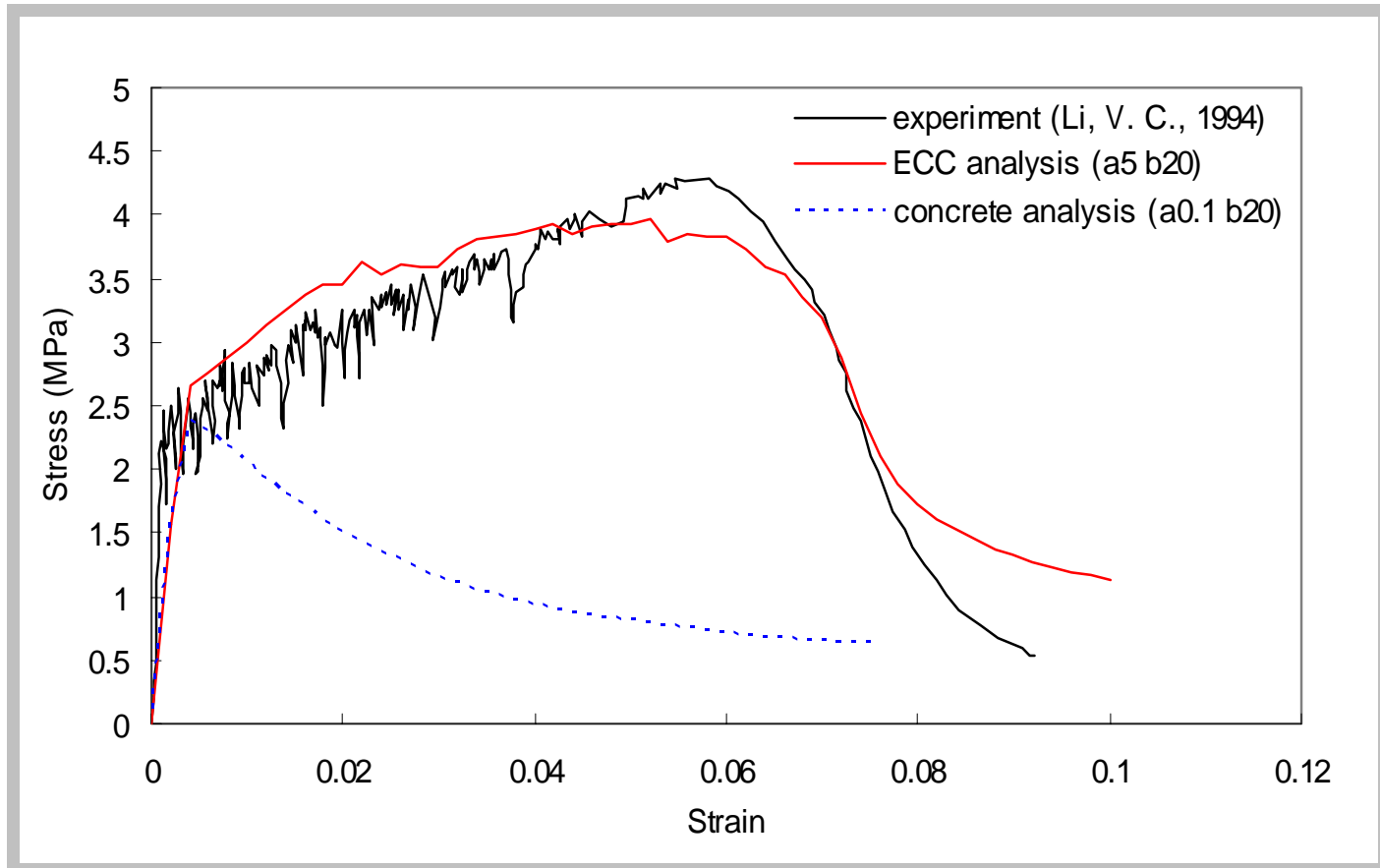
$$k\left(\frac{\sigma}{f_t}\right) = f_{t0} [(1 + a_t) \exp(-b_t' \frac{\sigma}{f_t}) - a_t \exp(-2b_t' \frac{\sigma}{f_t})]$$

$a_t < 1$      $\implies$     Concrete

$a_t > 1$      $\implies$     ECC



➤ **Result for 2% polyethylene fiber contained ECC**

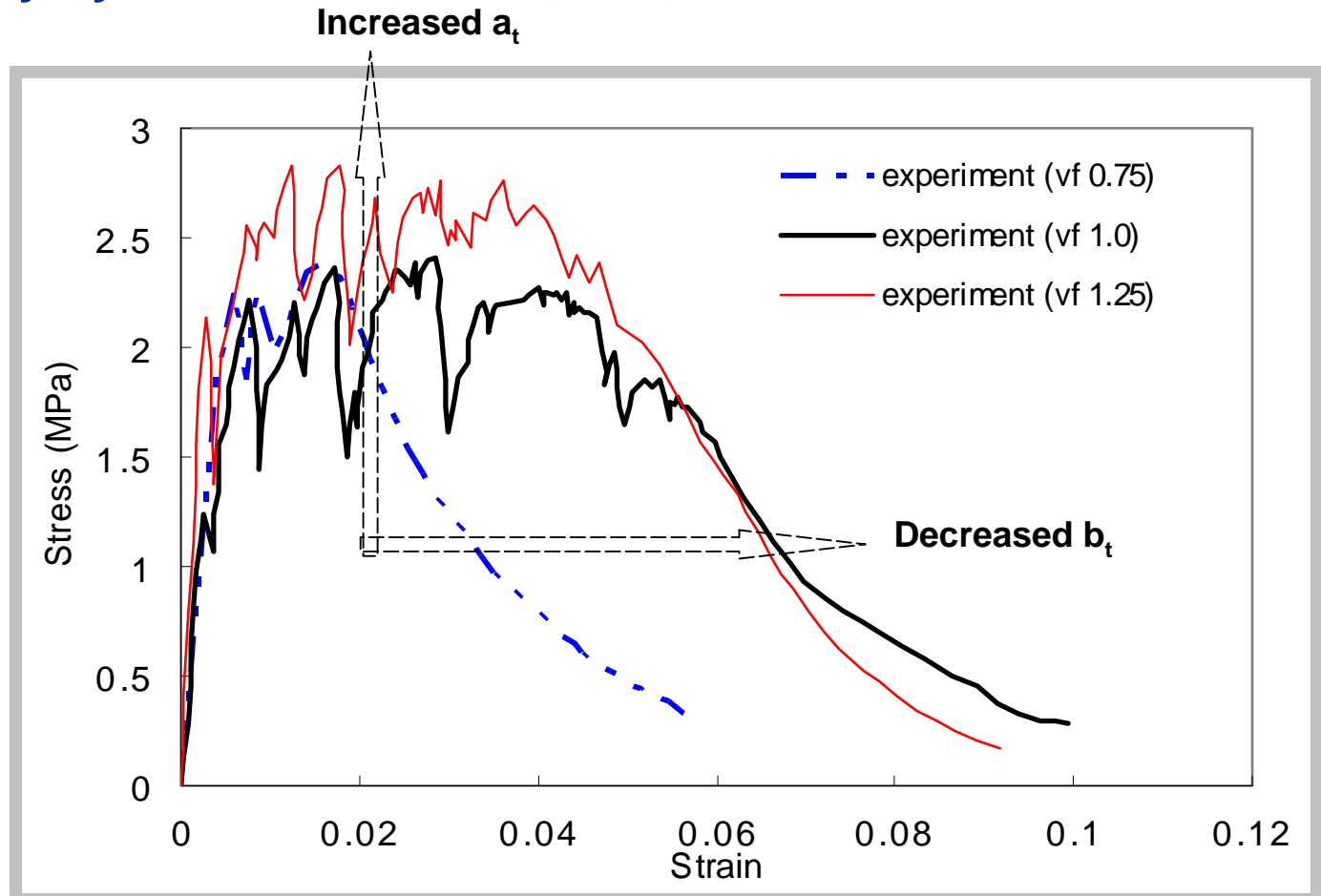


$a_t = 0.1$  : Concrete

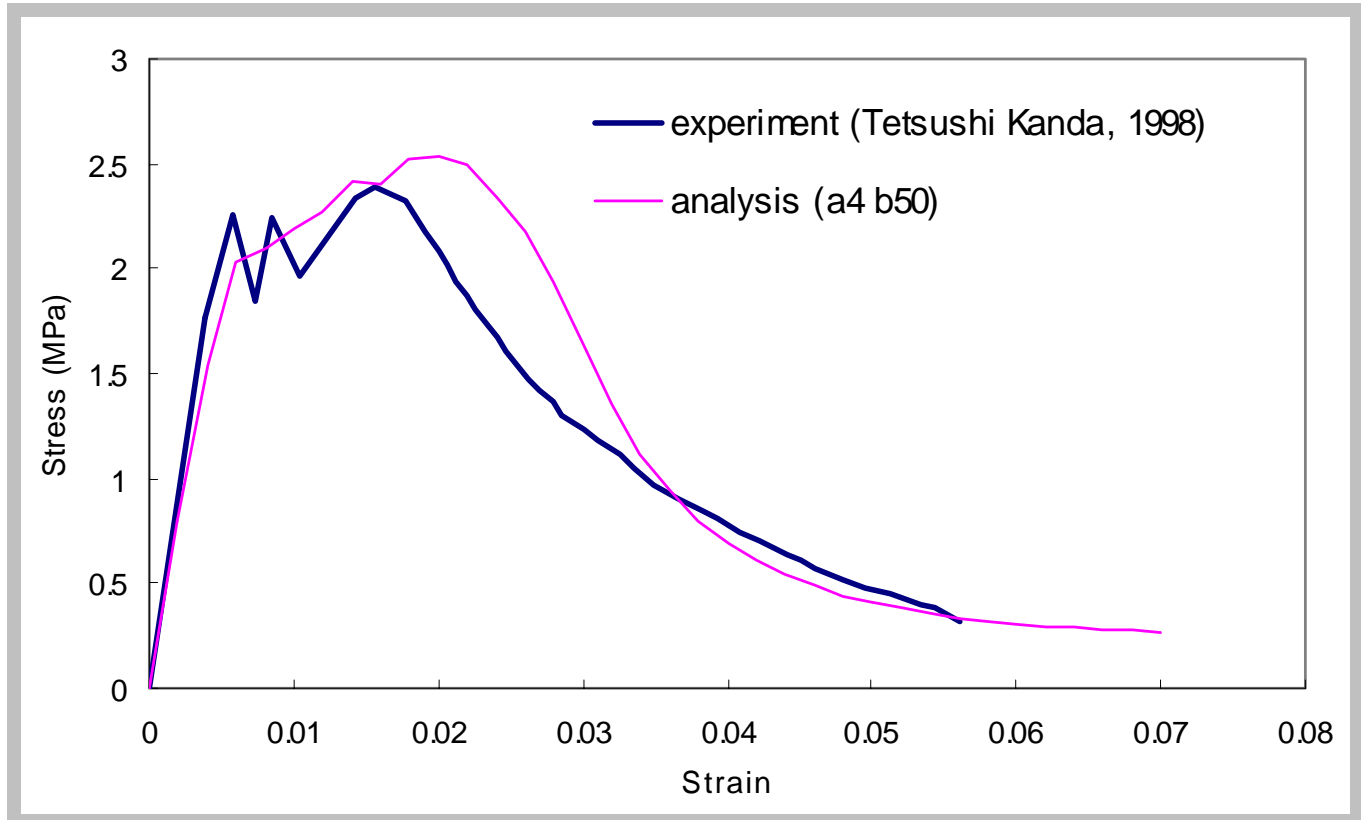
$a_t = 5$  : ECC



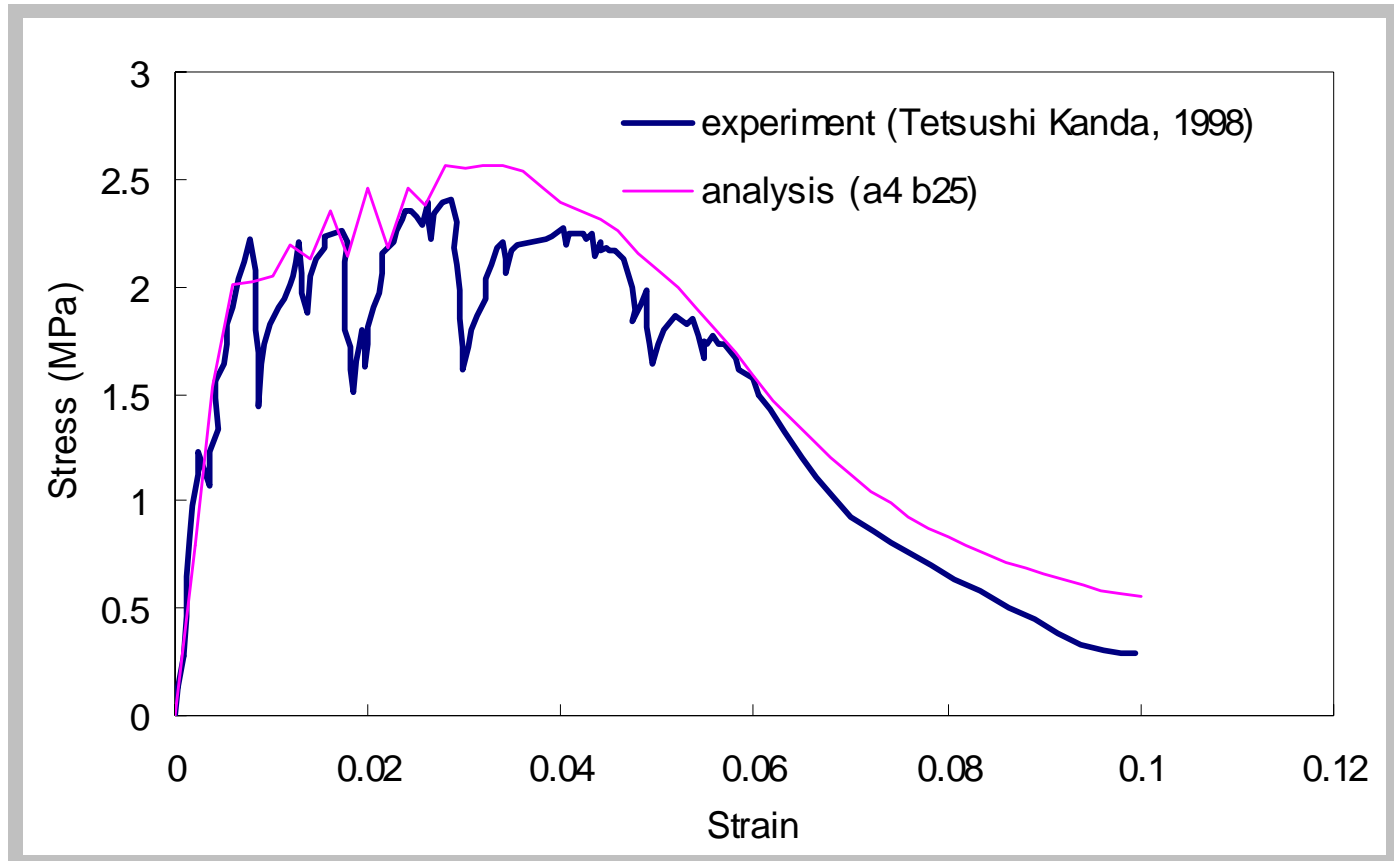
- Experimental result (Tetsushi Kanda, 1998)
- polyethylene fiber : 0.75% , 1% , 1.25%



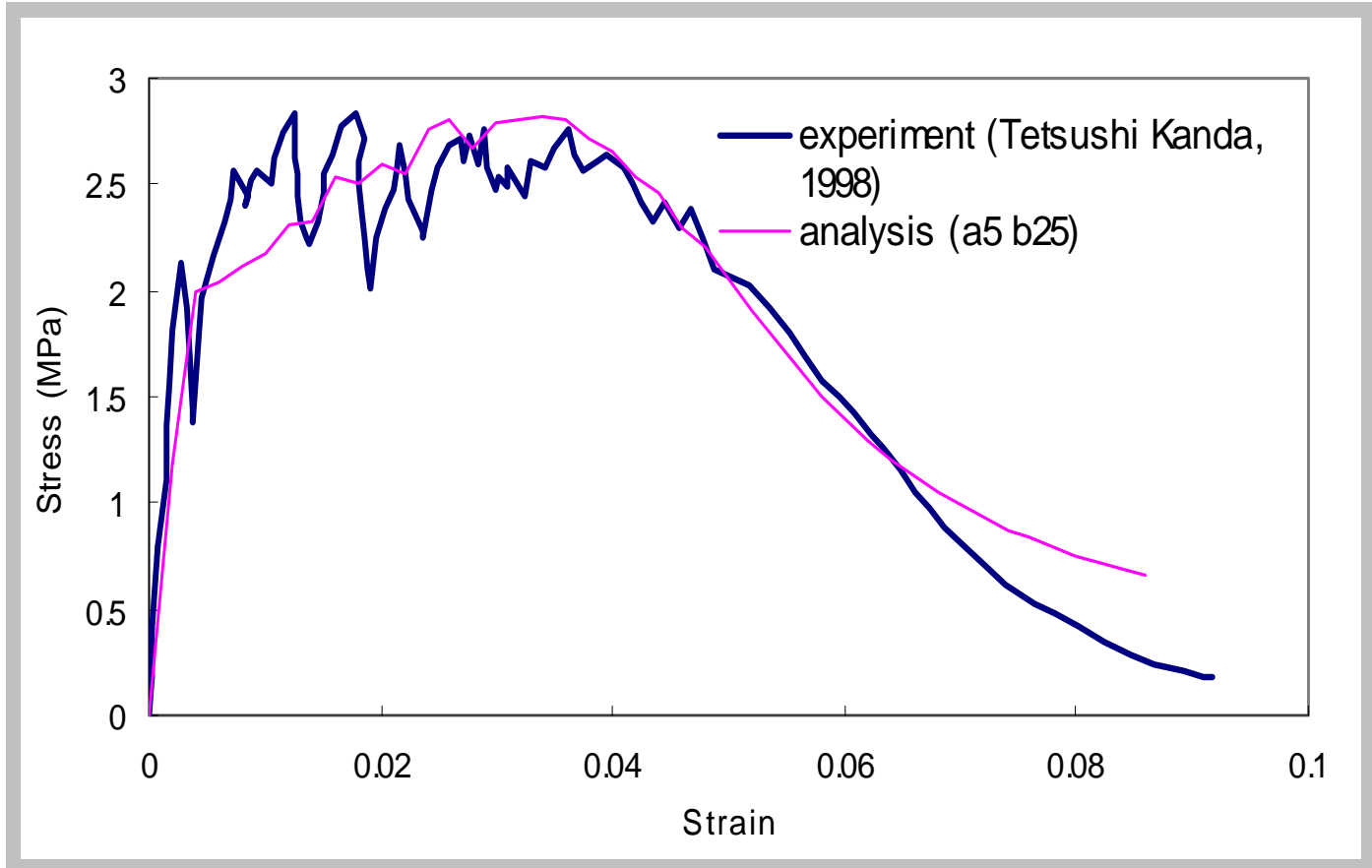
■ 0.75% ECC



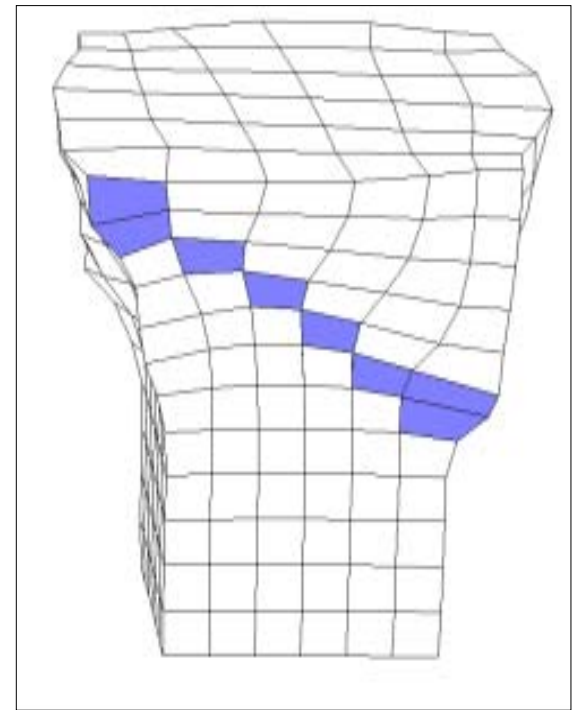
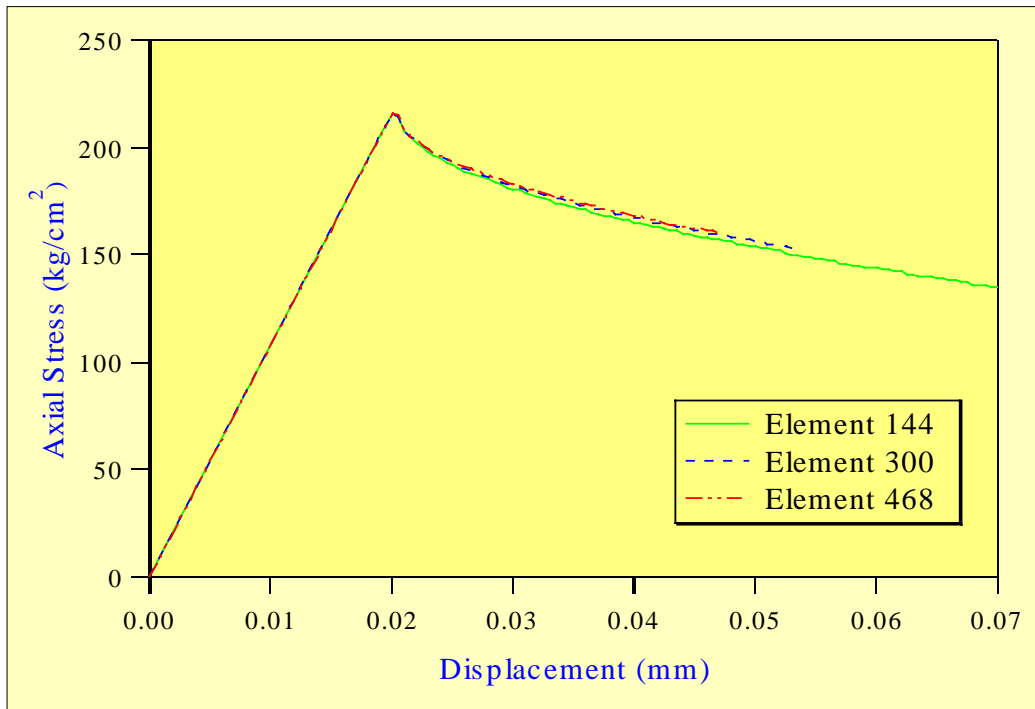
■ 1% ECC



