Adjustment and Reconciliation of Data or Design Values: Application of Fuzzy Optimization Concept

Shinya Kikuchi, Ph.D., PE, Professor
Department of Civil and Environmental Engineering
University of Delaware
Newark, Delaware 19716 USA
Kikuchi@ce.udel.edu

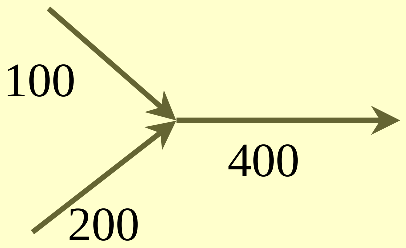
August 2004

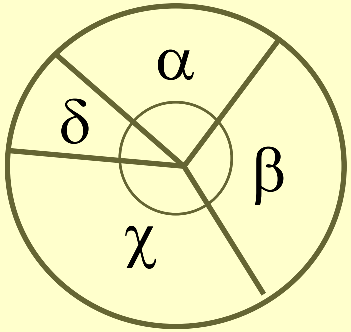
Nature of the Problem

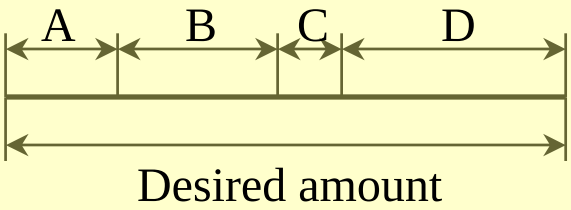
➢ **Inconsistency problem:**

Given: Available data does not support the underlying principle or required relationships.

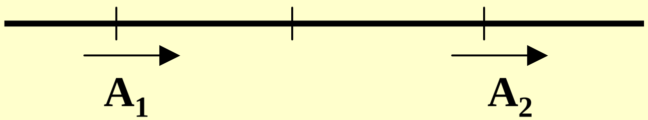
Objective: Adjust or reconcile the values so that the values are consistent with the principles and other relationships.

