Failure Analysis of RC Structures using Volume Control Technique
-COE Intensive Course-

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Outline

- **Introduction**
  - Characteristic of failure in concrete structures
  - Homogenized crack model
  - Volume control method

- **Modeling for cracked RC and ECC**
  - Constitutive equations of RC/ Layered shell element
  - Modeling of ECC as re-strengthening material

- **Analysis results and comparison**
  - RCCV subjected to internal pressure
  - RC tank, RC slab, and RC box culvert subjected to various loading
  - RC hollow column subjected to lateral loading
  - Verification with PCCV

- **Conclusion and future work**
Performance of deteriorated/repaired RC structures?

**Governing equations**

\[ \alpha_i \frac{\partial \theta}{\partial t} + \text{div} J_i - Q_i = 0 \]

**Durability Analysis**

- **START**
  - Size, shape, mix proportions, initial and boundary conditions
  - **Next iteration**
    - Hydration computation
      - Temperature, hydration level of each component
    - Conservation Laws satisfied?
      - yes
        - Corrosion rate, tension stiffening factor
          - at each time step
    - Output
      - Corrosion model
        - Corrosion rate, amount of O\(_2\) consumption
      - Continuum Mechanics
        - Deformation, compatibility, momentum conservation
    - Structural analysis (FEM etc.)
      - Tension stiffening factor \( \alpha \)
        \[ \sigma_t = f_t \left( \frac{\varepsilon_{tu}}{\varepsilon_t} \right)^\alpha \]

**Serviceability/Safety Analysis**

- **Performance degradation/upgrade**
  - Max. Load
  - Due to re-strengthening
  - Service-life decrease

**Bi-model porosity Distribution, Interlayer porosity**

**Microstructure computation**

- Bi-model porosity distribution, interlayer porosity

**Pore pressure Computation**

- Pore pressures, RH and moisture distribution

**Chloride Transport and equilibrium**

- Dissolved and bound chloride concentration

**Carbon dioxide Transport and equilibrium**

- Gas and dissolved CO\(_2\) concentration

**Deformation, Compatibility, Momentum Conservation**

- **Size, shape, mix proportions, initial and boundary conditions**

- **Hydration computation**

- **Conservation Laws satisfied?**

- **Corrosion model**

- **Continuum Mechanics**

- **Structural analysis (FEM etc.)**

- **Tension stiffening factor \( \alpha \)**

- **Performance degradation/upgrade**

- **Max. Load**

- **Due to re-strengthening**

- **Service-life decrease**
Characteristics of concrete fracture and analysis

- Material instability

Micro cracks localize Major macro cracks

Numerical problem in concrete fracture analysis:
- Loss of ellipticity of governing equation
- Ill-posed boundary value problem

Softening behavior
- Decreased load resistant capacity after peak
- Localized strain

Numerical drawback (Mesh sensitivity)
Structural instability

Sources of Non-linearity; Material, Geometrical, Boundary and Contact

Load control

Displacement control

Tangential solution from point A
Final solution

Circular path

Arc-length control method
Effort to solve material instability in progressive fracture analysis of concrete using FEM

- Discrete crack model and smeared crack model (in 1980’s)

- Embedded discontinuities approximations (in 1990’s)
  - Continuum approximation
    - Strain-stress relationship
    - Discontinuity: strain localization zone
  - Discrete approximation
    - Displacement jump-traction
    - Discontinuity: crack

- Weak discontinuities
  - Strain localization band width is finite ($k \gg 0$)

- Strong discontinuities
  - Strain localization band width is very small ($k \to 0$)

Homogenized crack model
Effort to solve structural instability in progressive failure of concrete shell structures using FEM

Failure analysis of RC shell structures subjected to various loadings

Load Control Method: difficulty to obtain post-peak ultimate behavior of RC structures

Displacement Control Method: difficulty to select a representative point for displacement control in 3D

Layered shell utilizing in-plane constitutive models of RC

Volume Control Method

- Remove the drawback of load control method
- Overcome the limitation for displacement control method
Volume Control Method with Pressure Node