■ 1.25% ECC

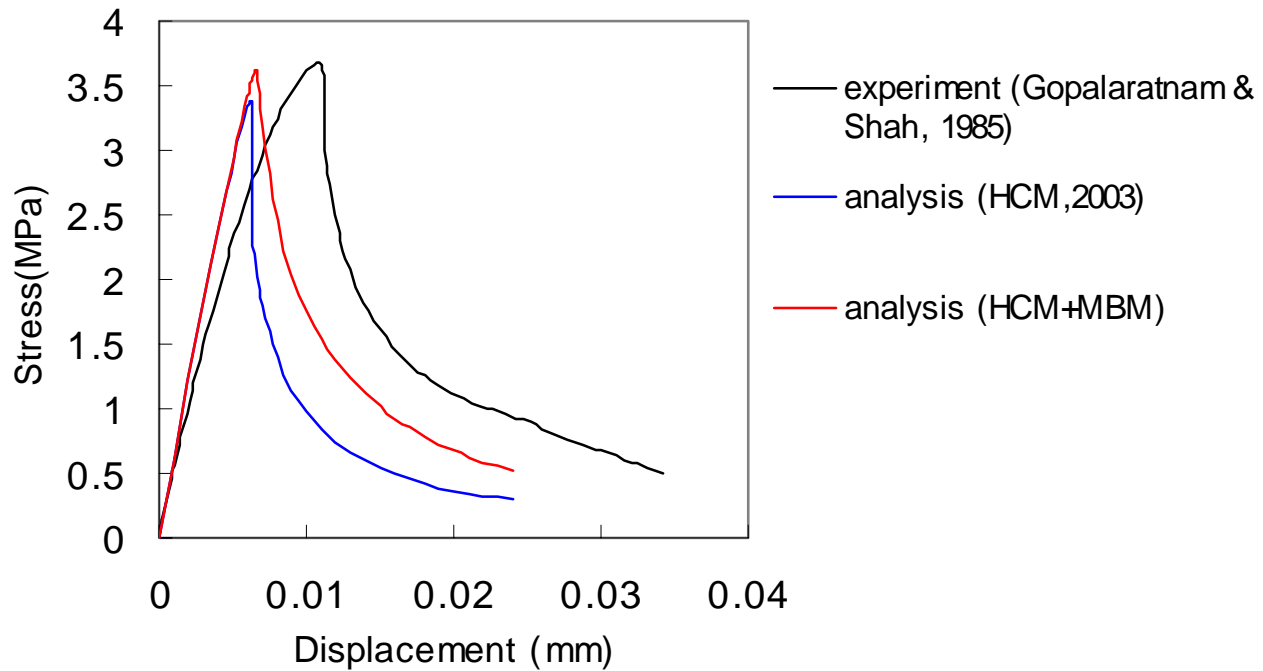


➤ Mesh sensitivity check on softening behavior

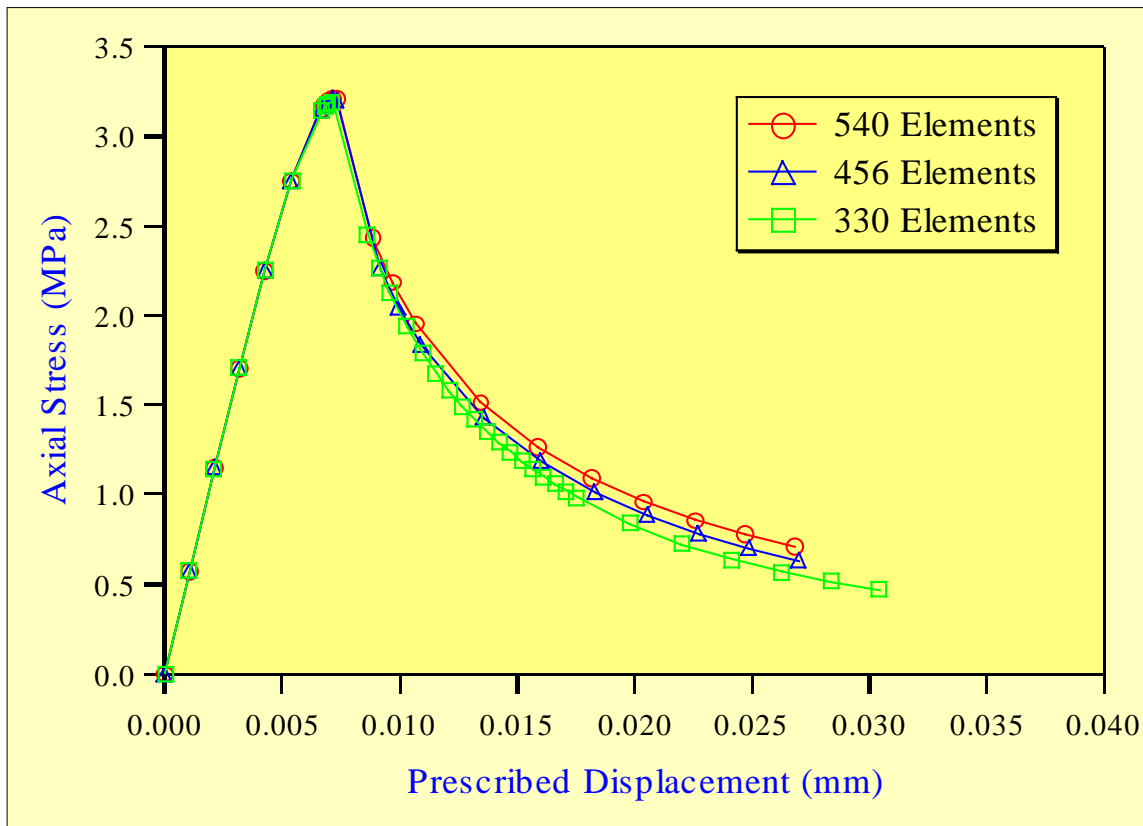




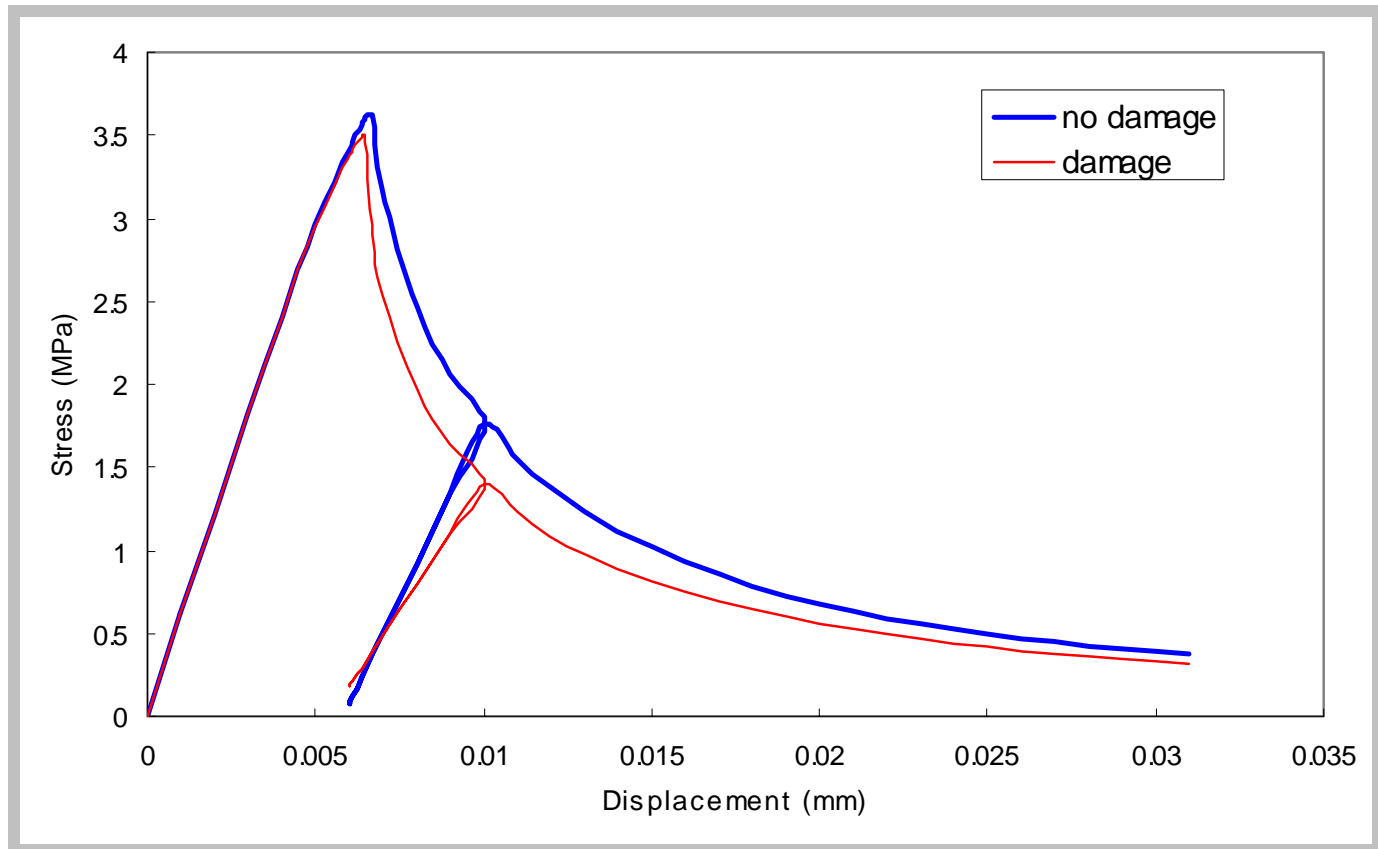
➤ Comparison with experimental result



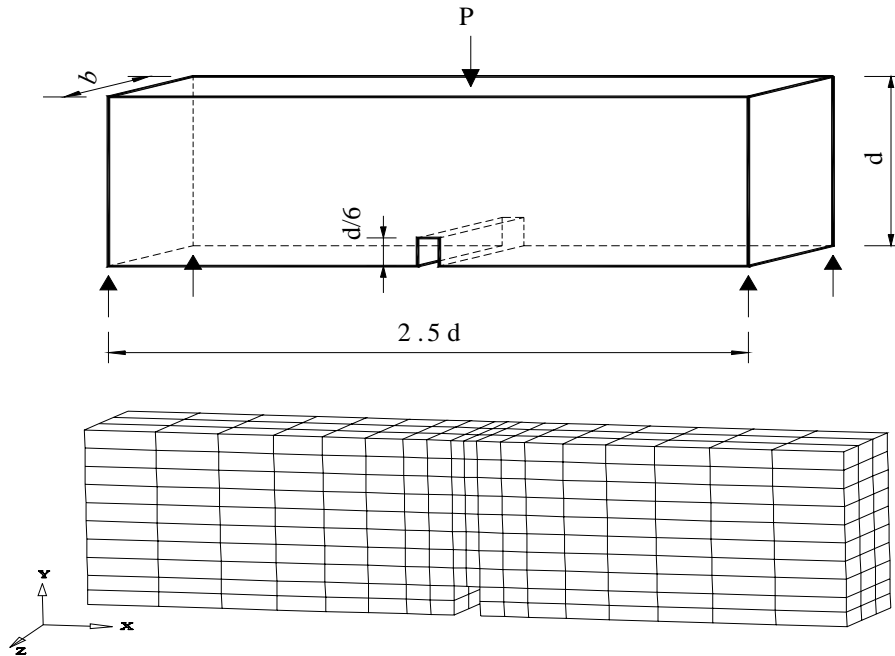
➤ Mesh sensitivity check for tension



## ➤ Tension failure with damage



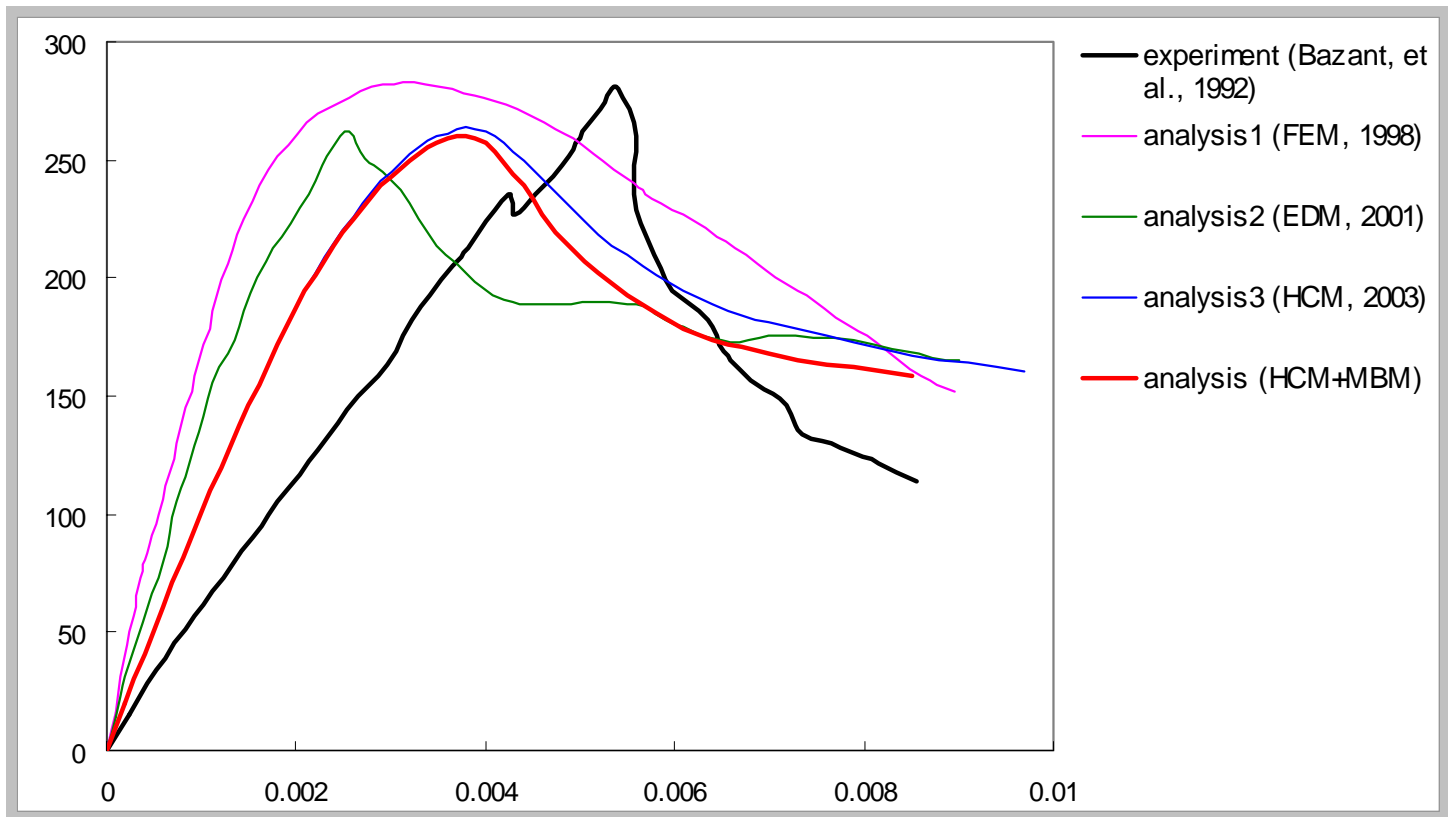
## ➤ Flexural failure



- 528 elements
- 2977 nodes

Specimen (mm)	Notch (mm)	$f_t$ (MPa)	$E_c$ (GPa)	$V_c$	$K_N$ (GPa/m)	$K_S$ (GPa/m)
190.5×76.2×38	6.5×12.7	2.97	28.06	0.2	2,680	1,680

## ➤ Results

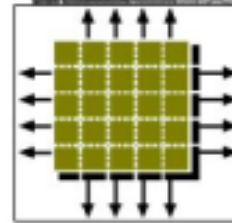
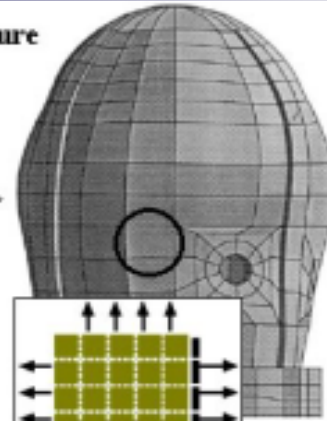
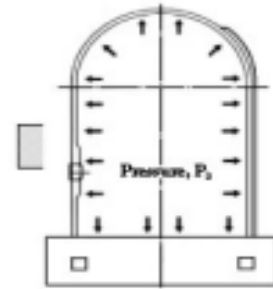