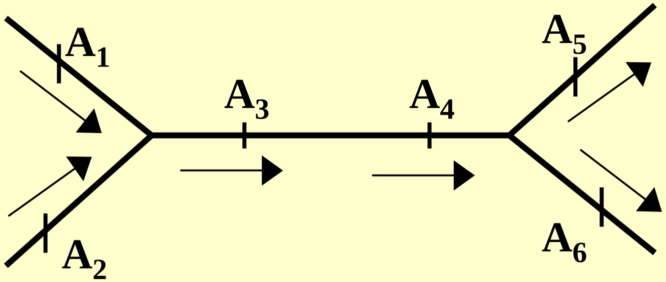


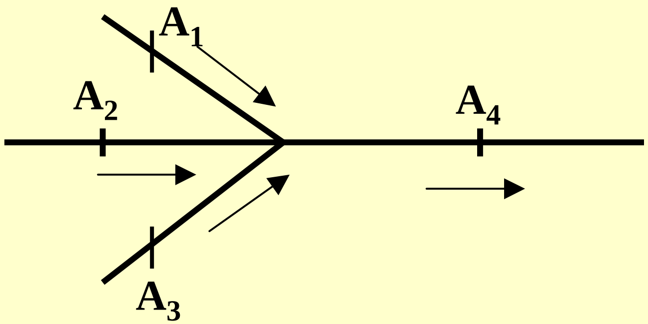


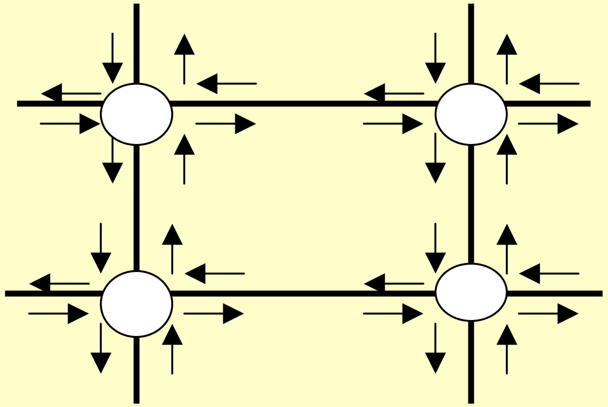


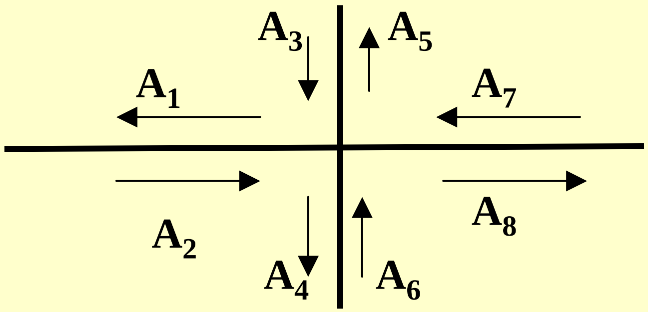
Inconsistency Problem Examples: Measured Outflow and Inflow Volumes

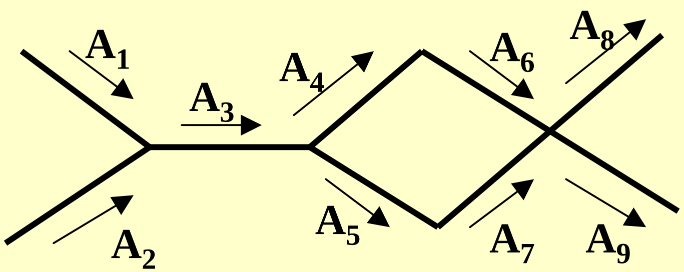












Nature of the Problem in General

Requirements for the values

$f_1\ (x_1, x_2, x_3\ldots) = Z_1$

$f_2\ (x_1, x_2, x_3\ldots) \approx Z_2$

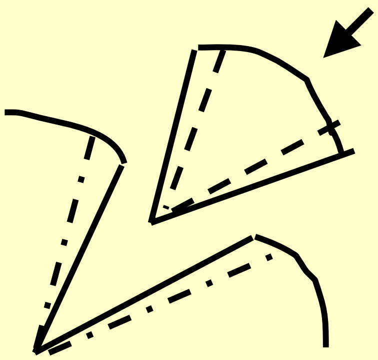
$f_3\ (x_1, x_2, x_3\ldots) >\approx Z_3$