- Pressure Node: the uniform change of applied pressure on the shell element ($\Delta p$) (Song and Tassoulas, IJNME, 1993)

\[
\Delta V = \int_{b^e} n^T \cdot \Delta u \, db^e = \left( \int_{b^e} n^T \cdot N \, db^e \right) \Delta U
\]

\[
\int_{b^e} N^T (t + \Delta t) \, db^e = - (\Delta t + \Delta \rho) \int_{b^e} N^T n \, db
\]

\[
K_e \Delta U = - (\Delta t + \Delta \rho) \int_{b^e} N^T n \, db^e - F_e
\]

\[
\begin{bmatrix}
K_e & \int_{b^e} N^T n \, db^e \\
\int_{b^e} N^T n \, db^e & 0
\end{bmatrix}
\begin{bmatrix}
\Delta U \\
\Delta p
\end{bmatrix}
= \begin{bmatrix}
\rho \int_{b^e} N^T n \, db^e - F_e \\
\Delta V
\end{bmatrix}
\]
Path dependant pseudo-volume control technique

- **Pseudo Volume** (Song et. al, J. Str. Eng. 2002)

- **Path-dependent Volume** (Song et. al, Nuclear Eng. and Design, 2003)
Algorithm for path dependant volume control technique

Using tangent Stiffness Matrix

\[ K^{i+1} = \beta K^i \]

is updated according to Loading history

Volume increment \( \Delta V^\text{ext}_m \) is controlled according to \( \beta \)

\[ V_m = v + \Delta V^\text{ext}_m \]

\( (\Delta V^\text{ext}_m)_{\text{next}} \rightarrow \Delta V^\text{ext}_m \)

Checking \((U^E)_k\) and \((U^A)_k\) at each loading steps

\( \| (U^E)_k \| < \) tolerance\( \| (U^A)_k \| \)

\( (d^A) \) is modified according to comparison experiment displ. with analysis displ.

\( (d^A) \) is updated according to Loading history

\[ \frac{\partial P}{\partial V_{\text{next}}} > 0 \quad \text{Loading} ; V_{\text{next}} \rightarrow P_{\text{next}} \]

\[ \frac{\partial P}{\partial V_{\text{next}}} < 0 \quad \text{Unloading} ; V_{\text{next}} \rightarrow P_{2\text{next}} \]

\[ \frac{\partial P}{\partial V_{\text{next}}} \quad \text{Reloading} ; V_{\text{next}} \rightarrow P_{3\text{next}} \]

\[ \text{deformed volume, } V_m^* \]

\[ \text{Initial volume, } V \]

Load. Yonsei Univ.
Solution procedure
path-dependant
volume control method
Layered shell element

- Degenerated, isoparametric, serendipity, quadratic shell element with drilling degree of freedom
- Geometrical nonlinearity is considered by adopting total Lagrangian formulation
- In plane constitutive laws applied to each layer of element consists of RC layers and PL layers
Constitutive law for each layer

- Concrete under compression: Elasto-plastic fracture model (Maekawa et al.)
- Cracked concrete: Smeared fixed crack model
- Concrete under shear: Crack density model (Maekawa and Li)

Shear locking + Membrane locking

Reduced integration (2 x 2 Gaussian quadrature)
In-plane constitutive models of cracked concrete

- Local strain of concrete
- Crack location
- Tension Stiffening
- Shear Stress transferred
- Shear along Cracks
- Compression Softening
- Multi-system
- Local Response
- Mean Response
- Cracked concrete
- Damage zone
- Mean stress
- Average tensile stress
- Average tensile strain
- Shear stress transferred
- Mean shear strain
- Compression strength reduction
- Mean normal strain
- Crack width
- Comp. stress
- Comp. strain
Cracking criteria of concrete

Cracking is affected by past loading history.