# RC panel of simulating RCCV wall subjected to biaxial tension

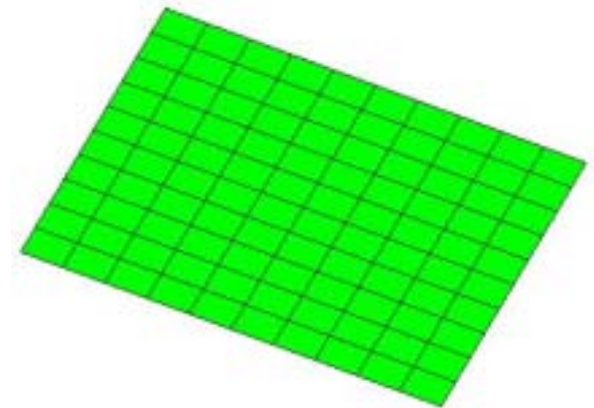


RCCV Structure

subjected to internal pressure



RC containment wall element subjected to biaxial tension



Modeling as shell element

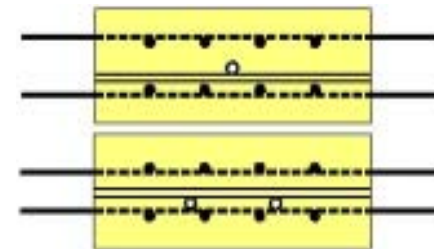


RC panel specimen

## Specification

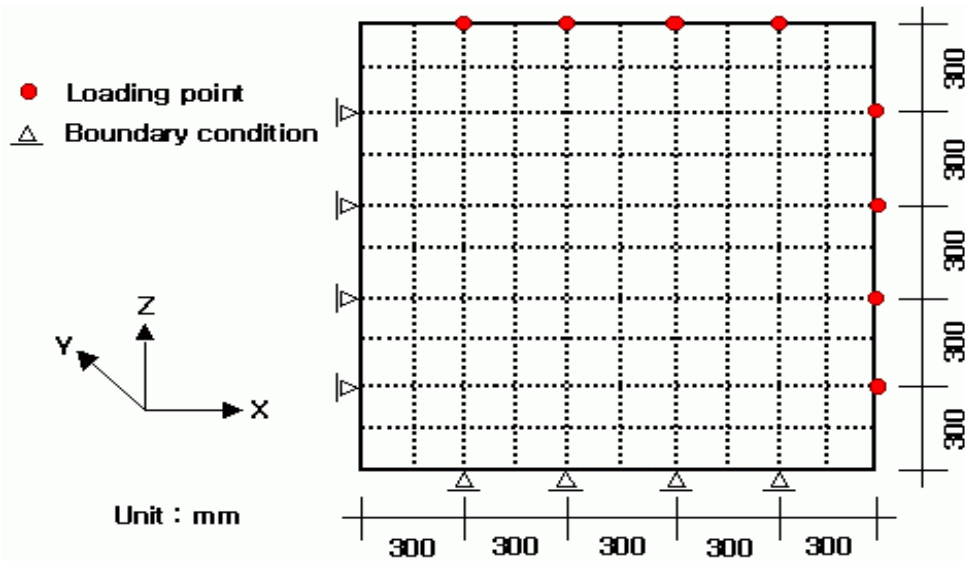
< HICT and KAERI, 2001 >

- Compressive strength of concrete ( $f_c$ ): 41.9MPa
- Tensile strength of concrete ( $f_t$ ): 2.87MPa
- Modulus of elasticity of concrete ( $E_c$ ): 23828MPa
- Yield strength of reinforcement ( $f_y$ ): 410MPa
- Modulus of elasticity of reinforcement ( $E_s$ ): 205744MPa

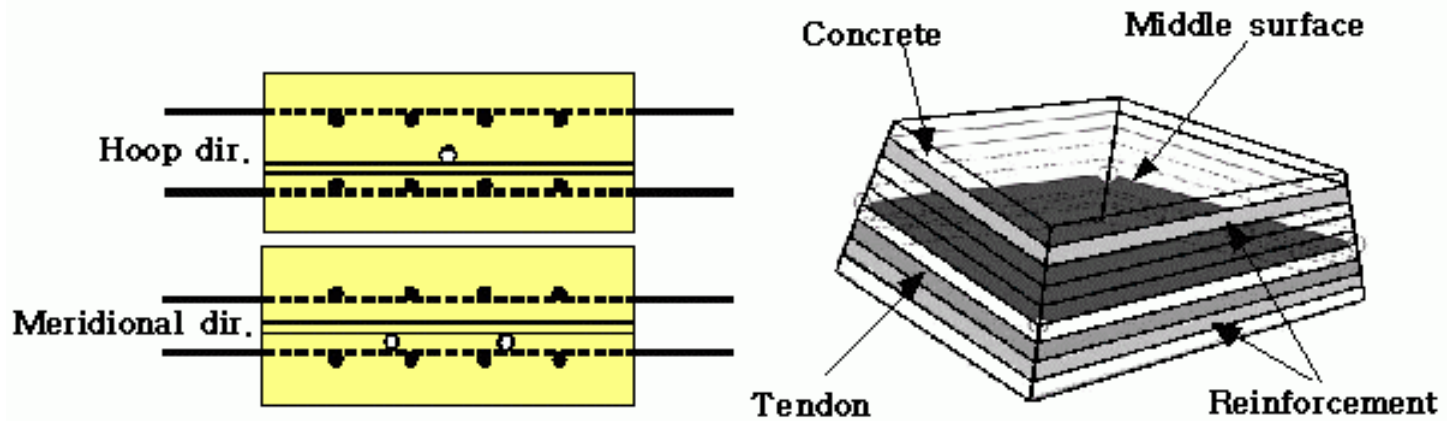


Layered shell element

# FEM MESH and Boundary condition

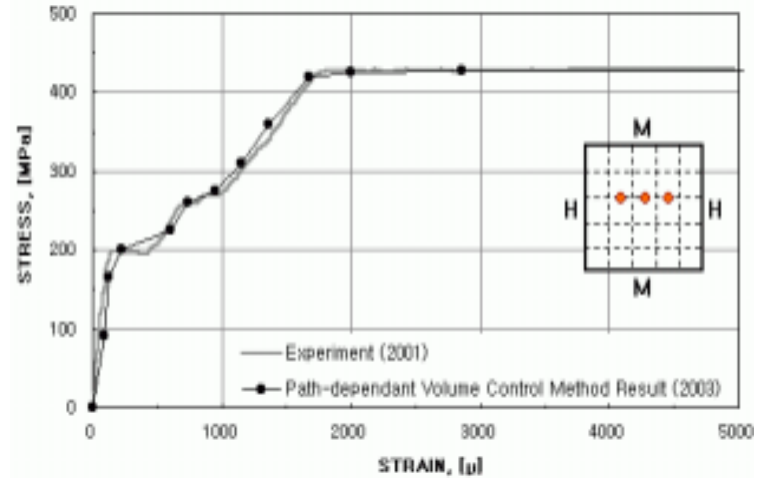
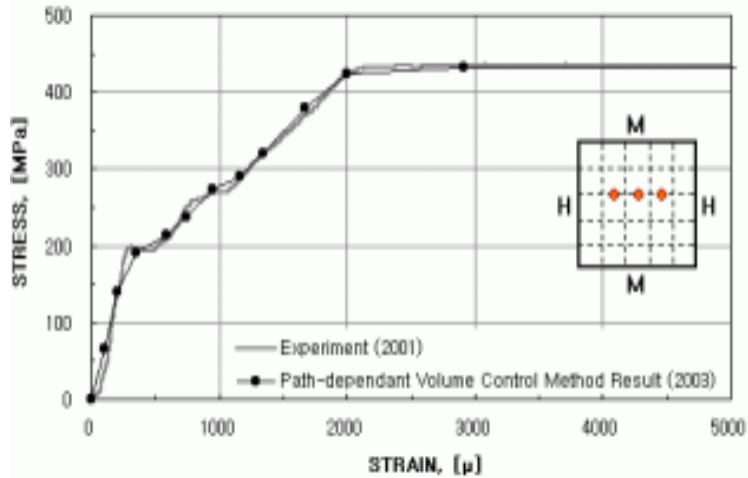


RC panel is discretized  
as 10 x 10 mesh

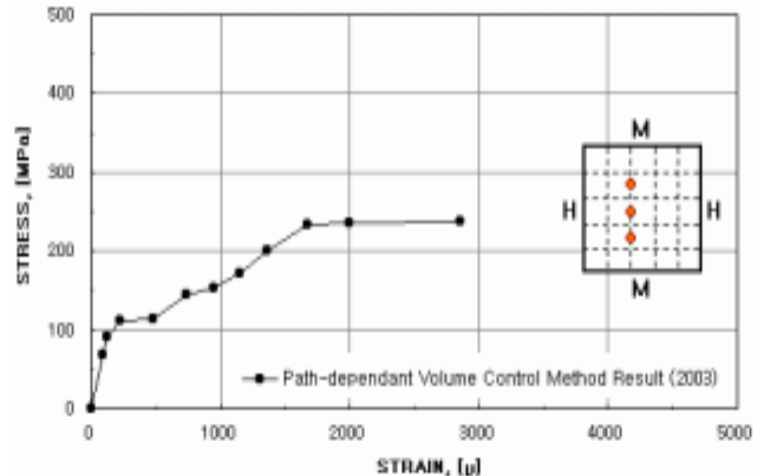
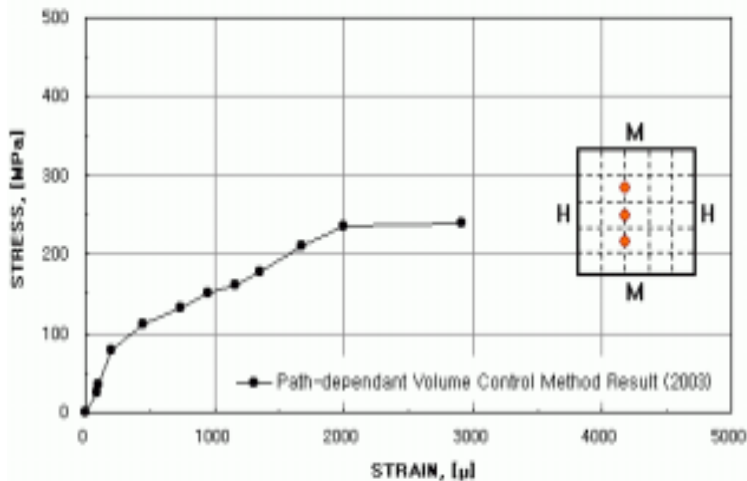


# Stress-strain curve of rebars

## Hoop direction



## Meridional direction



Top of RC panel

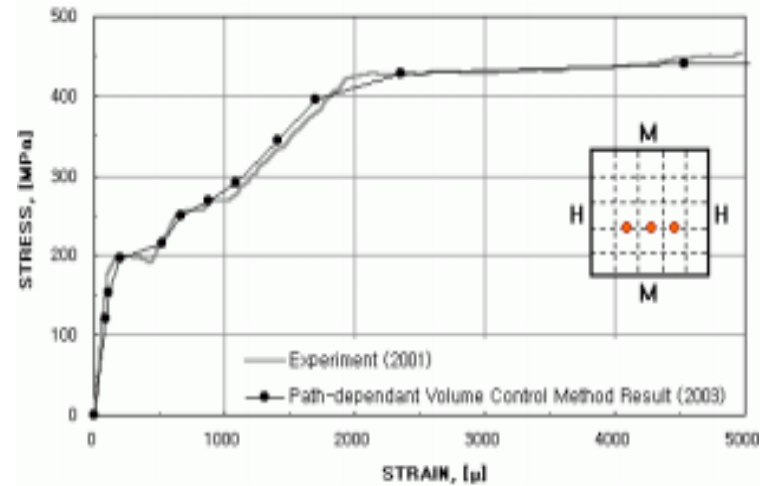
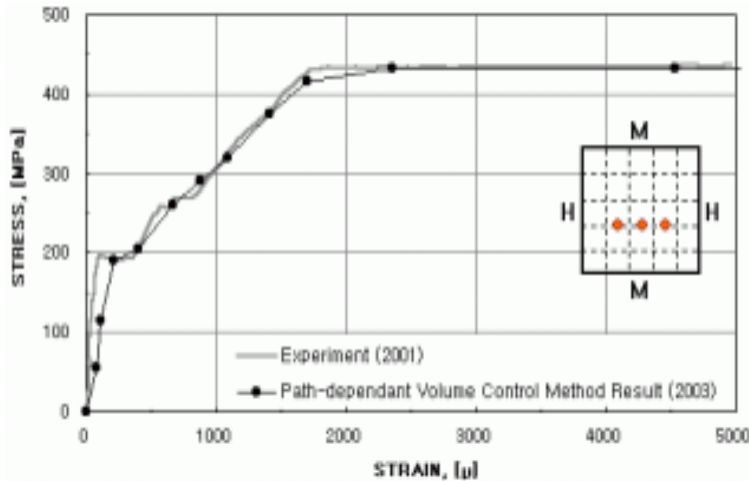
Bottom of RC panel



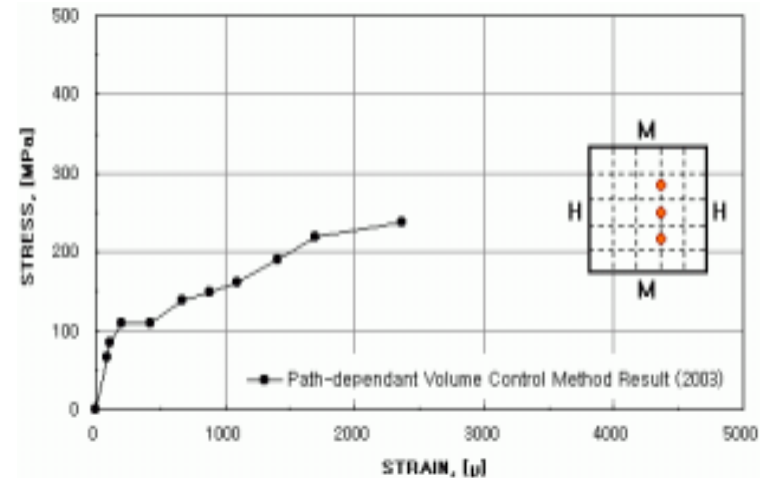
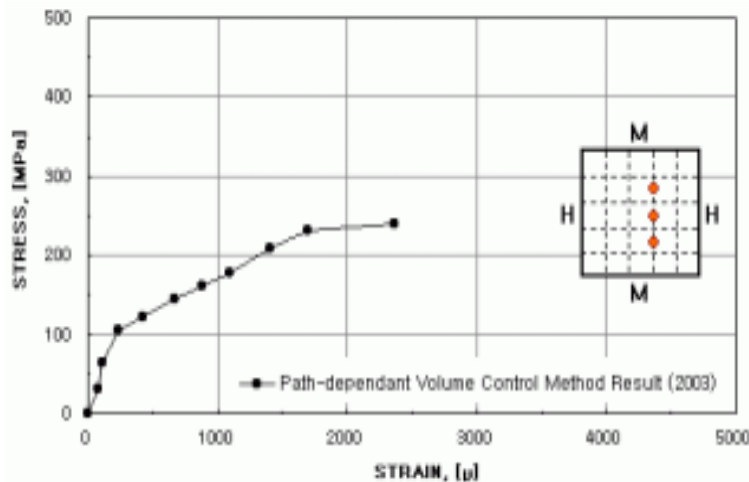


# Stress-strain curve of rebar

## Hoop direction



## Meridional direction



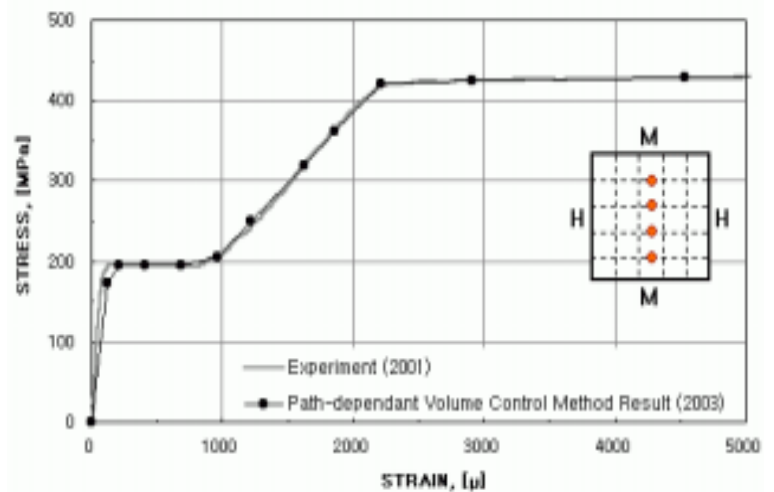
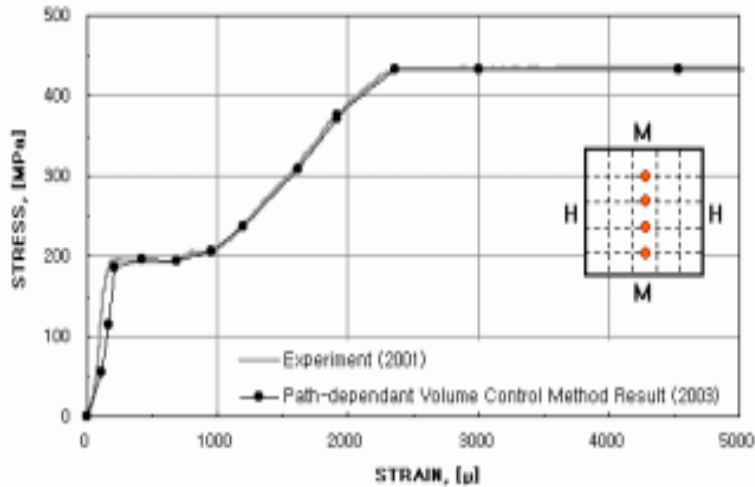
Top of RC panel

Bottom of RC panel

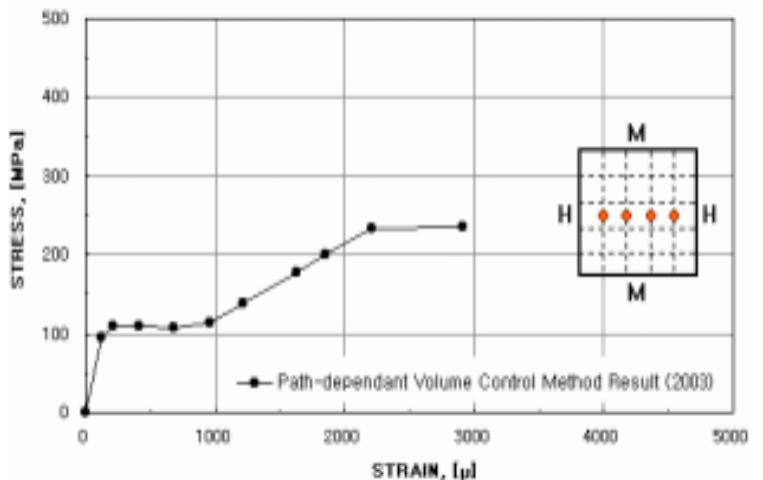
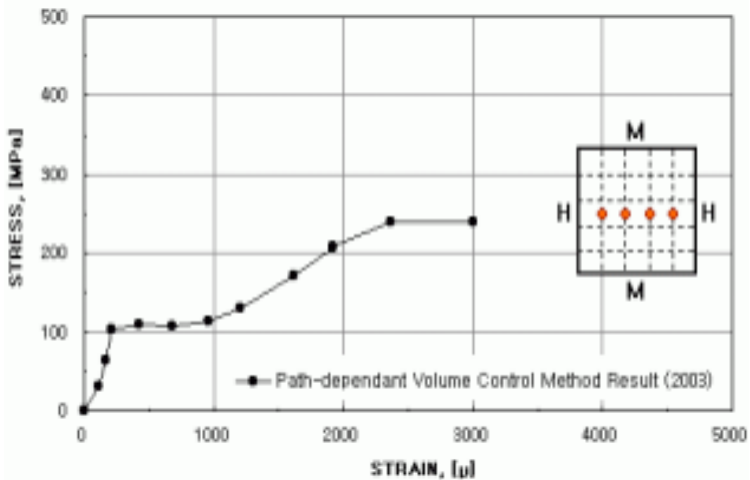