Obtained data $(x_1', x_2', x_3'..)$

$f_1\ (x_1', x_2', x_3'\ldots) \neq Z_1$

$f_2\ (x_1', x_2', x_3', \ldots)$ (Not $\approx$) Z_2

$f_3\ (x_1', x_2', x_3'\ldots)$ (Not $>\approx$) Z_3

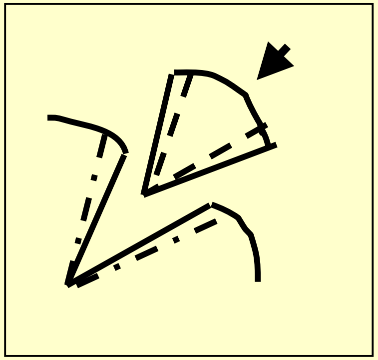


Adjustment problem

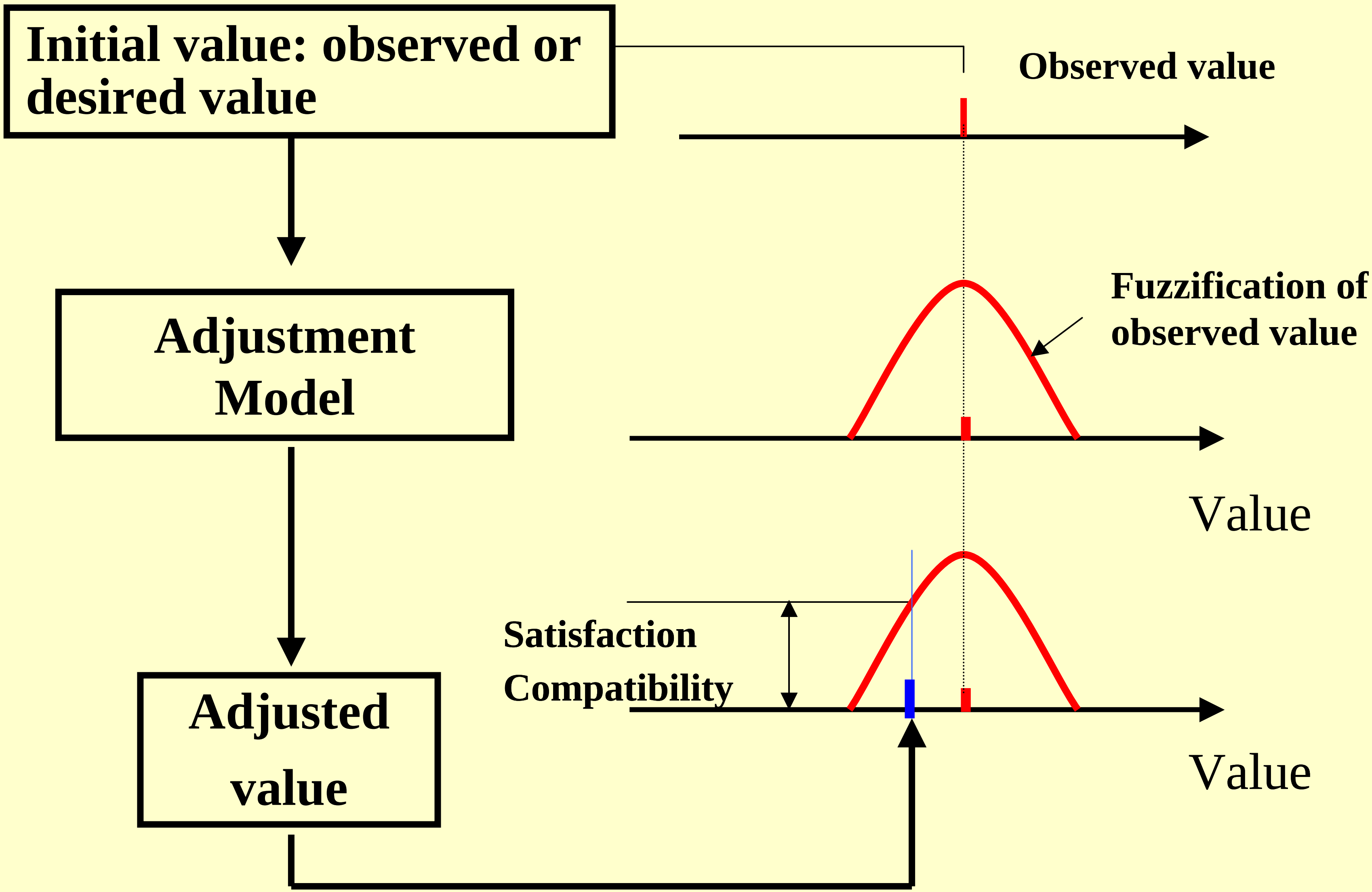
Objective: Find $\mathbf{x_1, x_2, x_3, \ldots}$

Requirements for the Model

- **Respects the initial values as much as possible.**
- **Incorporates the analyst's knowledge (hunch, accuracy and reliability of data collection, nature of relationships).**
- **Has the logical basis (explainable).**
- **Handles a large complicated situation (consistency).**