By taking account influence of continuum fracture in past compression, cracking criterion can be defined in the space of biaxial principal stresses

\[
\frac{\sigma_1}{(R_f \cdot f_t)} = 1.0 \quad \text{Compression-tension domain}
\]

\[
\left( \frac{\sigma_1}{(R_f \cdot f_t)} \right) + 0.26 \left( \frac{\sigma_2}{\sigma_1} \right) = 1.0 \quad \text{Tension-tension domain}
\]

Where, \( \sigma_1, \sigma_2 \): principal stress (\( \sigma_1 > \sigma_2 \))

\( f_t \): uniaxial tensile strength

\( R_f \): tensile strength reduction factor

< Failure envelope in tension-compression domain >

< Normalized tensile strength and fracture parameter >
Concrete model under tensile stress is unrelated to spacing of cracks, direction of reinforcing bars and reinforcement ratio.

Tension stiffening effect is known to increase overall stiffness of RC in tension compared with that of single reinforcing bar.

\[
\sigma_t = f_t \left( \frac{\varepsilon}{\varepsilon_{tu}} \right)^c
\]

Where, \(\sigma_t\) : average tensile stress, \(\varepsilon\) : average tensile strain
\(f_t\) : uniaxial tensile strength, \(\varepsilon_{tu}\) : cracking strain
\(c\) : stiffening parameter
Reinforcing bar model is based on the assumed cosine distribution of bar stress and concrete tension stiffening.

Where, $f_{yt}(-f_{yt})$: Bare bar yield strength

$f_{yt1}$: Yield strength of bar in concrete

$E_s$: Initial bar stiffness (before yielding)

$E_b$: Bauchinger's effect stiffness

$E_b = E_B = \frac{1}{E_s}$,\( E_s > 0 \)

$\varepsilon_0 = \varepsilon_p + f_{yt1} \left( \frac{1}{E_s} - \frac{1}{E_b} \right)$

$E_b = -E_s \log_{10}(10 \varepsilon_p)/6$
Multi-directional smeared crack approach

One-way active crack governs the overall nonlinearity as for in-plane cyclic shear
Two-way active cracks may control the overall nonlinearity as for out of plane cyclic action
Orthogonal two-way fixed crack model

Concrete

Already crack occurred?

Calculation of un-cracked concrete stress according to loading conditions

Reinforcement

Calculation of x, y directional reinforcement stress using reinforcement model

History renewal of concrete reinforcement

Combination of concrete stress reinforcement stress

Stress of reinforced concrete

Calculation of cracked concrete stress using constitutive models

- Compression model parallel to crack direction
- Tension model normal to crack direction
- Tension stiffening model for concrete in RC
- Tension softening model for plain concrete
- Modeling of unloading-reloading path
- Re-contact model
- Coupled compression-tension model
- Shear transfer model

Transform stress into global coordinate
ECC as durable overlays and repair layers

Engineered Cementitious Composites (ECC)  
V.C. Li et al. 1992~

- cementitious matrix + short random fibers
- conscious micromechanics-based design of material composition … Performance Driven Design Approach
- high performance with low fiber content (~2%)
High performance cementitious composites

- cementitious matrix + fibers
- multiple cracking

- high overall ductility in tension and shear with ease of processing and variability of shaping
- damage tolerance, durability, ...

Stress (MPa) vs. Strain (%) graph showing fracture localization and multiple cracking.
Characteristic of ECC behavior

**Mechanical properties:**
- High tensile strain capacity (~5%)
- Small crack width $O(10\text{~}100\ \mu\text{m})$

- Strain hardening
- Multiple cracking
- Localized failure
3-d homogenized crack model

- 3-D formulation of homogenized crack model

- Mixture rule (R.E.V)
  \[
  \sigma = \mu_i \sigma_i + \mu_j \sigma_j \quad \varepsilon = \mu_i \varepsilon_i + \mu_j \varepsilon_j \quad \mu_i + \mu_j = 1
  \]

- Equilibrium & compatibility
  \[
  \sigma_{ij} = \sigma_{ij} = \sigma_{ij} \quad \varepsilon_{ij} = \varepsilon_{ij} = \varepsilon_{ij} \quad \tau_{ij} = \tau_{ij} \quad \varepsilon_{ij} = \varepsilon_{ij} \quad \tau_{ij} = \tau_{ij} \quad \gamma_{ij} = \gamma_{ij} = \gamma_{ij}
  \]