# Stress-strain curve of rebar

## Hoop direction



## Meridional direction



Top of RC panel

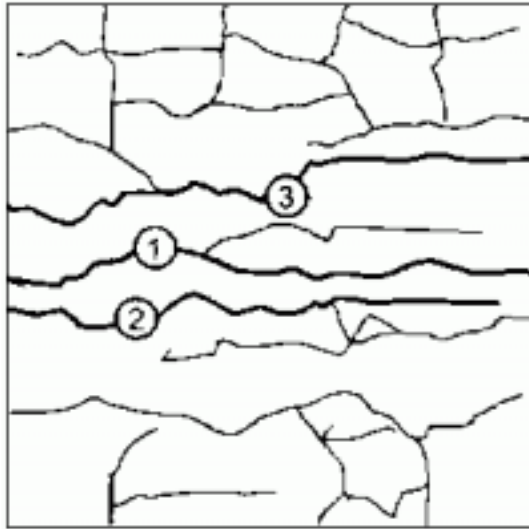
Bottom of RC panel



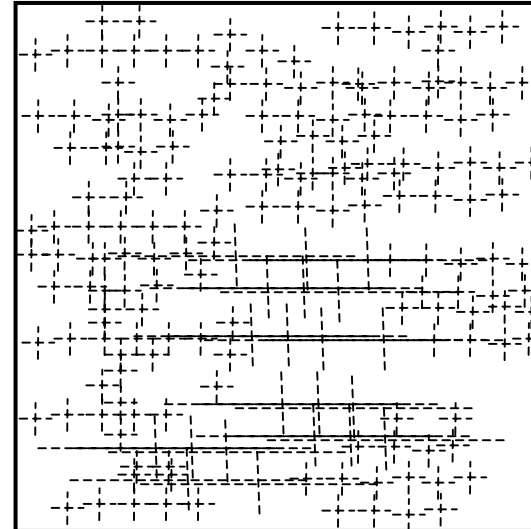
# Crack patterns of RC panel



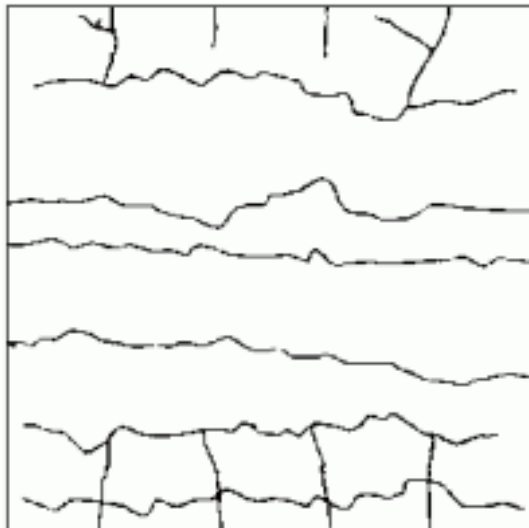
Top of RC panel by experiment



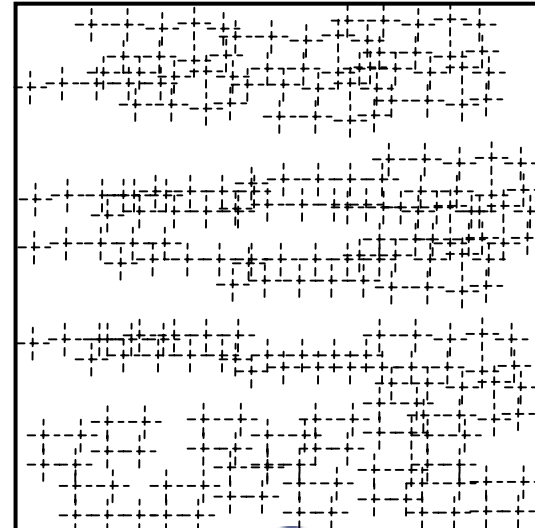
Top of RC panel by volume control analysis



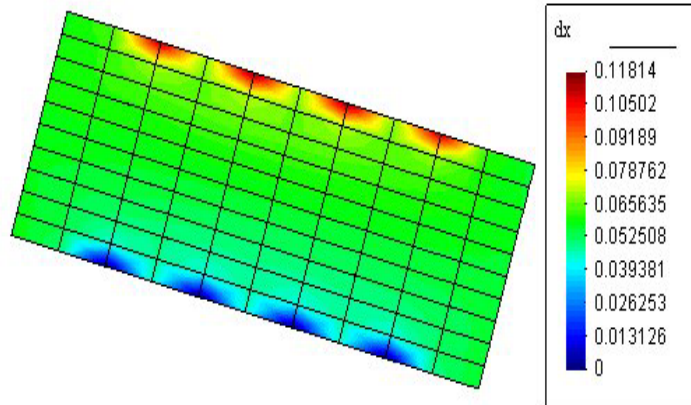
Bottom of RC panel by experiment



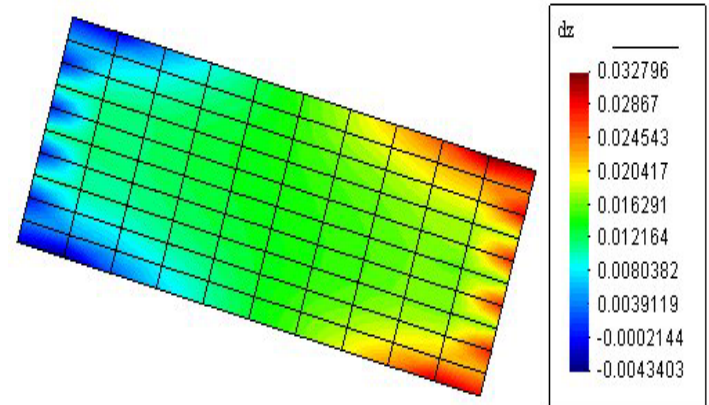
Bottom of RC panel by volume control analysis



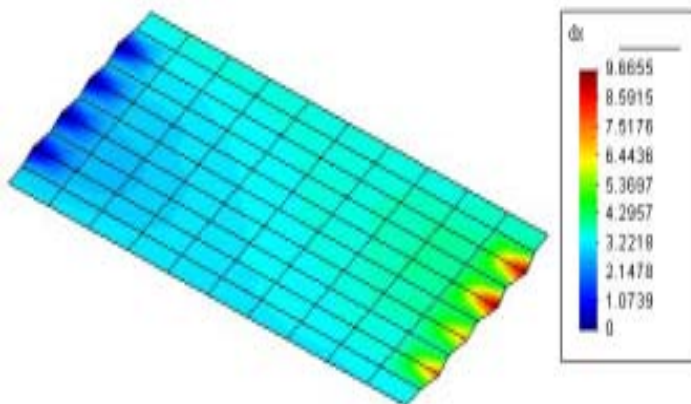
# Deformed shape of RC panel



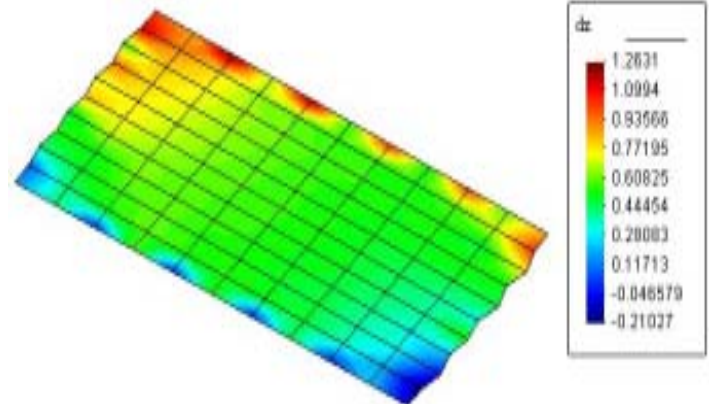
Hoop dir. displacement. at 1<sup>st</sup> crack occurrence



Meridional dir. displacement at 1<sup>st</sup> crack occurrence



Hoop dir. displacement. at rebar yielding



Meridional dir. displacement at rebar yielding

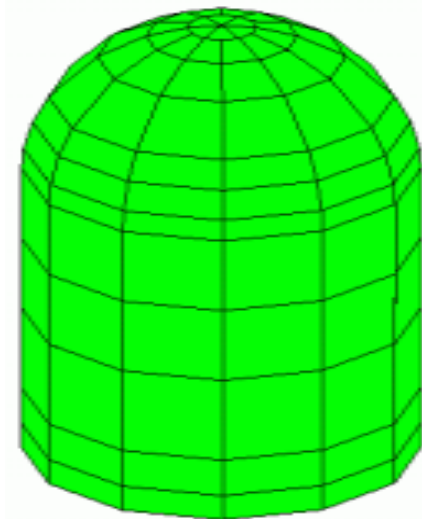
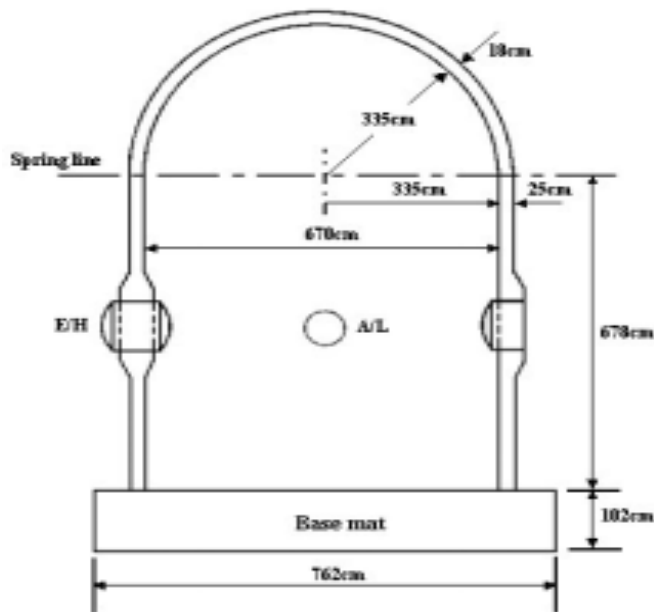


# RCCV subjected to internal pressure

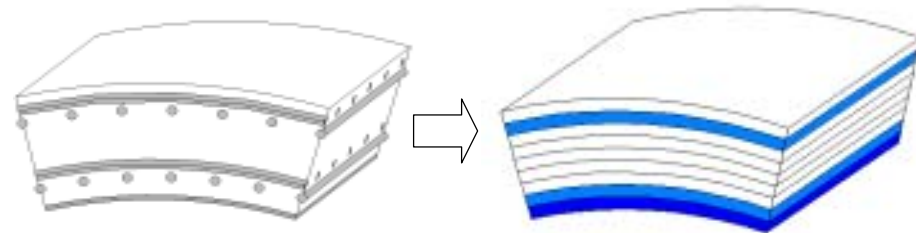
## Specification

< SNL, 2001 >

- Compressive strength of concrete ( $f_c$ ): 46.0MPa
- Tensile strength of concrete ( $f_t$ ): 3.45MPa
- Modulus of elasticity of concrete ( $E_c$ ): 33,100MPa
- Poisson's ratio of concrete ( $\nu_c$ ): 0.20
- Yield strength of reinforcement ( $f_y$ ): 450.0MPa
- Modulus of elasticity of reinforcement ( $E_s$ ): 214,000MPa
- Poisson's ratio of reinforcement ( $\nu_s$ ): 0.30



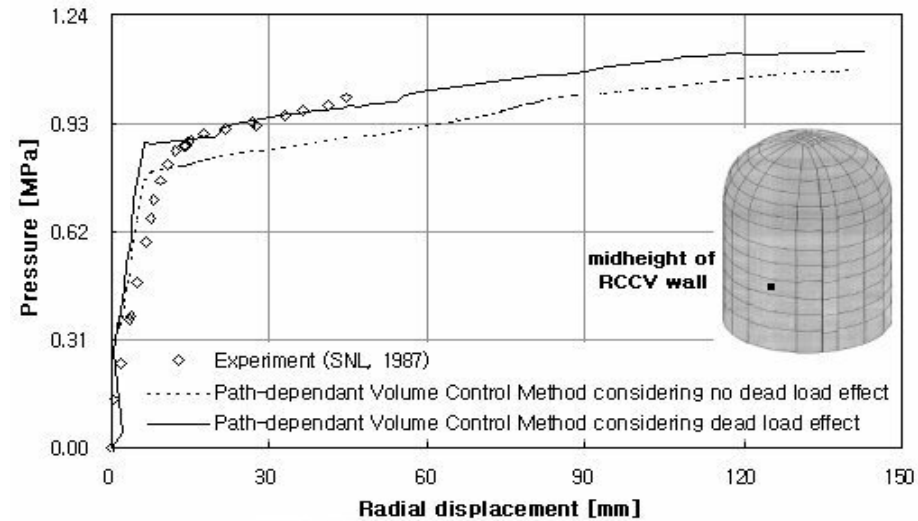
Modeling with or without considering foundation



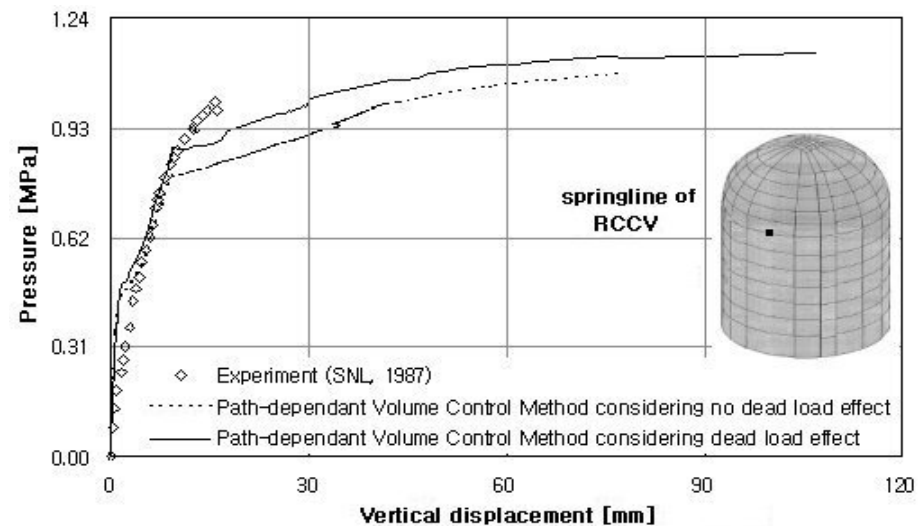
Layered shell element

# Global behaviors

## Radial displacement – Pressure relationship at mid-height



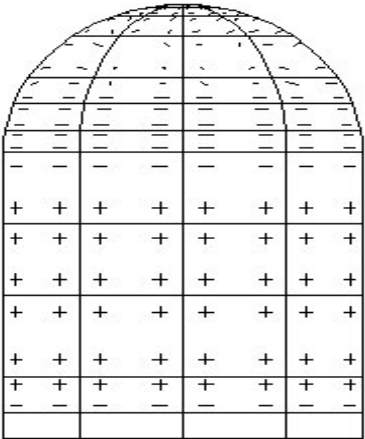
## Vertical displacement – Pressure relationship at spring-line



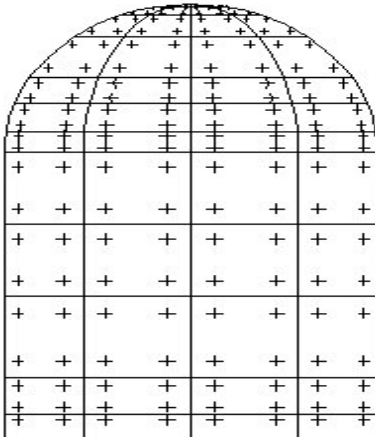
# Crack patterns of RCCV



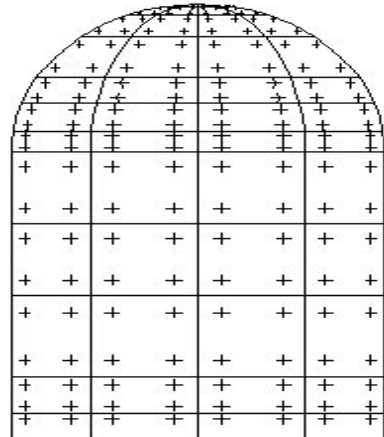
1.0Pd (0.31MPa)



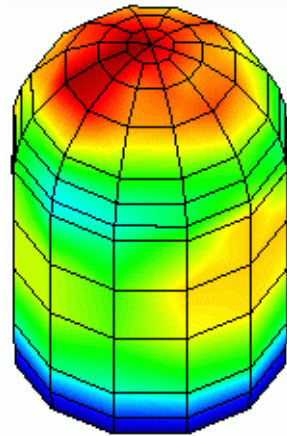
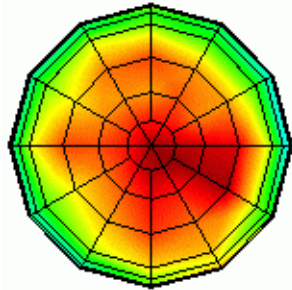
2.0Pd (0.62MPa)



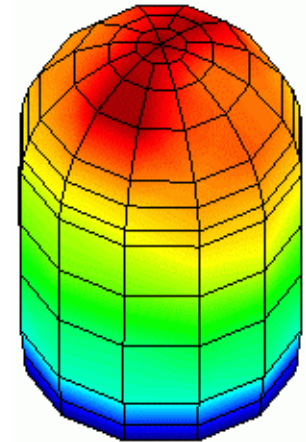
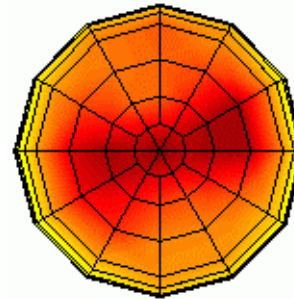
3.0Pd (0.93MPa)



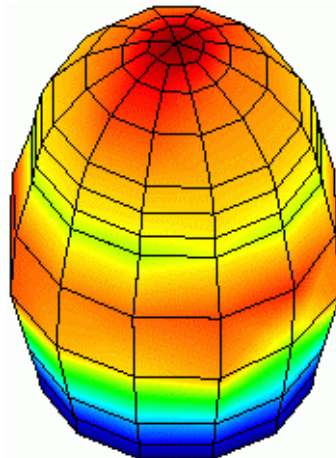
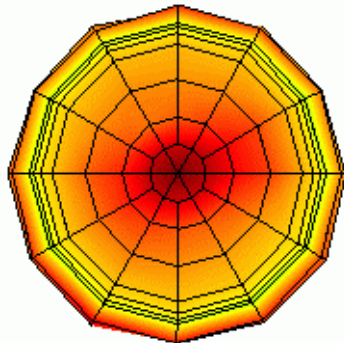
# Deformed shape of RCCV



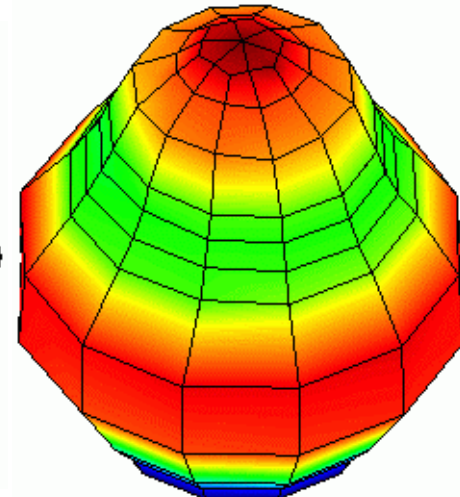
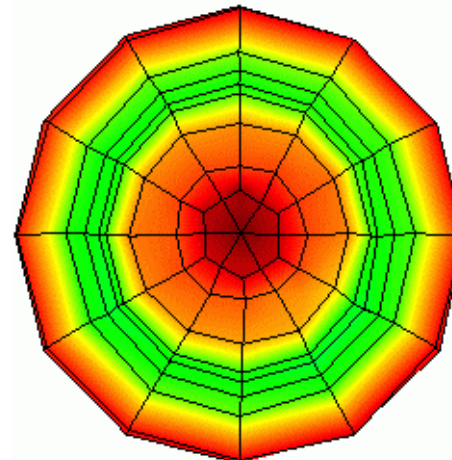
1.0Pd (0.31MPa)



2.0Pd (0.62MPa)



3.0Pd (0.93MPa)