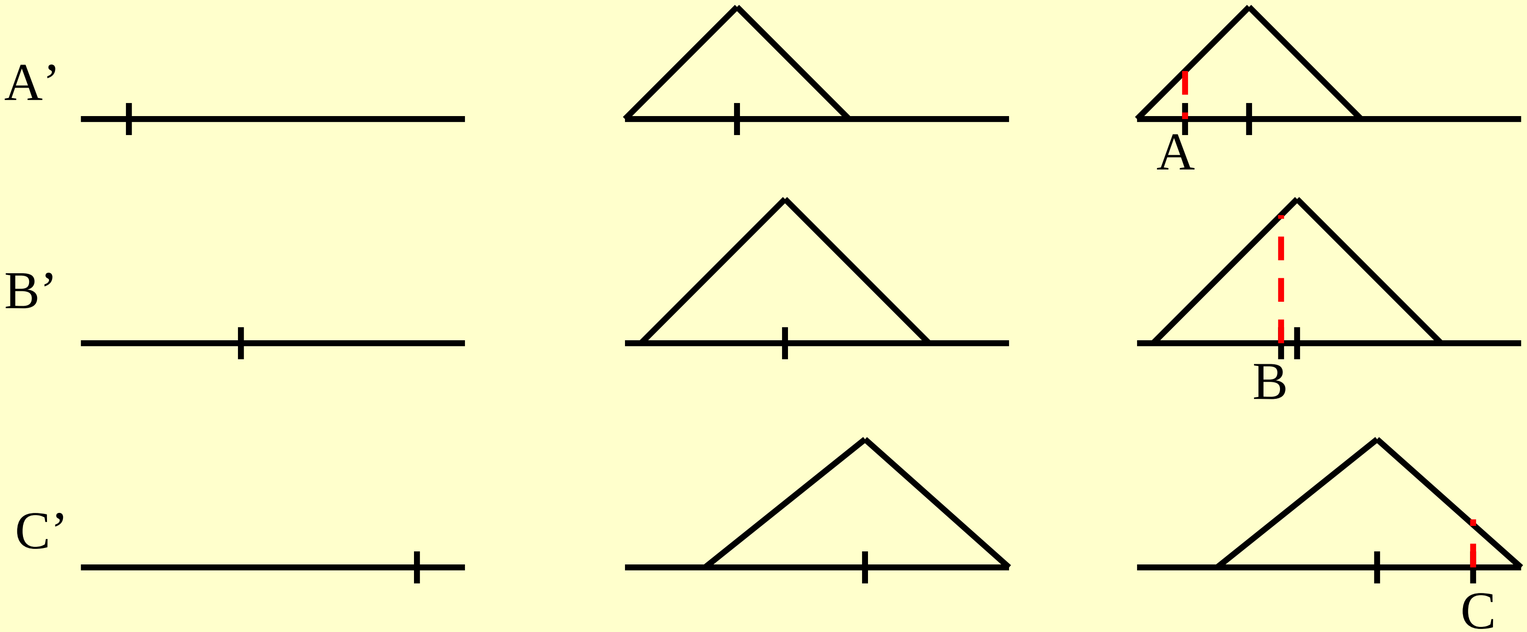


Basic Concept of Model Formulation



APPROACH

Data Adjustment **A + B = C**



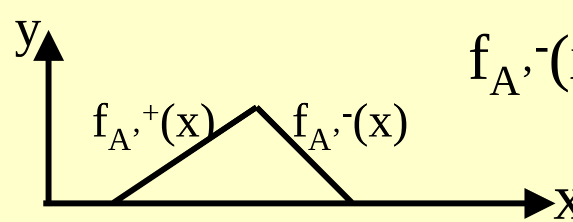
Max h

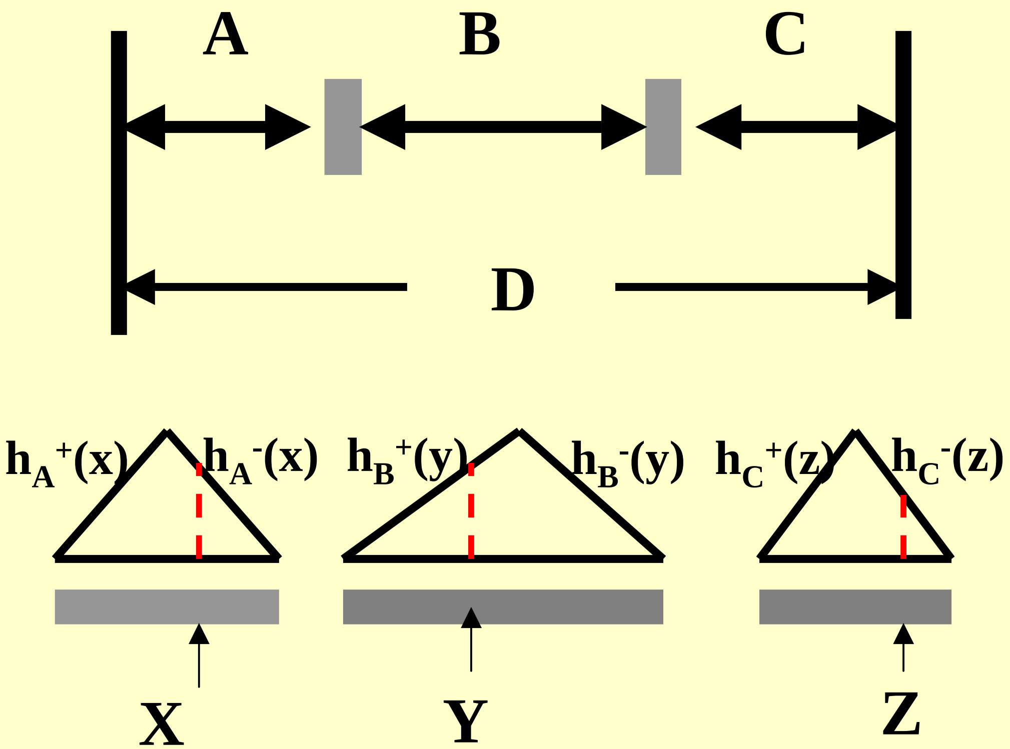
$h_{A'}(x) \geq h,\ \ h_{B'}(y) \geq h,\ h_{C'}(z) \geq h$