- Velocity discontinuity at crack surface

Representative elementary volume (REV)

- \( \sigma_i, \varepsilon_i \): concrete
- \( \sigma_j, \varepsilon_j \): crack
- \( \sigma^c, \varepsilon^c \): concrete with crack

\[
[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \quad [K] = [K^c] = \begin{bmatrix} K_N & 0 & 0 \\ 0 & K_{s1} & 0 \\ 0 & 0 & K_{s2} \end{bmatrix}
\]
If $t \ll H$, 

$$
\mu_i = \frac{BW(H - t)}{HBW} \approx 1
$$

$$
\mu_j = \frac{BWt}{HBW} \approx \frac{t}{H}
$$

$$
\frac{1}{g} [\delta] g \underbrace{t}_{\text{averaged crack strain}}
$$

Let $\mu$ be the ratio of the crack area and REV., i.e. $(\mu := \frac{1}{H})$

Then, $[\delta] \varepsilon = \mu_i [\delta] \varepsilon_i + \mu_j [\delta] \varepsilon_j$ can be written as

$$
= 1 \cdot [\delta] \varepsilon_i \underbrace{\frac{t}{H}}_{\text{t}} \cdot \frac{1}{g} \varepsilon_j
$$

$$
= [\delta] \varepsilon_i + \mu g
$$
• Structural relationship

\[
\sigma = [D] \varepsilon \quad \sigma = \mu_i \sigma + \mu_j \sigma
\]

\[
[\delta] \varepsilon \approx [\delta] \varepsilon + \mu g
\]

\[
[\delta] \varepsilon = [A] \varepsilon + [B] g
\]

\[
[A] = \begin{bmatrix}
-D_{21} & -K_{11} & -D_{23} & -D_{24} & -D_{25} & -D_{26} \\
C_1 & \mu C_1 & C_1 & C_1 & C_1 & C_1 \\
-D_{41} & -D_{42} & -D_{43} & -K_{22} & -D_{45} & C_1 \\
C_2 & C_2 & C_2 & \mu C_2 & C_2 & C_2 \\
-D_{51} & -D_{52} & -D_{53} & -D_{54} & -K_{33} & C_2 \\
C_3 & C_3 & C_3 & \mu C_3 & C_3 & C_3
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
0 & K_{12} + \mu D_{24} & K_{13} + \mu D_{25} \\
K_{21} + \mu D_{42} & 0 & K_{23} + \mu D_{45} \\
K_{31} + \mu D_{52} & K_{32} + \mu D_{54} & 0
\end{bmatrix}
\]

\[
C_1 = D_{22} + \frac{K_{11}}{\mu} \\
C_2 = D_{44} + \frac{K_{22}}{\mu} \\
C_3 = D_{55} + \frac{K_{33}}{\mu}
\]
• Structural relationship of (tensile) crack

\[ [\delta \varepsilon] = [S_1] \varepsilon + \mu g \]

\[ \varepsilon = [S_1] \varepsilon \quad \text{or} \quad [\delta \varepsilon] = [S] \varepsilon \]

i.e.,

\[ [\delta \varepsilon] = [S] \varepsilon + \mu g \]

\[ \therefore \mu g ([(\delta \varepsilon) - [S]) \varepsilon \]

\[ \mu \frac{1}{([\delta \varepsilon] - [S]) \varepsilon} \]

Let \[ [S_2] = \frac{1}{\mu} ([\delta \varepsilon] - [S]) \]

then \[ [S_2] \varepsilon \]
• Total strain relationship

\[
\begin{bmatrix} \delta \end{bmatrix} \varepsilon = [S] \varepsilon
\]

\[
[S] = ([I] + \frac{1}{\mu} [B])^{-1} ([A] + \frac{1}{\mu} [B] \delta)
\]

\[
\varepsilon = [S_1] \varepsilon
\]

\[
[S_1] = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\
0 & 0 & 1 & 0 & 0 & 0 \\
S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\
S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]
• Homogenized constitutive equation

\[
\sigma = \mu_i \sigma + \mu_i \sigma_i \approx \sigma_i = [D][S_1] \varepsilon
\]

\[
\sigma = [D^{eq}] \varepsilon \quad [D^{eq}] = [D][S_1]
\]

**Remark**

Crack width, \( t \), is removed in the final constitutive equation only expressed with \( \mu \). This is a solution for the mesh sensitivity problem without the introduction of additional length scale such as a characteristic length.