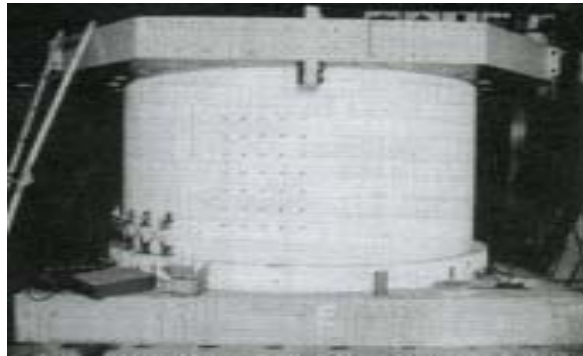


Ultimate pressure





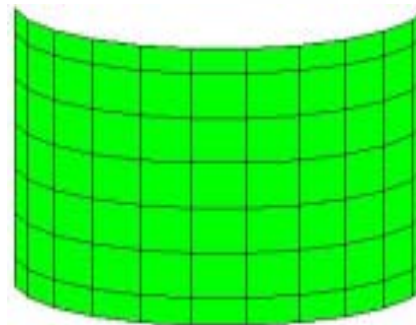
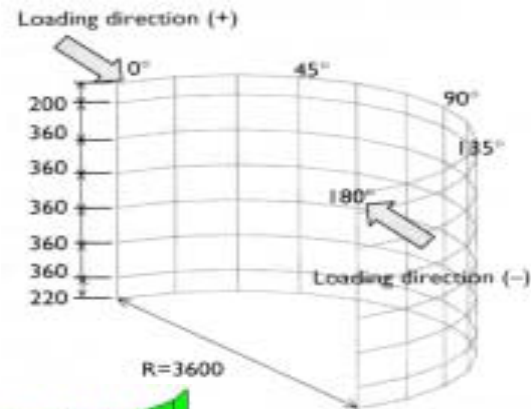
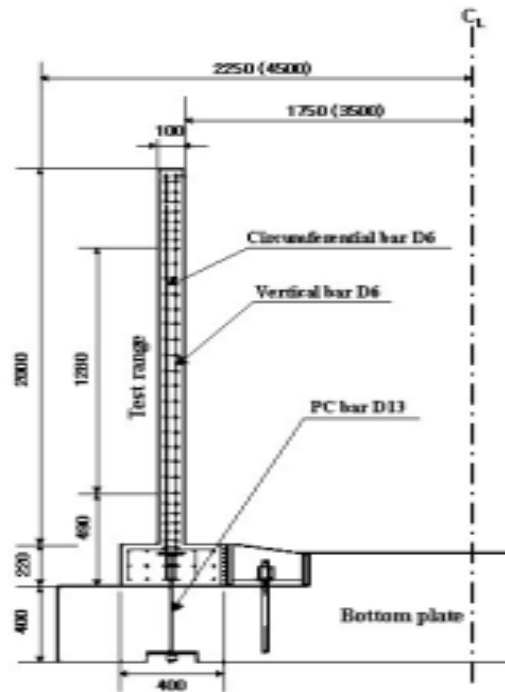
# RC tank subjected to reversed cyclic loading



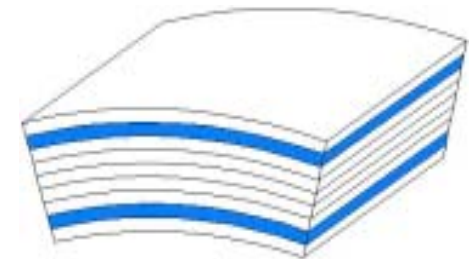
## Specification

< Harada et al., 2001 >

- Compressive strength of concrete ( $f_c$ ) : 28.0MPa
- Tensile strength of concrete ( $f_t$ ) : 2.20MPa
- Modulus of elasticity of concrete ( $E_c$ ) : 22,600MPa
- Yield strength of reinforcement ( $f_y$ ) : 384.0MPa
- Modulus of elasticity of reinforcement ( $E_s$ ) : 183,000MPa



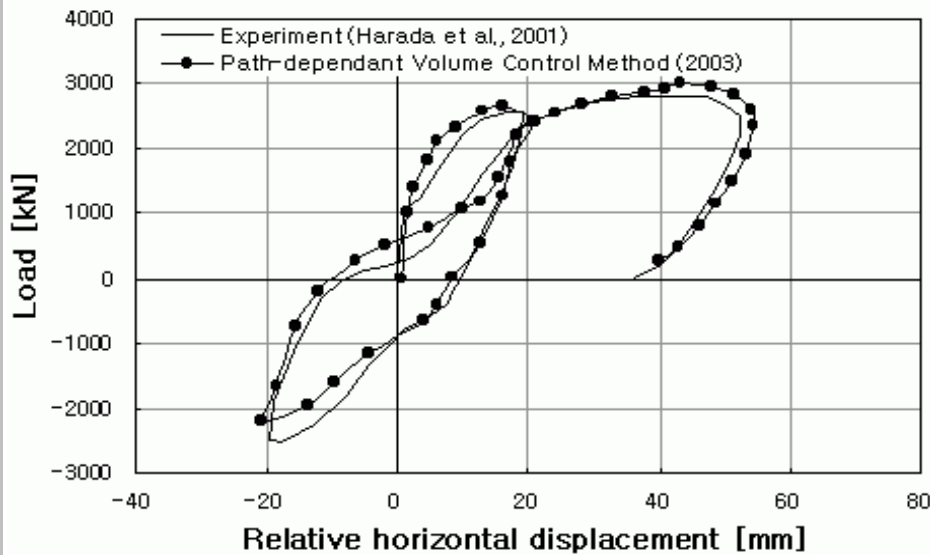
Modeling as shell element



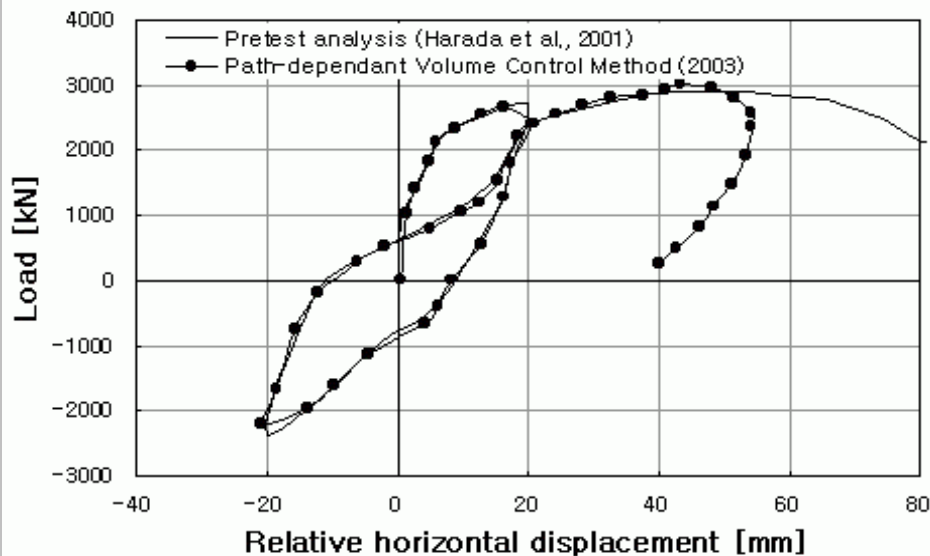
Layered shell element



# Relative horizontal displacement - load curve



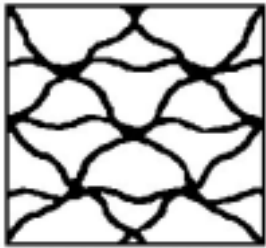
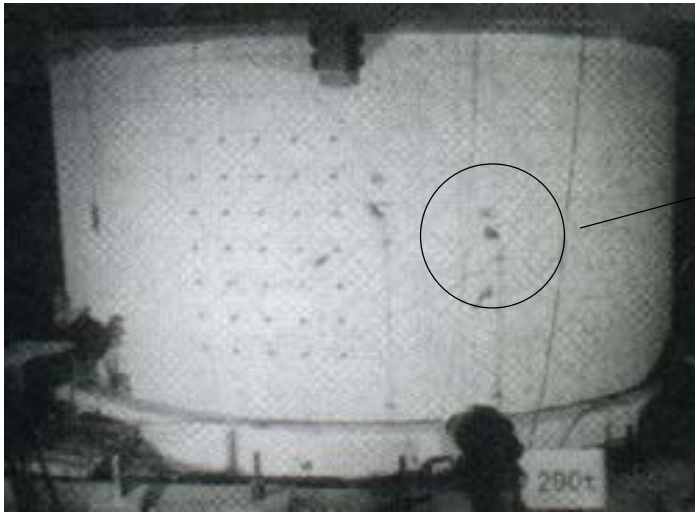
Comparison experiment result with path-dependant volume control result



Comparison path-dependant volume control result with pre-test analysis result

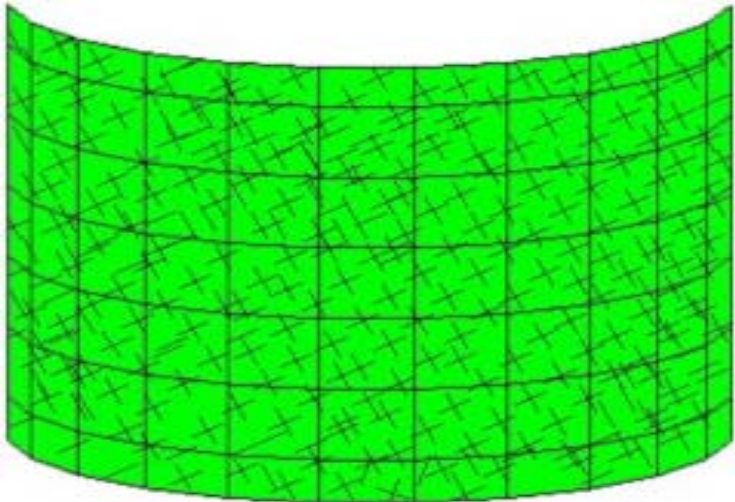


# Crack status of RC tank



Multi-directional cracks occurrence due to reversed cyclic loading

Crack status of RC tank by experiment

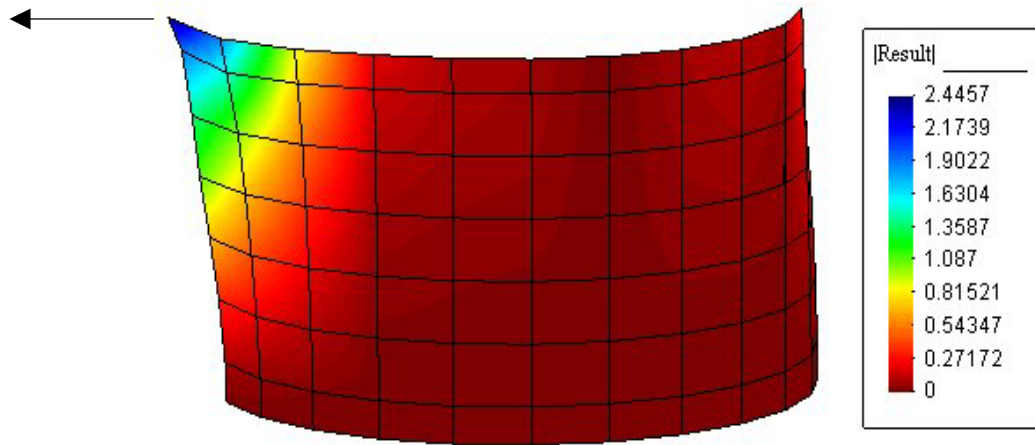


Crack status of RC tank by volume control analysis

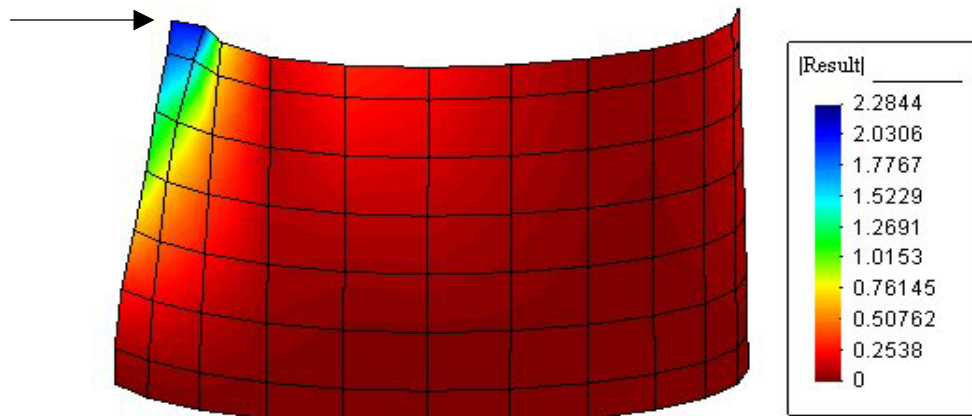


# Deformed shape of RC tank

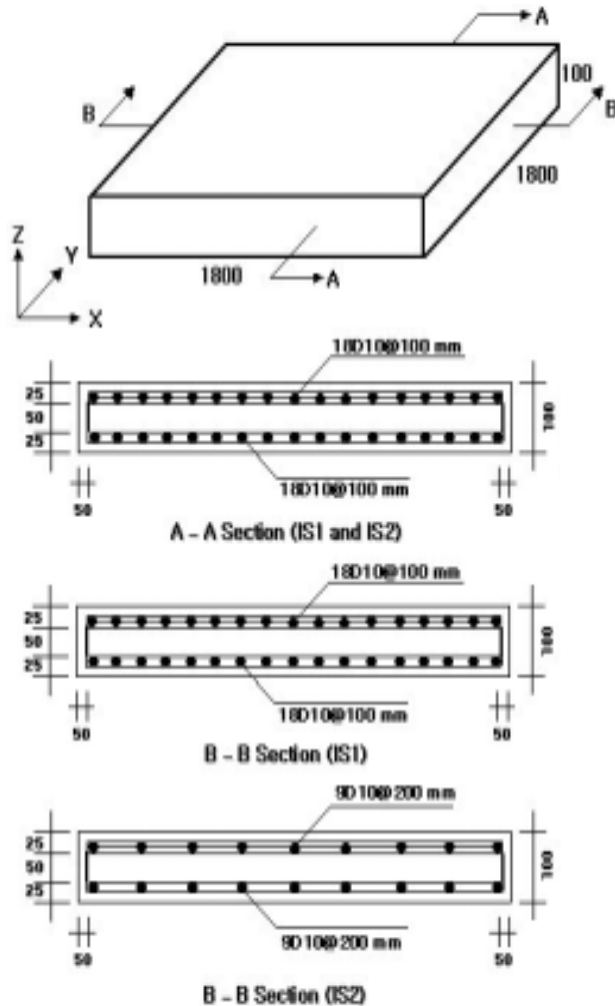
Horizontal load = 3,037 kN



Horizontal load = -2,200 kN



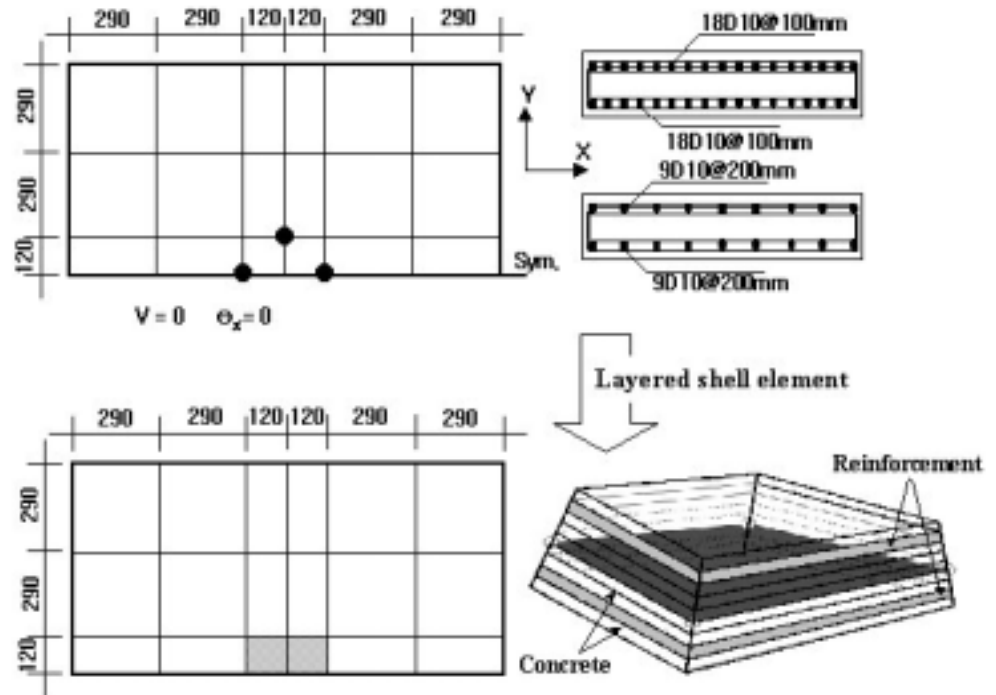
# RC slab subjected to out-of plane cyclic loading



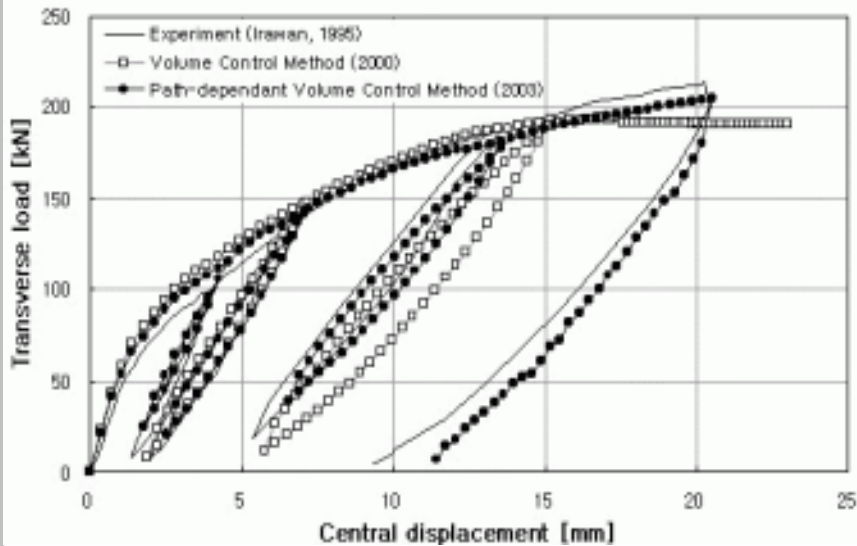
## Specification

< Irawan, 2001 >

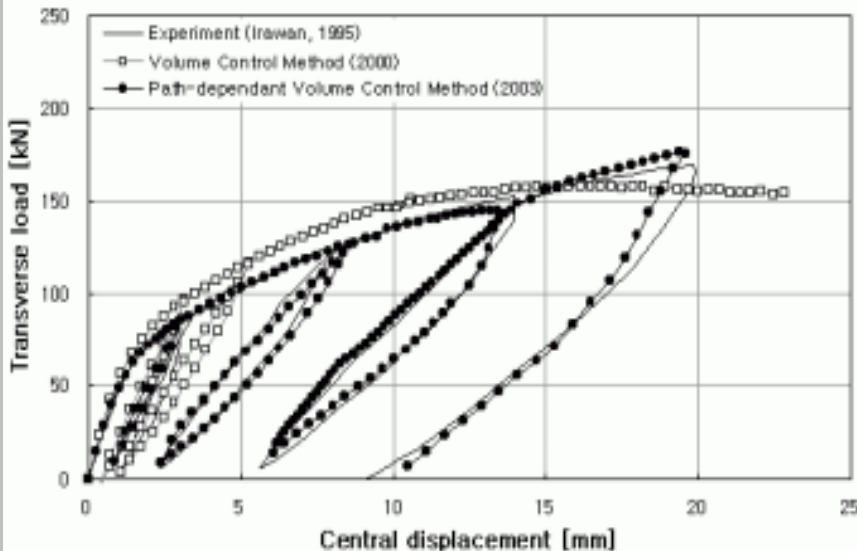
- Compressive strength of concrete ( $f_c$ ) : 37.0MPa
- Tensile strength of concrete ( $f_t$ ) : 3.70MPa
- Yield strength of reinforcement ( $f_y$ ) : 380.0MPa
- Modulus of elasticity of reinforcement ( $E_s$ ) : 206,000MPa



# Central displacement - load curve



Comparison existing volume control result with path-dependant volume control result (IS1)