$x + y = z$

$h_{A'}(x) \geq h \Rightarrow \quad f_{A'}^{+}(x) \geq h$

$f_{A'}^{-}(x) \geq h$





Question: Select X, Y, Z

$$X + Y + Z = D$$

Model Formulation

Max. h

Subject to:

$h_A^+(x) > h \Rightarrow a_1x + b_1 > h$

$h_A^-(x) > h \Rightarrow a_2x + b_2 > h$

$h_B^+(y) > h \Rightarrow a_3y + b_3 > h$

$h_B^-(y) > h \Rightarrow a_4y + b_4 > h$

$h_C^+(z) > h \Rightarrow a_5z + b_5 > h$

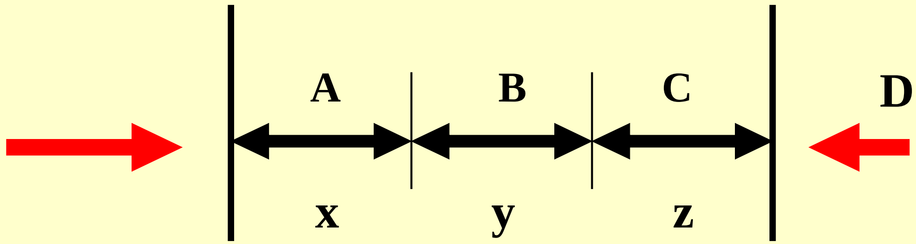
$h_C^+(z) > h \Rightarrow a_5z + b_6 > h$