→ Regularization of the continuum model
Constitutive equation for crack

Compression

- Bifurcation analysis for crack initiation

\[
F(n) = \det(n_i D_{ijkl}^e n_l)
\]

\[
[K] = \begin{bmatrix}
K_{N}^{ep} & 0 & 0 \\
0 & K_{S1}^{ep} & 0 \\
0 & 0 & K_{S2}^{ep}
\end{bmatrix}
\]

\[
t = \{ t_n^{cr}, t_{S1}^{cr}, t_{S2}^{cr} \}^T
\]

\[
\sigma_{yy} \quad \tau_{xy} \quad \tau_{yz}
\]

\[
= [\delta ] \sigma = [K] g
\]

\[
p(x,y,z)
\]

\[
\phi, \theta, \theta
\]

\[
x, y, z
\]
Failure criteria and softening curve (compression)

- **Drucker-Prager type**

\[ F = \alpha I_1 + \sqrt{J_2} - k(\bar{\sigma}, \bar{\epsilon}^p) = 0 \]

- **Hardening and softening function**

  1) Song and Na (1997)

\[ k(\bar{\epsilon}^p, \bar{\sigma}) = \sigma_0 + \bar{\sigma} \bar{\epsilon}^p + (\sigma_\infty - \sigma_0)[1 - e^{-\beta \bar{\epsilon}}] - \frac{3}{2} \alpha p \]

  2) Song et al. (2003), Farahat et al. (1995)

\[ k(W_p) = k_0 e^{-[(\beta W_p)^y - \xi]^2} \]

  3) Barcelona model

  - \( k(\epsilon^p) \) (J. Lubliner, 1996)

\[ k(\epsilon^p) = f_{N0}[((1 + a_N) \exp(-b_N \epsilon^p) - a_N \exp(-2b_N \epsilon^p))] \]

  - \( k(W_p) \) (Modified Barcelona Model, MBM)

\[ k(W_p) = f_{N0}[((1 + a_N) \exp(-b_N W_p) - a_N \exp(-2b_N W_p))] \]
Failure criteria and softening function (tension)

- **Failure criterion (Gopalaratnam and Shah, 1985)**
  \[ F = \sigma_1 - k(\cdot) \]

- **Hardening and softening function**
  1) Gopalaratnam and Shah (1985)
     \[ k(t) = f_t(e^{-\kappa t^2}) \]
  2) Song et al. (2003)
     \[ k\left(\frac{g}{\sqrt{y}}\right) = f_t(e^{-\kappa (\eta g \sqrt{y})^2}) \]
  3) Barcelona model
     - \( k(\varepsilon^p) \) (J. Lubliner, 1996)
       \[ k(\varepsilon^p) = f\varepsilon_0[(1 + a_1)\exp(-b_1 \varepsilon^p) - a_t \exp(-2b_1 \varepsilon^p)] \]
     - \( k(\frac{g}{\sqrt{y}}) \) (MBM)
       \[ k\left(\frac{g}{\sqrt{y}}\right) = f\varepsilon_0[(1 + a_1)\exp(-b_1 \frac{g}{\sqrt{y}}) - a_t \exp(-2b_1 \frac{g}{\sqrt{y}})] \]
Start

INPUT
Input data - geometry, boundary condition, material properties, etc

Load3D
Evaluates equivalent nodal forces

INCREM
Increments applied loads

STIF3D
Calculate element stiffness for elastic and elasto-plastic behavior

REACT
Calculate displacement and reactions

RESID3B
Compute residual stress, backward Euler integration scheme

YLD3D and FLOWPT
Determine flow vector, a, d, etc

INVAR
Evaluates invariant stress

CONVER
Check if solution has converged

OUTERS
Print results for this load increment

End
Hardening and softening curve for concrete and ECC (tension)