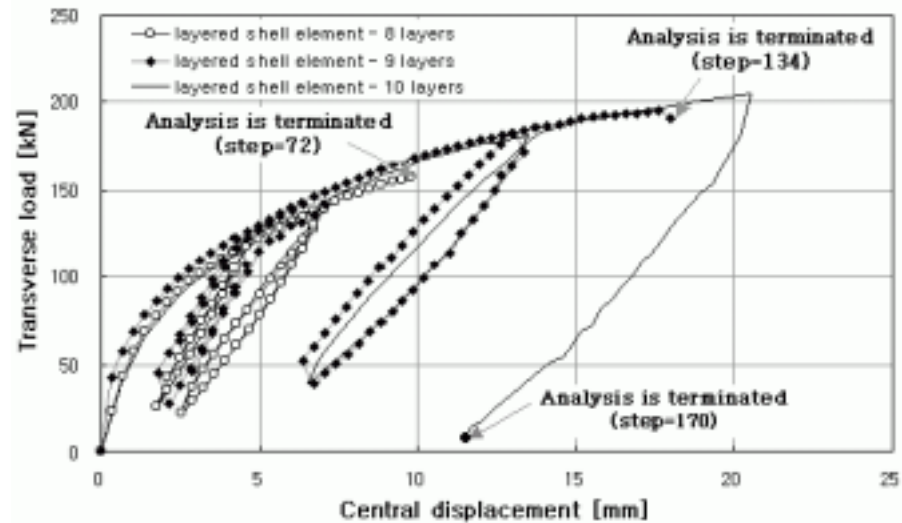


Comparison existing volume control result with path-dependant volume control result (IS2)

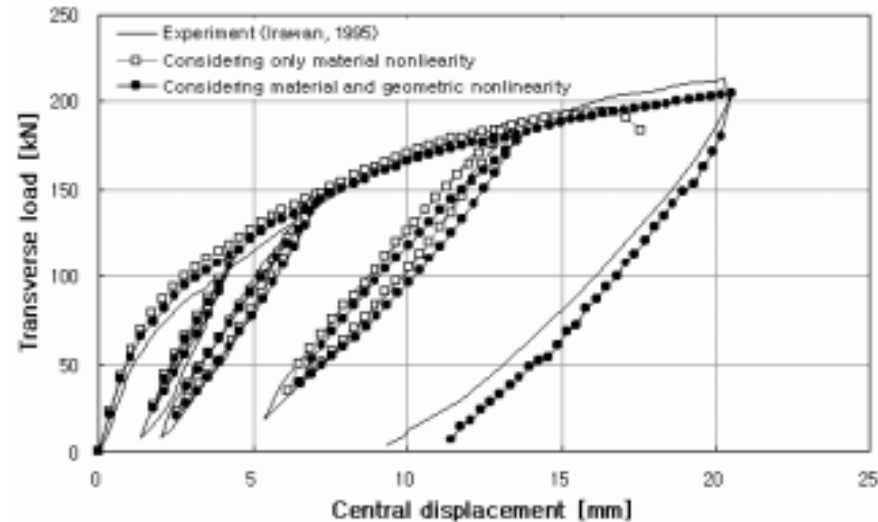




## Analysis result according to number of layered shell element

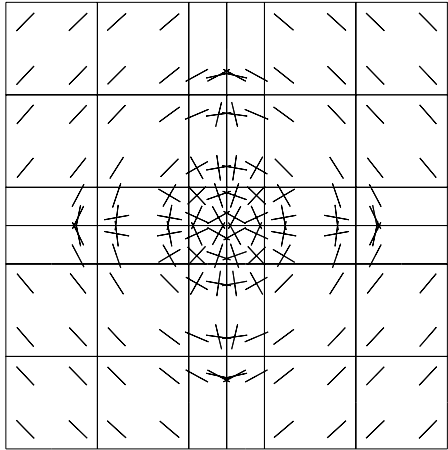
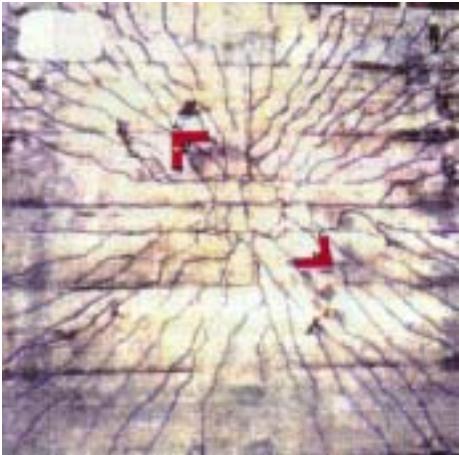


## Analysis result according to number of layered shell element

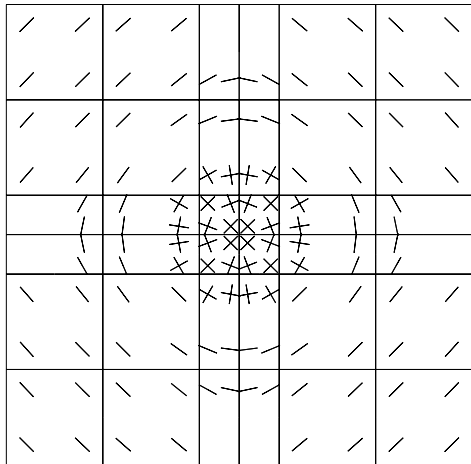
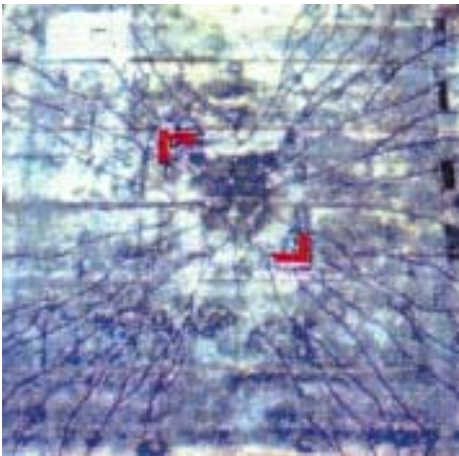


# Crack status of RC slab

Specimen IS1 : vertical load = 193 kN



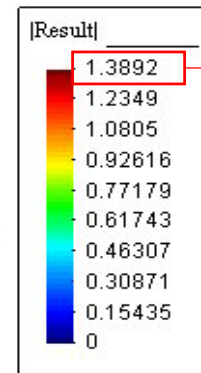
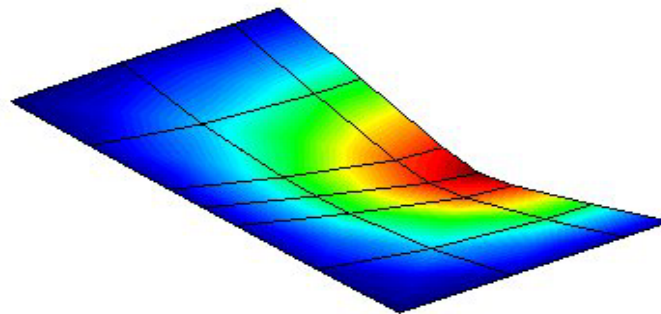
Specimen IS2 : vertical load = 193 kN





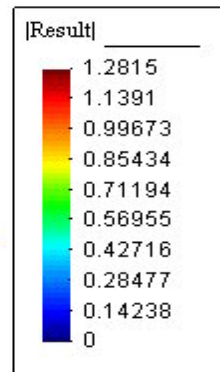
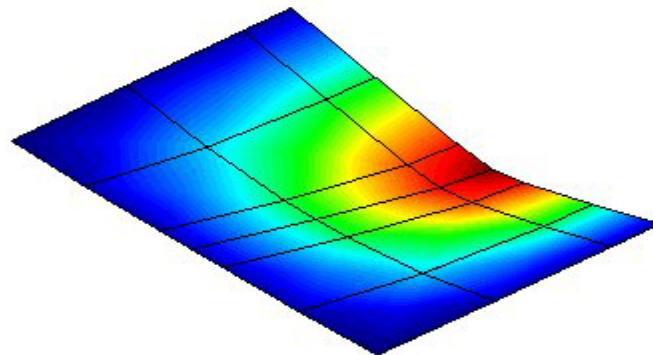
# Deformed shape of RC slab

Specimen IS1 : vertical load = 178 kN



Isotropic reinforcement arrangement

Specimen IS2 : vertical load = 144 kN



Stiffness of IS1 > Stiffness of IS2  
Due to reinforcement arrangement

Anisotropic reinforcement arrangement

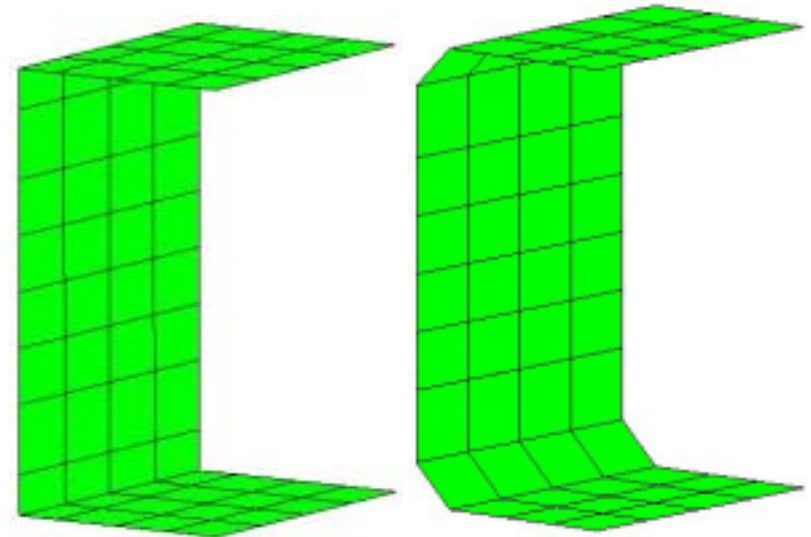
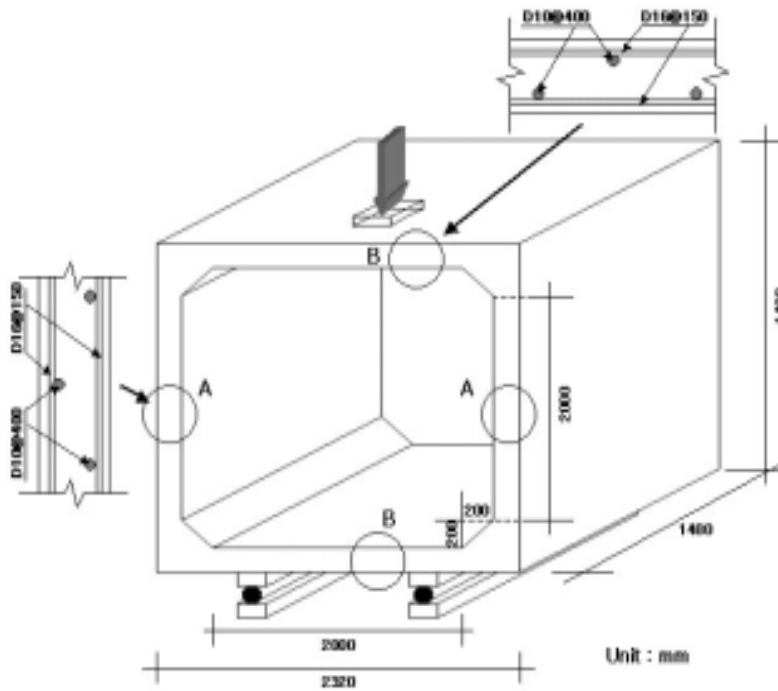


# RC box culvert subjected to cyclic loading

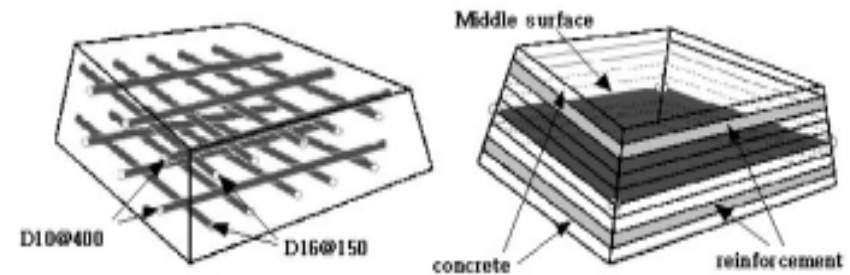
## Specification

< Irawan, 1995 >

- Compressive strength of concrete ( $f_c$ ) : 50.0MPa
- Yield strength of reinforcement ( $f_y$ ) : 500.0MPa



Modeling with and without considering haunch

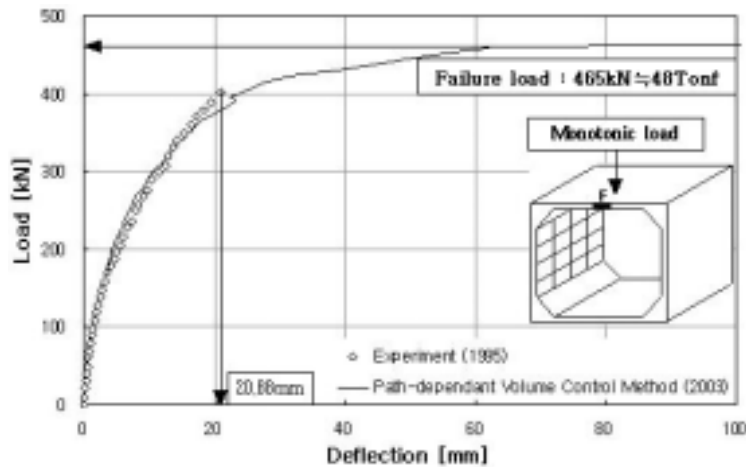


Layered shell element

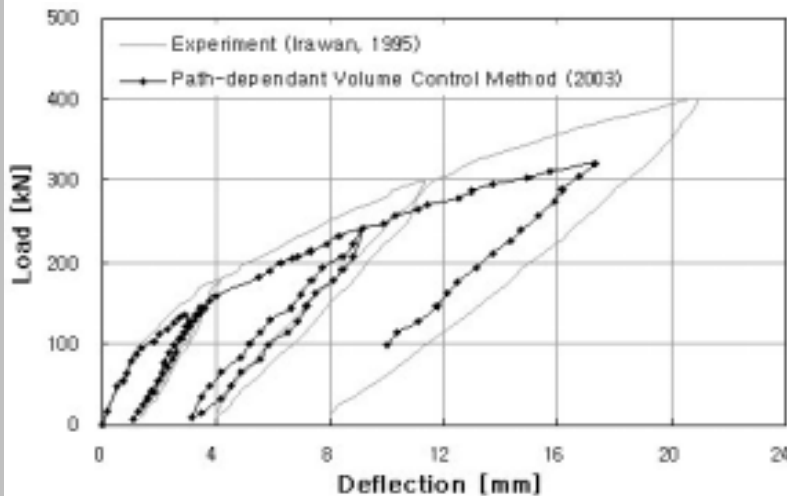
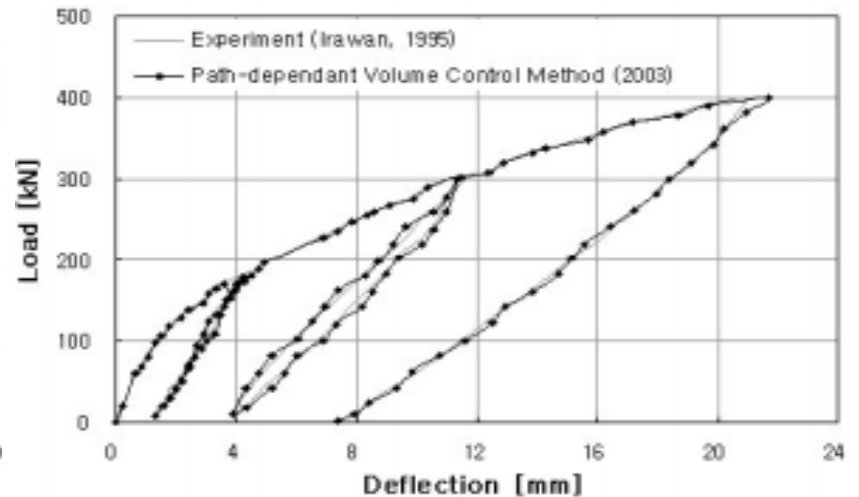


# Load-deflection curve of RC box culvert

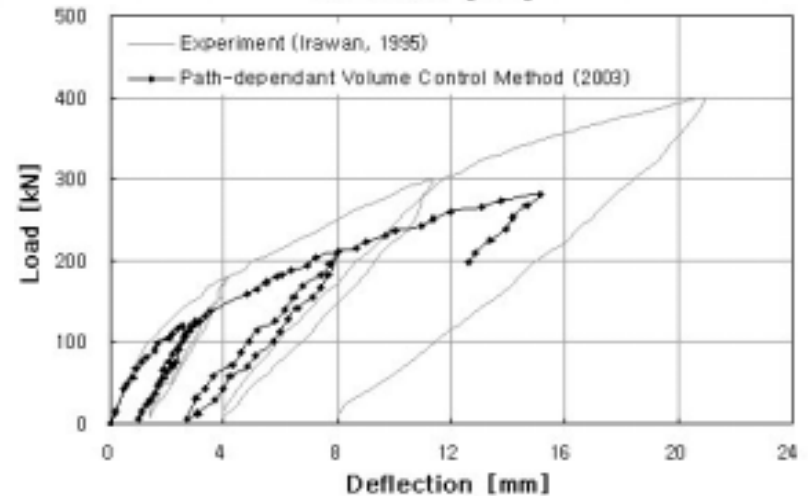
Static failure load : 48 tonf



Model of considering without considering haunch



Model of considering with haunch  
is reinforced concrete layer

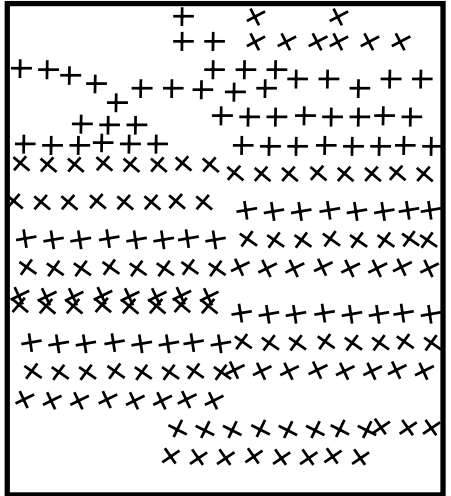


Model of considering with haunch  
is plain concrete layer

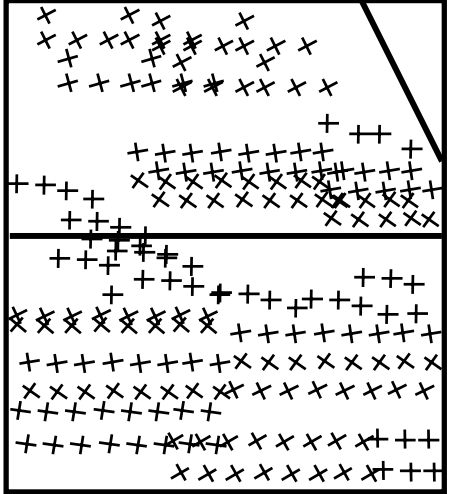
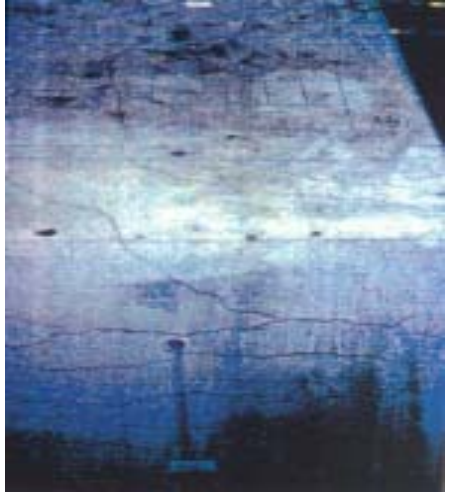