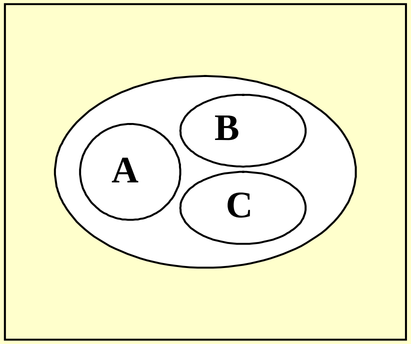
$x + y + z = D$

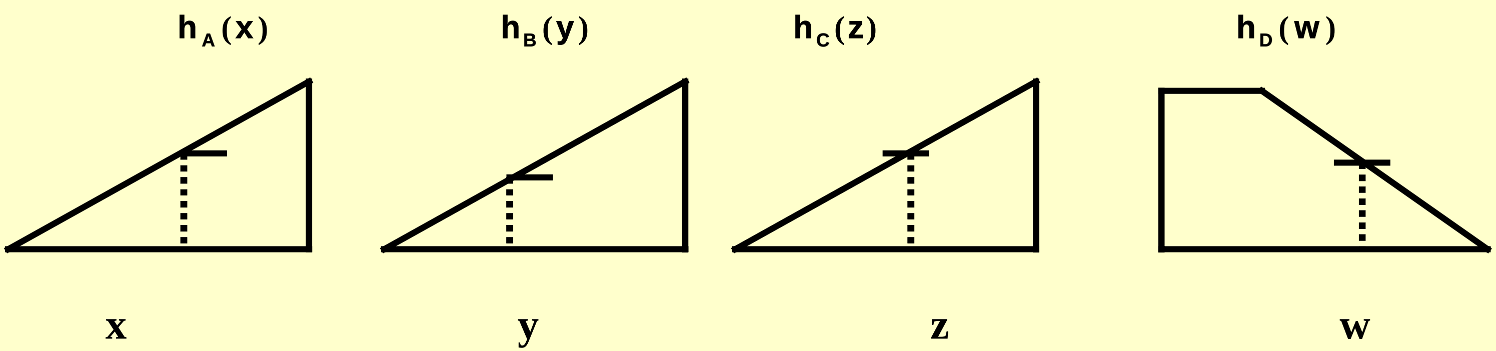
$x, y, z \geq 0$

Example 1

Resource Allocation



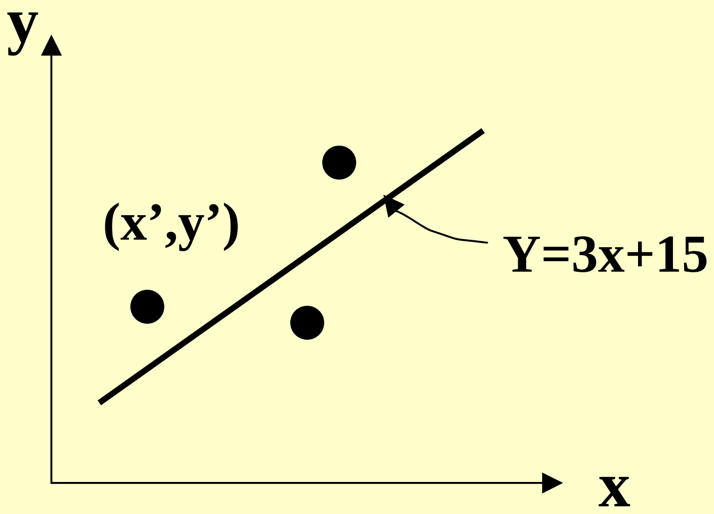


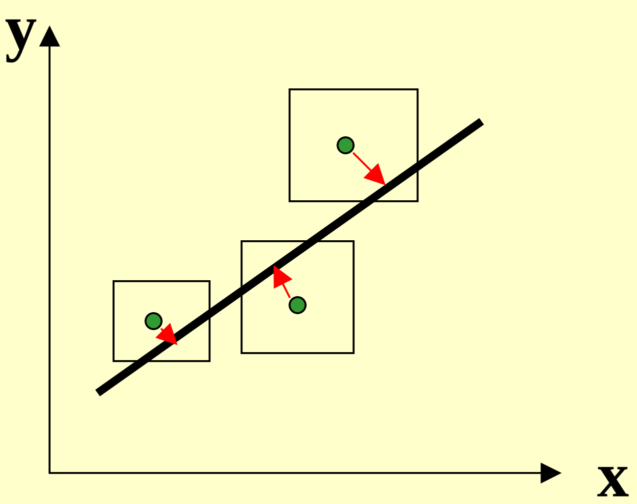


Problem : Select, x, y, z, and w

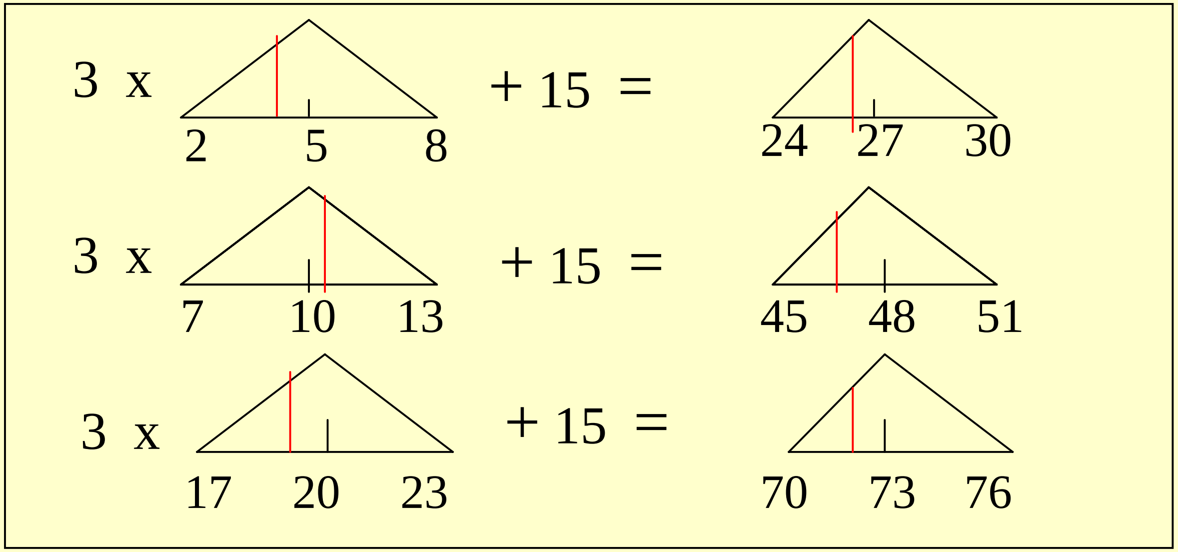
Such that Min { $h_A(x)$, $h_B(y)$ and $h_C(z)$,$h_D(w)$} is maximized