- J. Lubliner (1996)

\[ k\left(\frac{\rho}{\overline{p}}\right) = f_{t0}\left[(1 + a_t)\exp(-b_{t1}\frac{\rho}{\overline{p}}) - a_t\exp(-2b_{t1}\frac{\rho}{\overline{p}})\right] \]

1. \( a_t < 1 \quad \rightarrow \quad \text{Concrete} \)
2. \( a_t > 1 \quad \rightarrow \quad \text{ECC} \)
Result for 2% polyethylene fiber contained ECC

![Graph showing stress-strain relationship for concrete and ECC.]

1. \( a_t = 0.1 \): Concrete
2. \( a_t = 5 \): ECC
- Experimental result (Tetsushi Kanda, 1998)
- polyetylane fiber: 0.75%, 1%, 1.25%

![Graph showing stress-strain relationship for different fiber volumes](image)

- Increased $a_t$
- Decreased $b_t$
0.75% ECC

- Experiment (Tetsushi Kanda, 1998)
- Analysis (a4 b50)
1% ECC

实验 (Tetsushi Kanda, 1998)
分析 (a4 b25)
1.25% ECC

![Graph showing stress-strain relationship with experiment (Tetsushi Kanda, 1998) and analysis (a5 b25).]
Mesh sensitivity check on softening behavior

![Graph showing axial stress vs. displacement for different elements.](image)
Comparison with experimental result

- experiment (Gopalaratnam & Shah, 1985)
- analysis (HCM, 2003)
- analysis (HCM+MBM)
Mesh sensitivity check for tension

![Graph showing Axial Stress vs Prescribed Displacement for different element counts]
Tension failure with damage

![Stress vs. Displacement Graph]

- **Stress (MPa)**
  - Axes: 0 - 4
  - Markers: 0.005, 0.01, 0.015, 0.02, 0.025, 0.03, 0.035

- **Displacement (mm)**
  - Axes: 0 - 0.035

- **Lines:**
  - Blue: no damage
  - Red: damage

- **Legend:**
  - no damage
  - damage
Flexural failure

- 528 elements
- 2977 nodes

<table>
<thead>
<tr>
<th>Specimen (mm)</th>
<th>Notch (mm)</th>
<th>$f_t$ (MPa)</th>
<th>$E_c$ (GPa)</th>
<th>$V_c$</th>
<th>$K_N$ (GPa/m)</th>
<th>$K_S$ (GPa/m)</th>
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<tr>
<td>190.5×76.2×3 8</td>
<td>6.5×12.7</td>
<td>2.97</td>
<td>28.06</td>
<td>0.2</td>
<td>2,680</td>
<td>1,680</td>
</tr>
</tbody>
</table>
Results

- experiment (Bazant, et al., 1992)
- analysis1 (FEM, 1998)
- analysis2 (EDM, 2001)
- analysis3 (HCM, 2003)
- analysis (HCM+MBM)
RC panel of simulating RCCV wall subjected to biaxial tension

RC panel specimen

Modeling as shell element

Layered shell element

Specification

- Compressive strength of concrete ($\sigma_c$): 41.9MPa
- Tensile strength of concrete ($\sigma_t$): 2.87MPa
- Modulus of elasticity of concrete ($E_c$): 23828MPa
- Yield strength of reinforcement ($\sigma_y$): 410MPa
- Modulus of elasticity of reinforcement ($E_s$): 205744MPa
FEM MESH and Boundary condition

RC panel is discretized as 10 x 10 mesh
Stress-strain curve of rebars

Hoop direction

Meridional direction

Top of RC panel

Bottom of RC panel
Stress-strain curve of rebars

Hoop direction

Meridional direction

Top of RC panel

Bottom of RC panel

CMME Lab. Yonsei Univ.
Stress-strain curve of rebars

Hoop direction

Meridional direction

Top of RC panel

Bottom of RC panel
Crack patterns of RC panel

Top of RC panel by experiment

Top of RC panel by volume control analysis

Bottom of RC panel by experiment

Bottom of RC panel by volume control analysis
Deformed shape of RC panel

Hoop dir. displacement at 1st crack occurrence

Meridional dir. displacement at 1st crack occurrence

Hoop dir. displacement at rebar yielding

Meridional dir. displacement at rebar yielding
RCCV subjected to internal pressure