# Crack status of RC box culvert

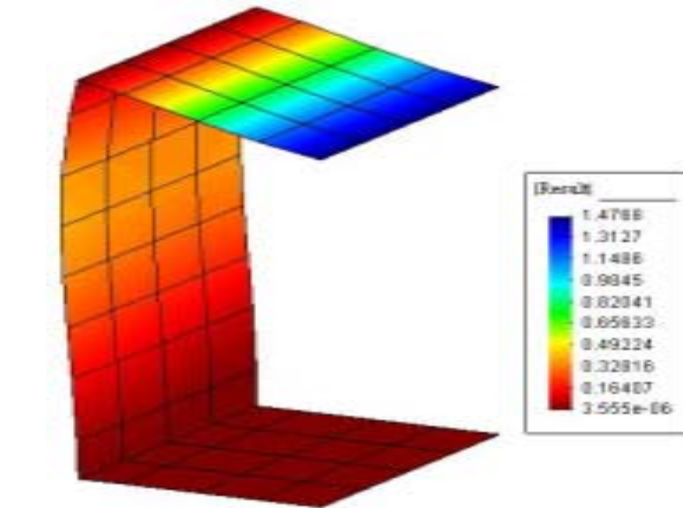
Wall of RC box culvert



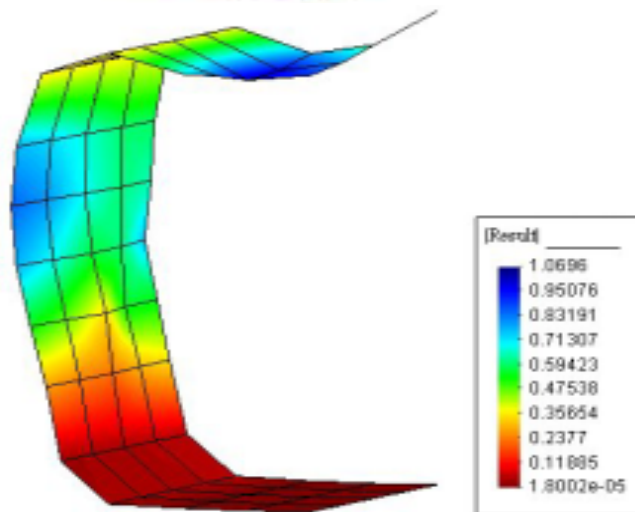
Top slab and wall of RC box culvert



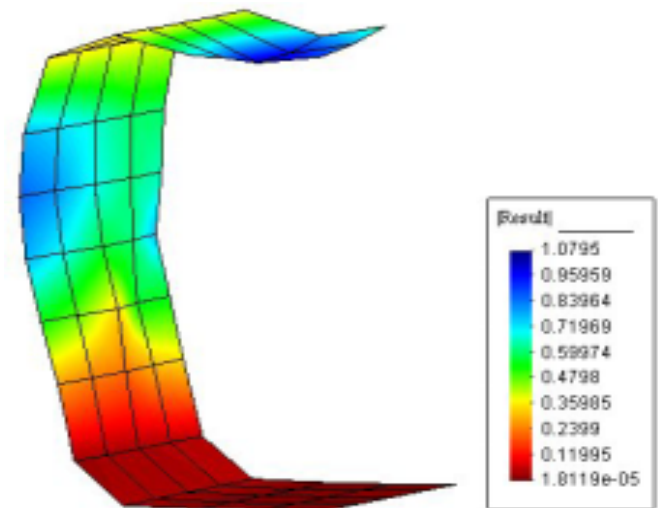
# Deformed shape of RC box culvert



Model of considering without haunch



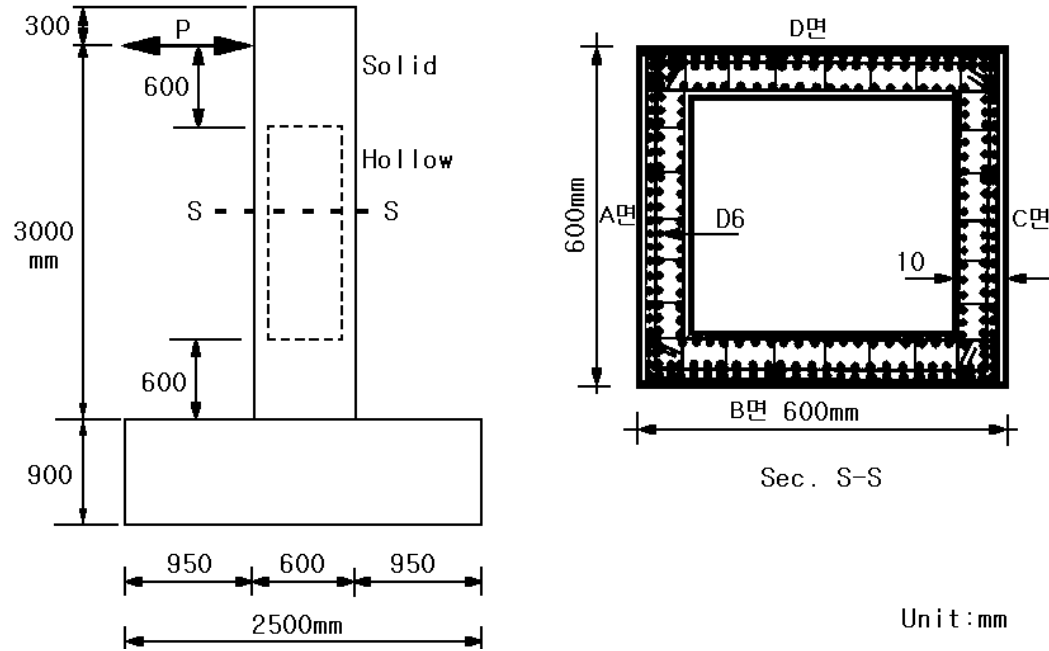
Model of considering with haunch  
is reinforced concrete layer



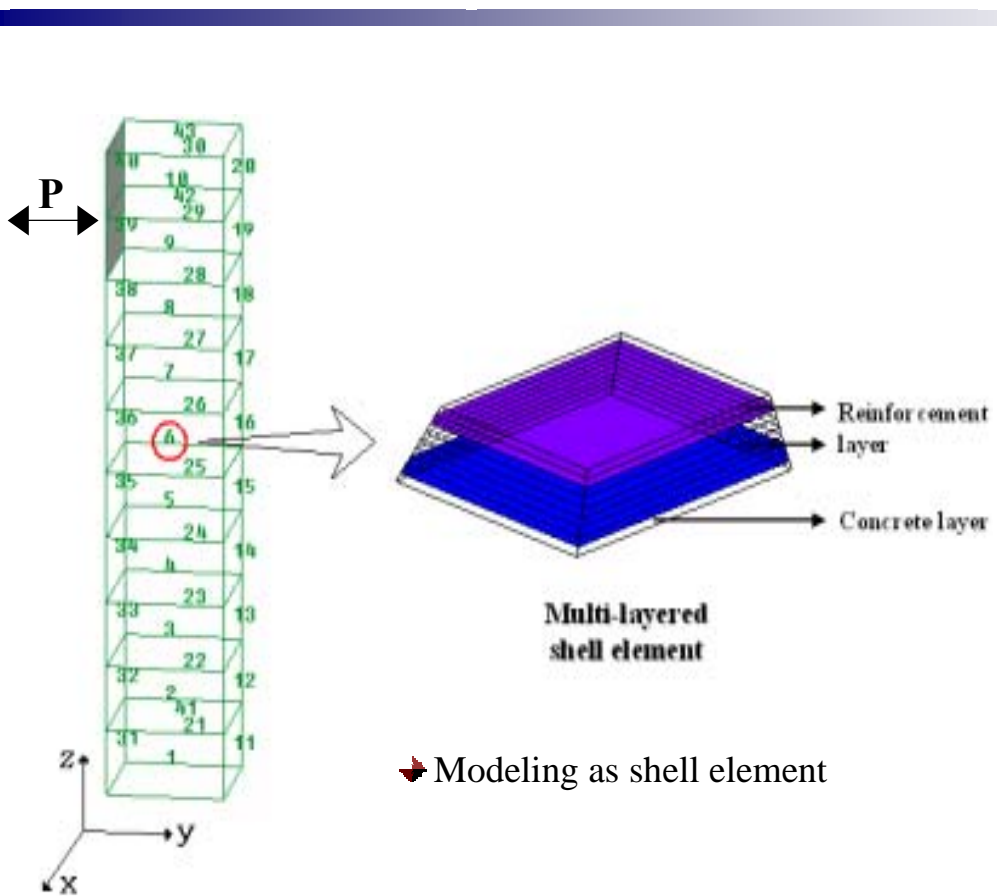
Model of considering with haunch  
is plain concrete layer



# RC hollow column under lateral loading

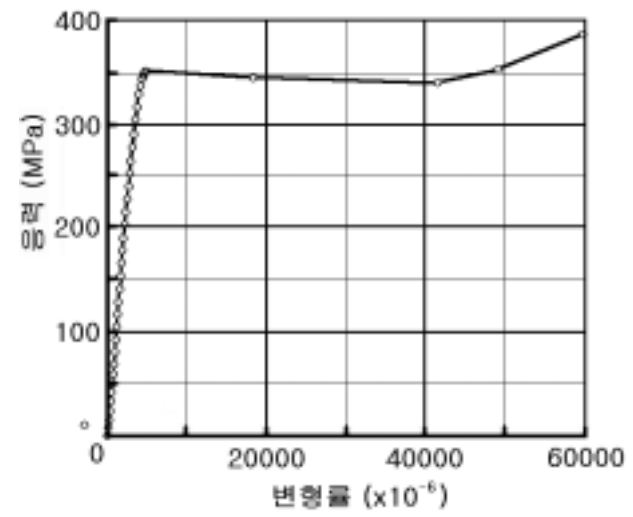


Concrete		Reinforcement	
$f_c$	76.5 MPa	$f_y$	350 MPa
$E_c$	31.4 GPa	$f_u$	421 MPa
	0.21	$E_s$	200 MPa
$f_t$	2.63 MPa		

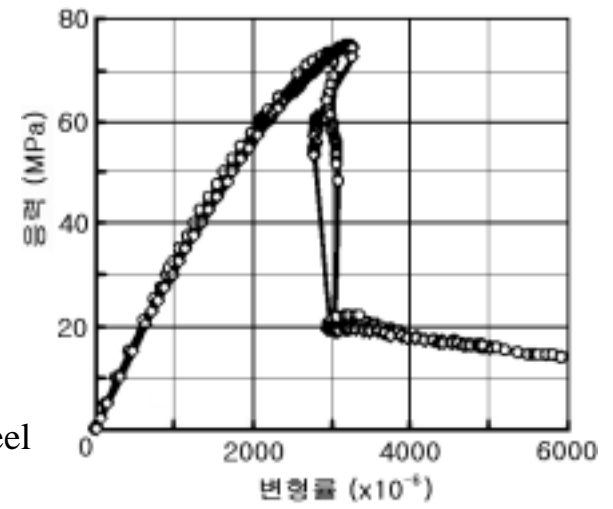


➔ Modeling as shell element

➔ Stress-strain curve of concrete and reinforcing steel  
(Masukawa et al., 1997)

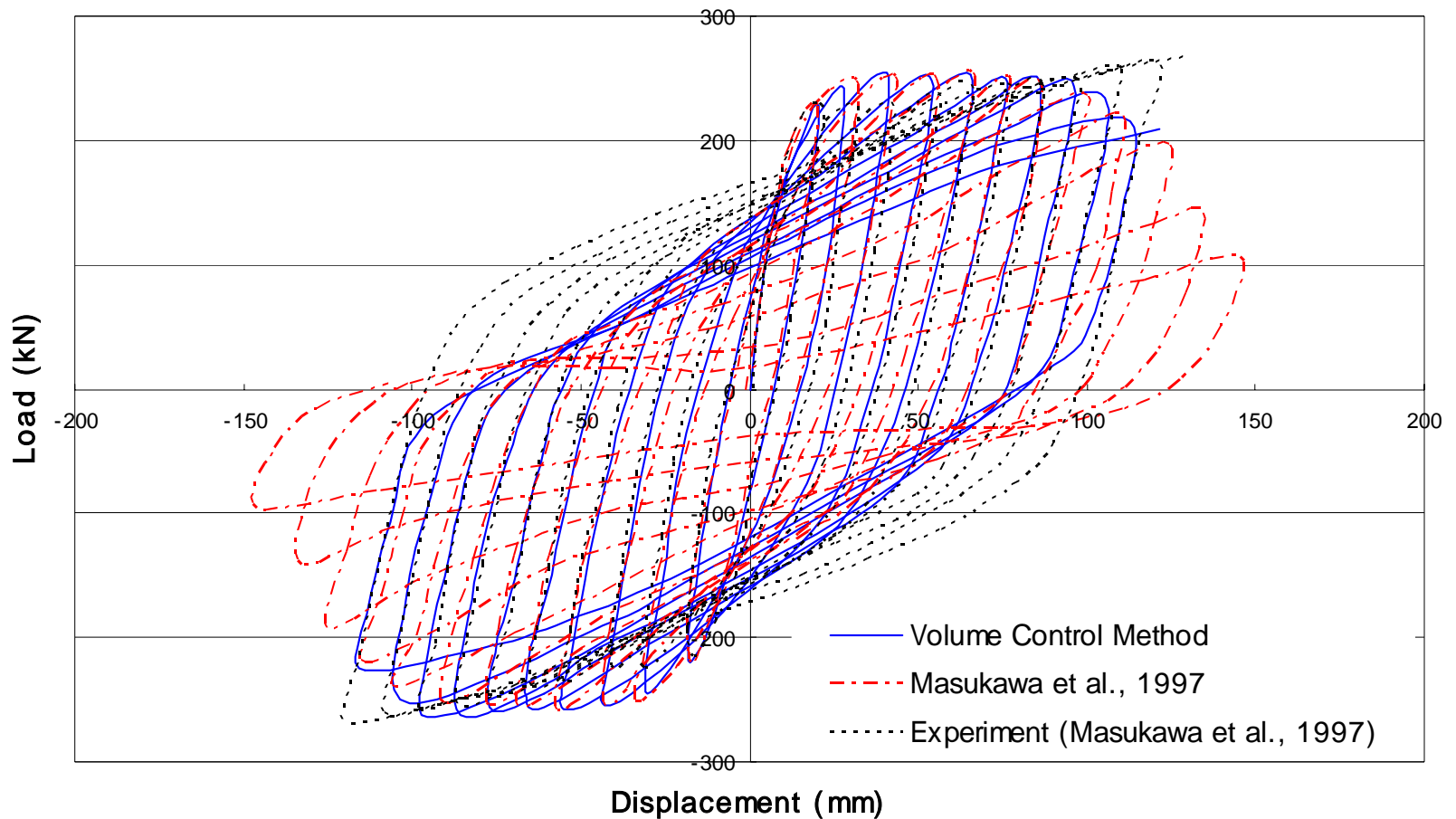


(a) Steel



(b) Concrete

# Load-displacement curve

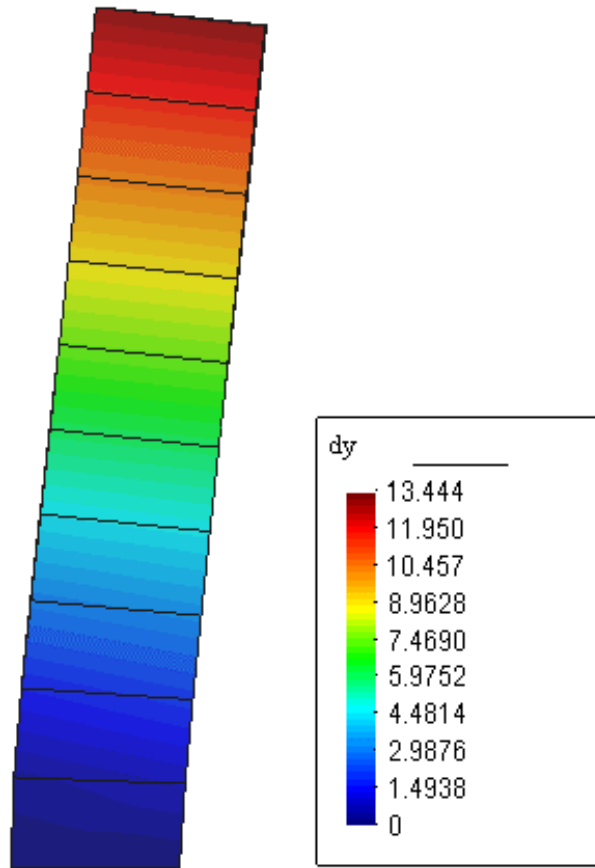


➡ Masukawa et al., “Development of RC column members in use of high strength reinforcement”, Proceedings of JCI, Vol. 19, No. 2, pp. 557-564, 1997

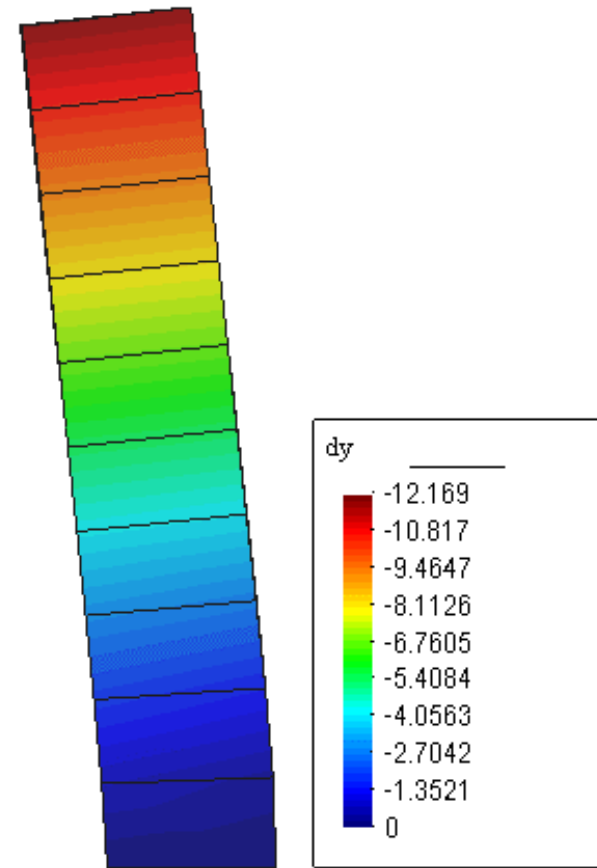




# Contour

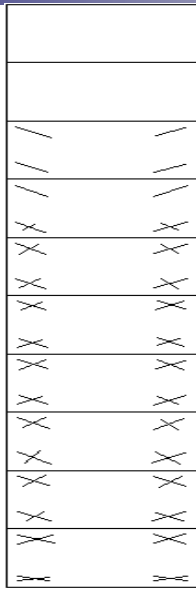
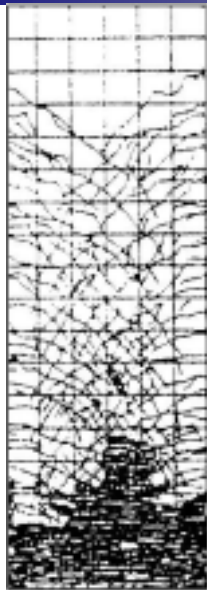


(a) At final compression

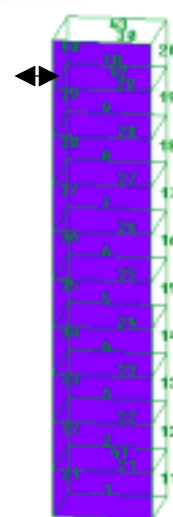
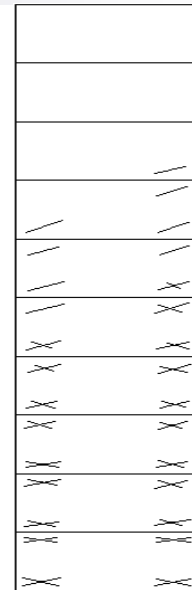
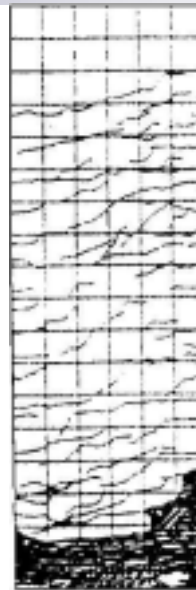