**Specification**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Compressive strength of concrete (σ_c)</td>
<td>46.0 MPa</td>
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<tr>
<td>Tensile strength of concrete (σ_t)</td>
<td>3.45 MPa</td>
</tr>
<tr>
<td>Modulus of elasticity of concrete (E_c)</td>
<td>33,100 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio of concrete (ν_c)</td>
<td>0.20</td>
</tr>
<tr>
<td>Yield strength of reinforcement (σ_y)</td>
<td>450.0 MPa</td>
</tr>
<tr>
<td>Modulus of elasticity of reinforcement (E_s)</td>
<td>214,000 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio of reinforcement (ν_s)</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Modeling with or without considering foundation

Layered shell element
Global behaviors

Radial displacement – Pressure relationship at mid-height

Vertical displacement – Pressure relationship at spring-line
Crack patterns of RCCV

1.0Pd (0.31MPa)

2.0Pd (0.62MPa)

3.0Pd (0.93MPa)
Deformed shape of RCCV

1.0Pd (0.31MPa)    2.0Pd (0.62MPa)

3.0Pd (0.93MPa)    Ultimate pressure
RC tank subjected to reversed cyclic loading

< Harada et al., 2001 >

- Compressive strength of concrete ($\sigma_c$) : 28.0MPa
- Tensile strength of concrete ($\sigma_t$) : 2.20MPa
- Modulus of elasticity of concrete ($E_c$) : 22,600MPa
- Yield strength of reinforcement ($\sigma_y$) : 384.0MPa
- Modulus of elasticity of reinforcement ($E_s$) : 183,000MPa

Specifying the material properties and the loading direction. Modeling as shell element and layered shell element.
Relative horizontal displacement - load curve

Comparison experiment result with path-dependant volume control result

Comparison path-dependant volume control result with pre-test analysis result
Crack status of RC tank

Multi-directional cracks occurrence due to reversed cyclic loading

Crack status of RC tank by experiment

Crack status of RC tank by volume control analysis
Deformed shape of RC tank

Horizontal load = 3,037 kN

Horizontal load = -2,200 kN
RC slab subjected to out-of-plane cyclic loading

Specification

- Compressive strength of concrete ($\sigma_c$): 37.0MPa
- Tensile strength of concrete ($\sigma_t$): 3.70MPa
- Yield strength of reinforcement ($\sigma_y$): 380.0MPa
- Modulus of elasticity of reinforcement ($E_s$): 206,000MPa

< Irawan, 2001 >
Central displacement - load curve

Comparison existing volume control result with path-dependant volume control result (IS1)

Comparison existing volume control result with path-dependant volume control result (IS2)
Analysis result according to number of layered shell element

Analysis is terminated (step=134)

Analysis is terminated (step=72)

Analysis is terminated (step=170)

Analysis result according to number of layered shell element

- Experiment (Iwan, 1995)
- Considering only material nonlinearity
- Considering material and geometric nonlinearity

Transverse load [kN]
Central displacement [mm]
Crack status of RC slab

Specimen IS1 : vertical load = 193 kN

Specimen IS2 : vertical load = 193 kN
Deformed shape of RC slab

Specimen IS1: vertical load = 178 kN

Isotropic reinforcement arrangement

Specimen IS2: vertical load = 144 kN

Anisotropic reinforcement arrangement

Stiffness of IS1 > Stiffness of IS2
Due to reinforcement arrangement
RC box culvert subjected to cyclic loading

**Specification**

*< Irawan, 1995 >*

- Compressive strength of concrete \((\sigma_c)\) : 50.0MPa
- Yield strength of reinforcement \((\sigma_y)\) : 500.0MPa

Modeling with and without considering haunch

Layered shell element
Load-deflection curve of RC box culvert

Static failure load: 48 tonf

Model of considering without considering haunch

Model of considering with haunch is reinforced concrete layer

Model of considering with haunch is plain concrete layer
Crack status of RC box culvert