(a) At final tension

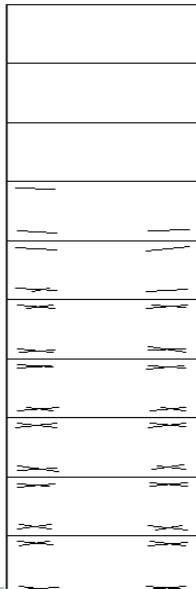
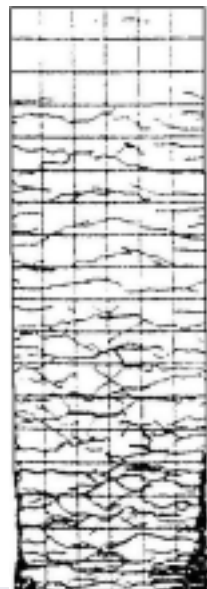
# Crack patterns



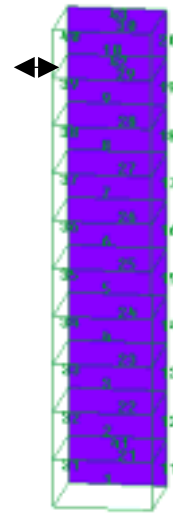
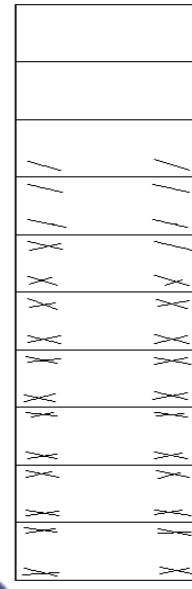
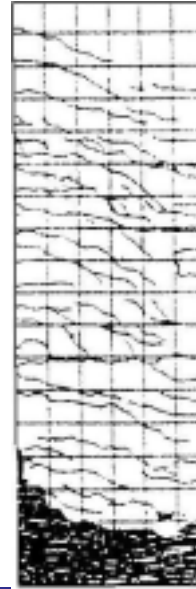
A



B



C



D



# Analysis of 1/4 prestressed concrete containment vessel(PCCV)



Sandia National Laboratories, Albuquerque, New Mexico, 2000



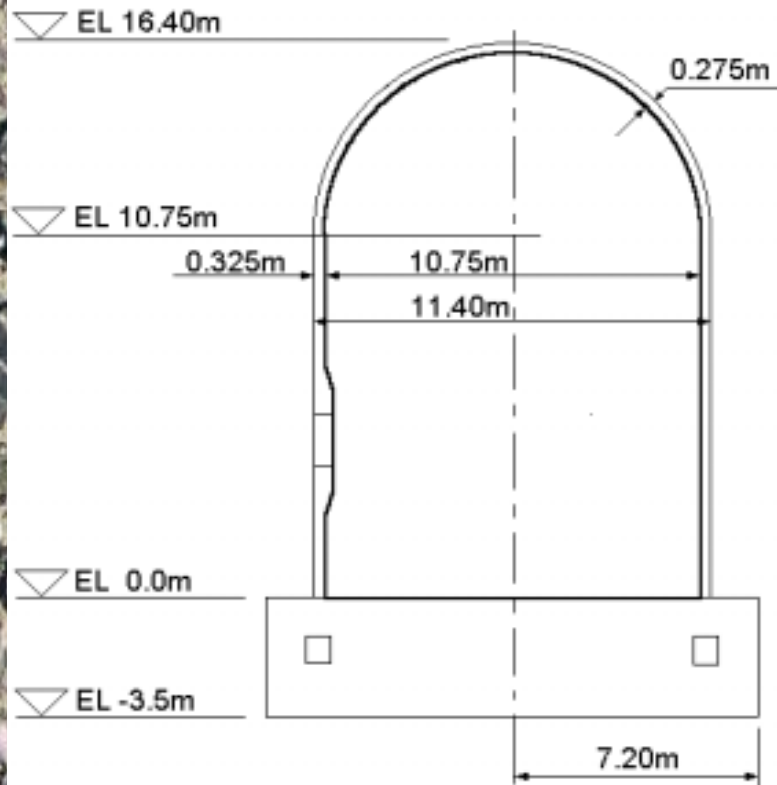


Tendon layout



Unbonded tendon

# Characteristics of Model Test and Analysis



1/4 Scale PCCV model

- The first model test to satisfy material and design details of design code (ASME Sec. III, Div.2)
  - limit state test (LST) and structural failure mechanism test (SFMT)  
Large scale model including openings and steel liners
- 3D finite element modeling including tendons, rebars and openings using DIANA
  - Pre-test analysis
  - Post-test analysis
- Introduction of volume control technique

# Model Tests (Failure Test due to Internal Pressure)

## ■ Limit state test (LST)

◆ Pressurized by nitrogen gas

◆ Structural soundness test & leakage test

◆  $1.5P_d$ ,  $2.0P_d$ ,  $2.5P_d$  and  $3.3P_d$

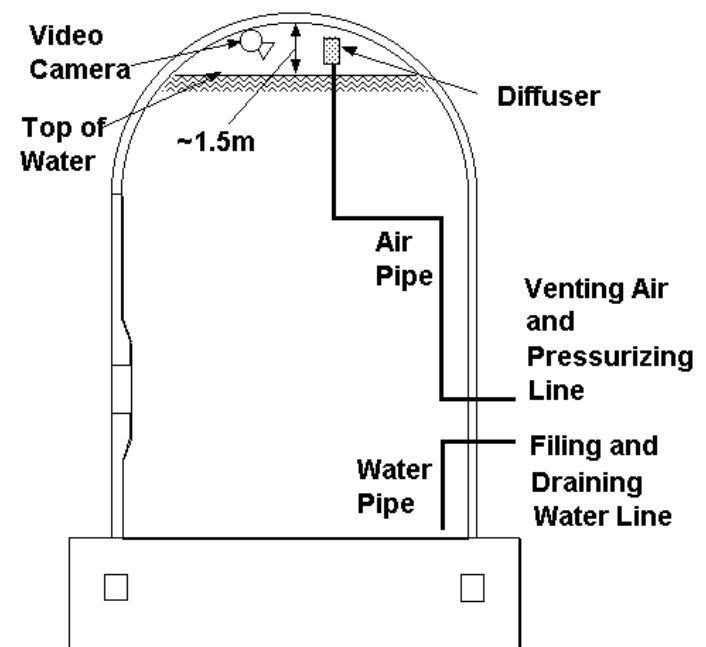
( $P_d = 0.39 \text{ MPa}$ )

◆ Functional failure due to leakage was occurred at  $3.3P_d$  due to tearing of liner

## ■ Structural failure mechanism test (SFMT)

◆ Pressurized by water

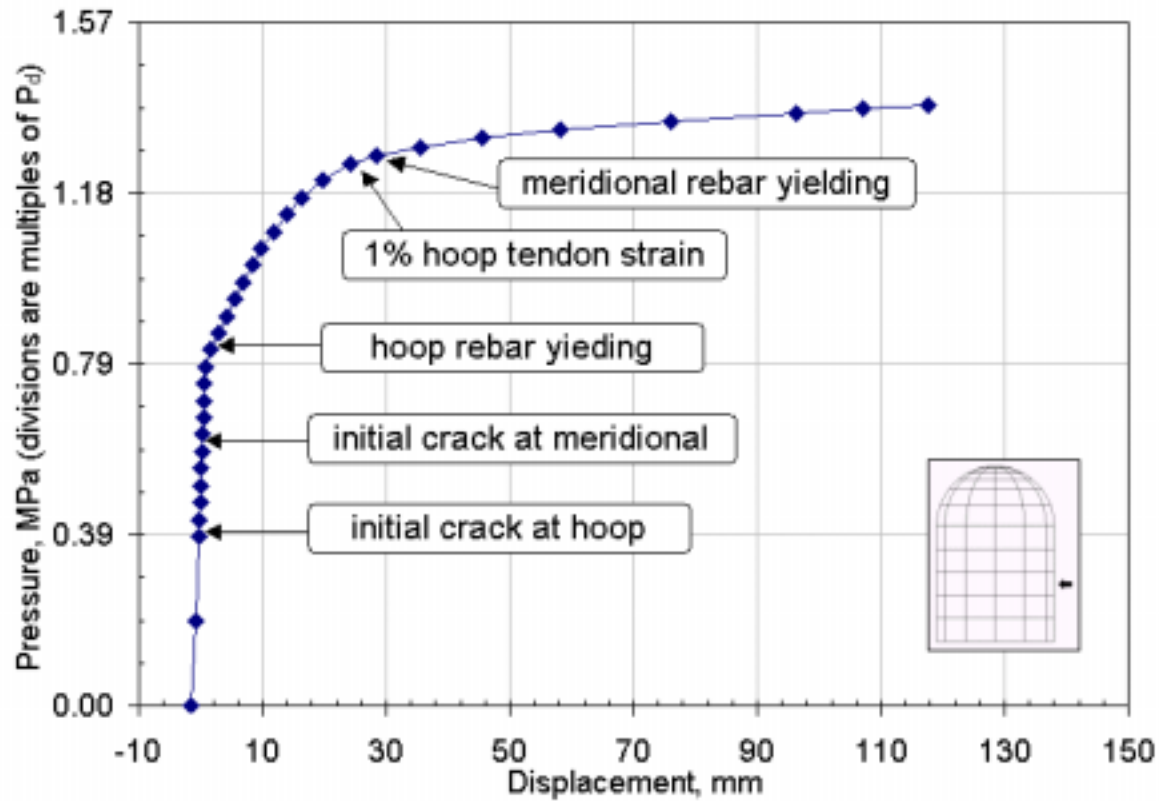
◆ Structural failure at  $3.6P_d$



# Structural Failure Mechanism Test (SFMT)



# LST test result

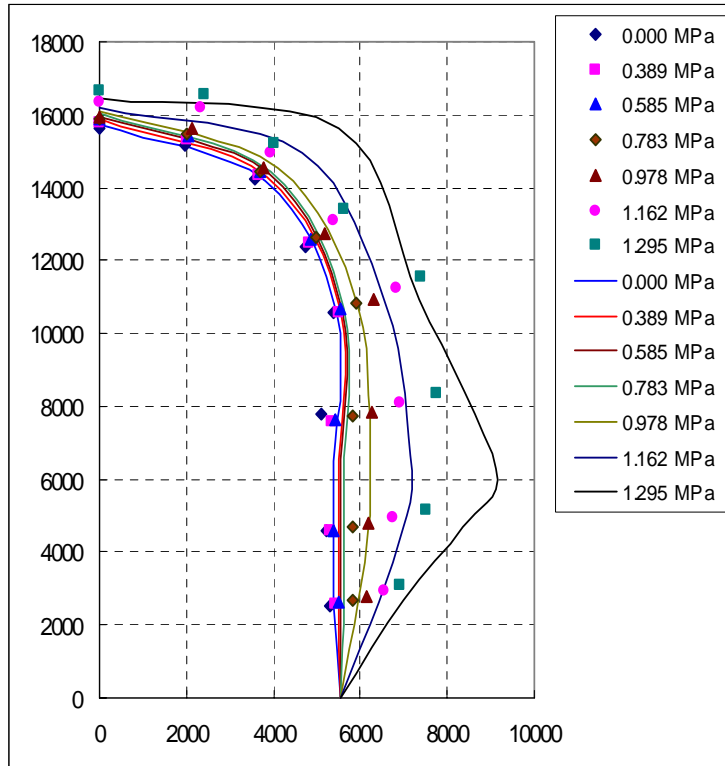


**-Hoop directional deformations govern PCCV behaviors**



# Test result and comparison with the analytical results

## ■ Deformation profile at 135° section (magnified ×100)



LST

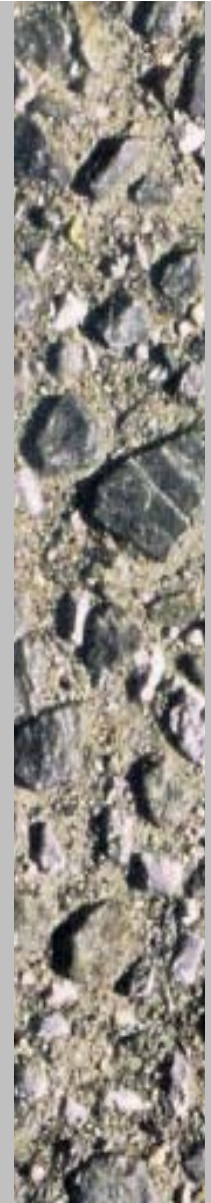
Analysis

◆ For higher pressure, analysis generally predicts larger deformations than those by the test

◆ The analysis comparably well predicts the global behavior



## ■ LST results



Pressure/Pd	Leakage Rate*	Observation
1.5	0.5	no leakage
2.0		no evidence of distress
2.5	1.6	liner strain: 2% evidence of liner tear
3.0		difficult to increase pressure
3.1	100	
3.3		
Final	900	

\* volume change (%) per day (V/Day)

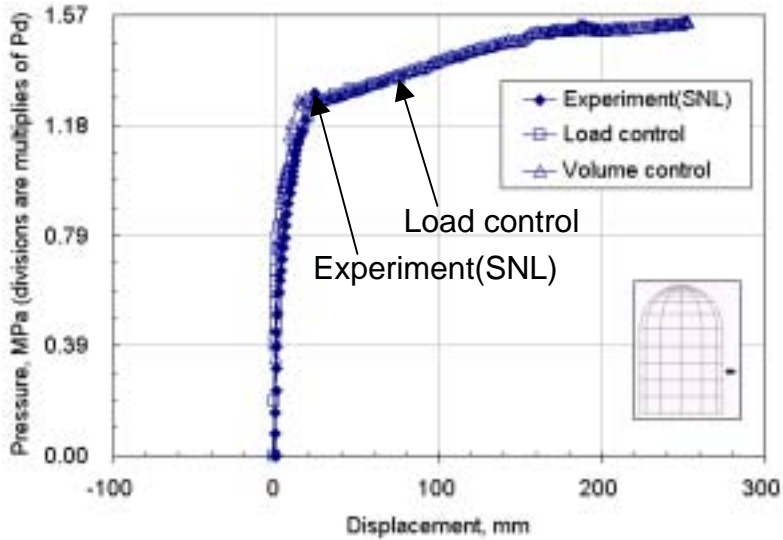
\*\*Permissible leakage rate for design pressure:

(pressurized water reactor with steel liner) 0.1% V/Day

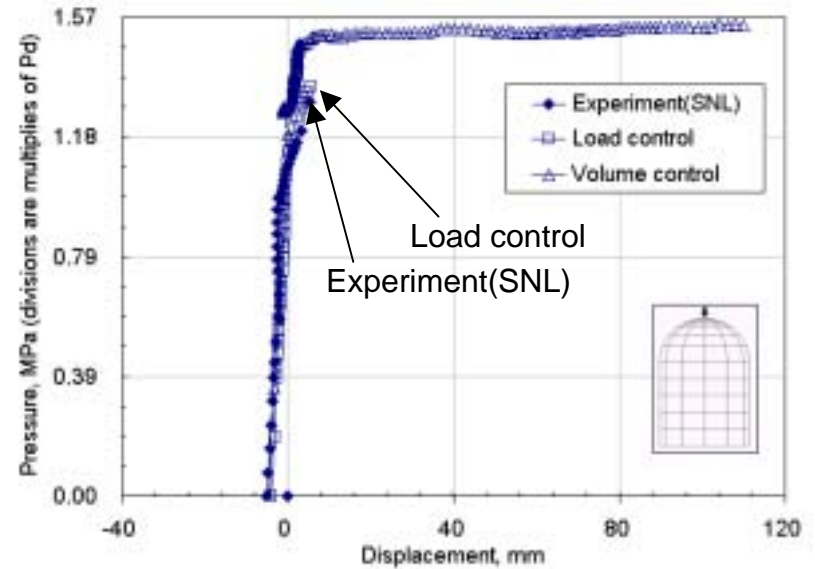


# Comparison

## ■ Deformations



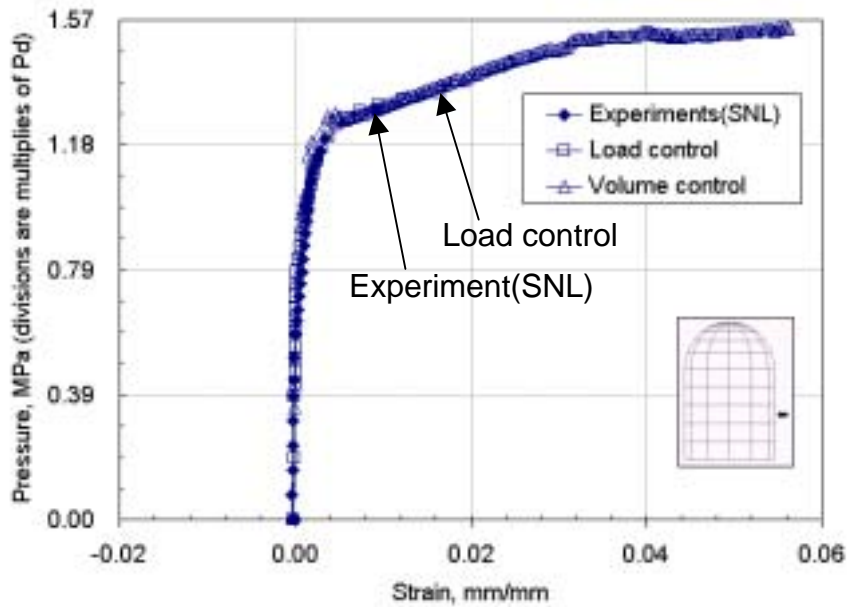
Radial displ. at midheight



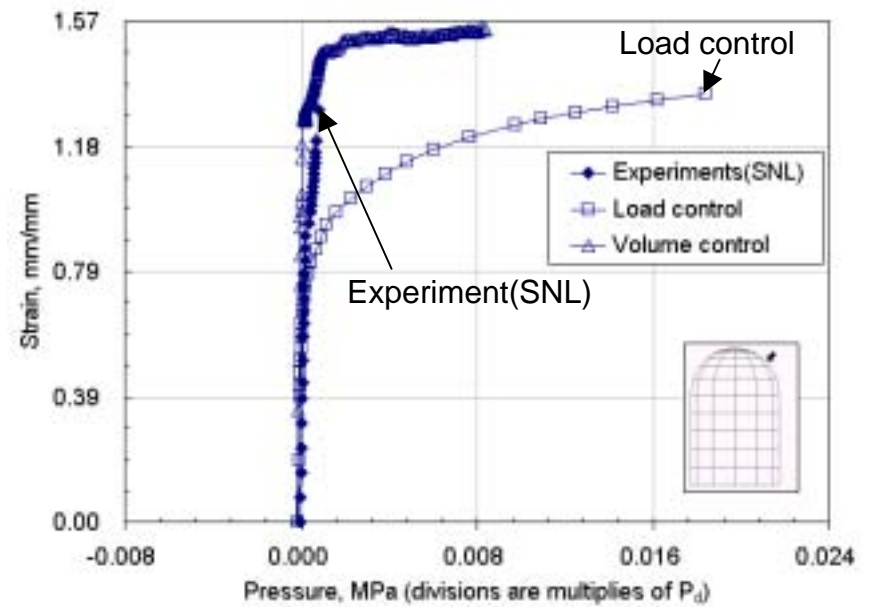
Vertical displ. at dome apex

- More stable solution is possible with volume control technique

## ■ Rebar stains



Outer rebar hoop strain at midheight



Outer rebar hoop strain at dome 45°

# Conclusions and future work

- For the failure analysis of RC shell structures using FEM, both material and structural instability problems can be solved effectively by the homogenized crack model and the volume control technique.
- In-plane constitutive laws of cracked concrete and modified Barcelona model can be useful for the modeling of the layered RC shell element and ECC repaired layers.
- Failure analysis or performance evaluation of the deteriorated RC shell structures repaired with the ECC layers is now under carried out.





---

**Thank you for  
your kind attention!**

**song@yonsei.ac.kr**