Wall of RC box culvert

Top slab and wall of RC box culvert
Deformed shape of RC box culvert

Model of considering without haunch

Model of considering with haunch is reinforced concrete layer

Model of considering with haunch is plain concrete layer
RC hollow column under lateral loading

Concrete

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>76.5 MPa</td>
</tr>
<tr>
<td>$E_c$</td>
<td>31.4 GPa</td>
</tr>
<tr>
<td>$v$</td>
<td>0.21</td>
</tr>
<tr>
<td>$f_t$</td>
<td>2.63 MPa</td>
</tr>
</tbody>
</table>

Reinforcement

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_y$</td>
<td>350 MPa</td>
</tr>
<tr>
<td>$f_u$</td>
<td>421 MPa</td>
</tr>
<tr>
<td>$E_s$</td>
<td>200 MPa</td>
</tr>
</tbody>
</table>
Stress-strain curve of concrete and reinforcing steel (Masukawa et al., 1997)

Modeling as shell element
Load-displacement curve

(a) At final compression

(a) At final tension
Crack patterns

A

B

C

D
Analysis of 1/4 prestressed concrete containment vessel (PCCV)

Sandia National Laboratories, Albuquerque, New Mexico, 2000
Characteristics of Model Test and Analysis

- The first model test to satisfy material and design details of design code (ASME Sec. III, Div. 2)
  - limit state test (LST) and structural failure mechanism test (SFMT)
  - Large scale model including openings and steel liners
- 3D finite element modeling including tendons, rebars and openings using DIANA
  - Pre-test analysis
  - Post-test analysis
- Introduction of volume control technique

1/4 Scale PCCV model
Model Tests (Failure Test due to Internal Pressure)

- **Limit state test (LST)**
  - Pressurized by nitrogen gas
  - Structural soundness test & leakage test
  - $1.5P_d, 2.0P_d, 2.5P_d$ and $3.3P_d$
    ($P_d = 0.39$ MPa)
  - Functional failure due to leakage was occurred at $3.3P_d$ due to tearing of liner

- **Structural failure mechanism test (SFMT)**
  - Pressurized by water
  - Structural failure at $3.6P_d$
Structural Failure Mechanism Test (SFMT)
LST test result

-Hoop directional deformations govern PCCV behaviors
Test result and comparison with the analytical results

- **Deformation profile at 135° section** (magnified ×100)

- For higher pressure, analysis generally predicts larger deformations than those by the test

- The analysis comparably well predicts the global behavior
## LST results

<table>
<thead>
<tr>
<th>Pressure/Pd</th>
<th>Leakage Rate*</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>no leakage</td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td>no evidence of distress</td>
</tr>
<tr>
<td>2.5</td>
<td>1.6</td>
<td>liner strain: 2%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>evidence of liner tear</td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td>difficult to increase pressure</td>
</tr>
<tr>
<td>3.1</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Final</td>
<td>900</td>
<td></td>
</tr>
</tbody>
</table>

* volume change (%) per day (V/Day)

**Permissible leakage rate for design pressure:
(pressurized water reactor with steel liner) 0.1% V/Day
Comparison

- **Deformations**

  ![Graph 1](Image 1)

  Load control
  Experiment(SNL)

  Radial displ. at midheight

  ![Graph 2](Image 2)

  Load control
  Experiment(SNL)

  Vertical displ. at dome apex

  More stable solution is possible with volume control technique
Rebar stains

Outer rebar hoop strain at midheight

Outer rebar hoop strain at dome 45°
Conclusions and future work

- For the failure analysis of RC shell structures using FEM, both material and structural instability problems can be solved effectively by the homogenized crack model and the volume control technique.

- In-plane constitutive laws of cracked concrete and modified Barcelona model can be useful for the modeling of the layered RC shell element and ECC repaired layers.

- Failure analysis or performance evaluation of the deteriorated RC shell structures repaired with the ECC layers is now under carried out.
Thank you for your kind attention!
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