Example 2: Fitting to a Pre-determined Line



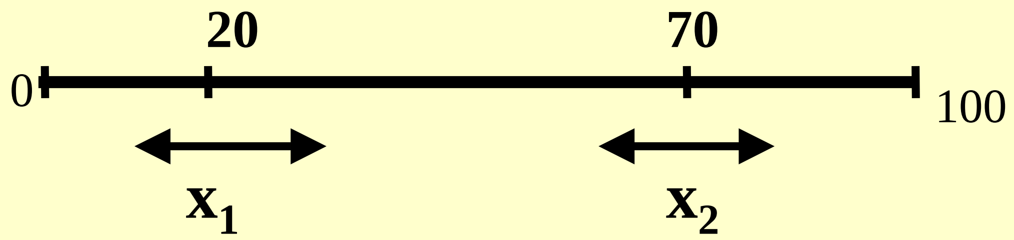


x'	y'	x	y
5	27	3.5 (h=0.5)	25.5 (h=0.5)
10	48	10.5 (h=0.8)	46.5 (h=0.5)
20	73	18.8 (h=0.6)	71.4 (h=0.7)



Overall h = 0.5

Example 3



I: x2 must be to the right of x1.

II: x1 should be much greater than 0.

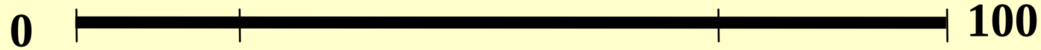
III: x2 should be much less than 100.

IV: x1 and x2 should be as far away as possible.

V: x1 should be near 20.

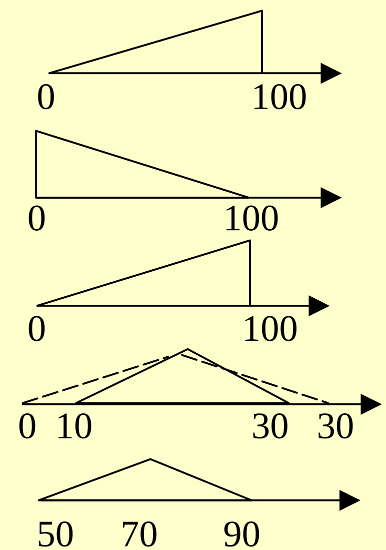
VI: x2 should be near 70.

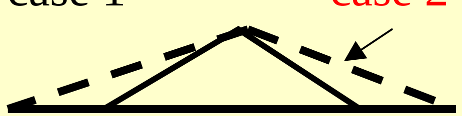
Example 3

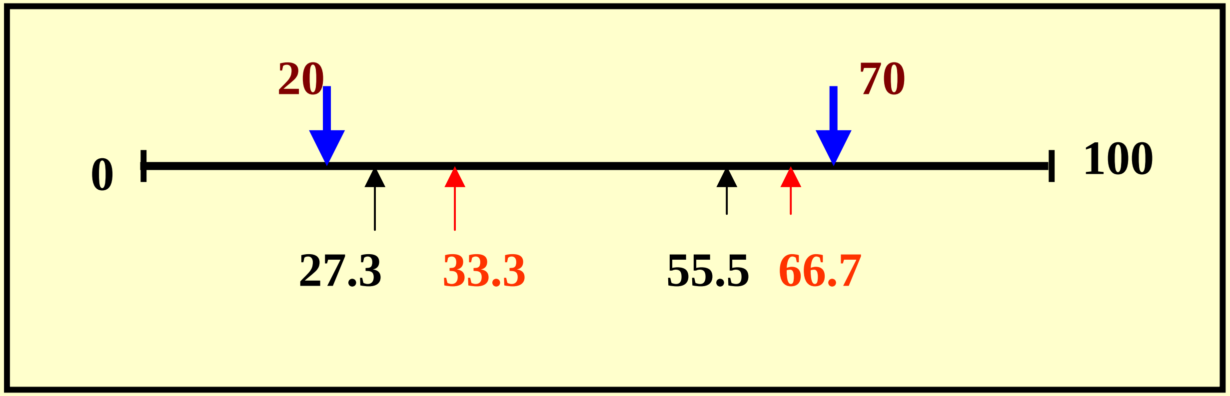


Formulation: Max h

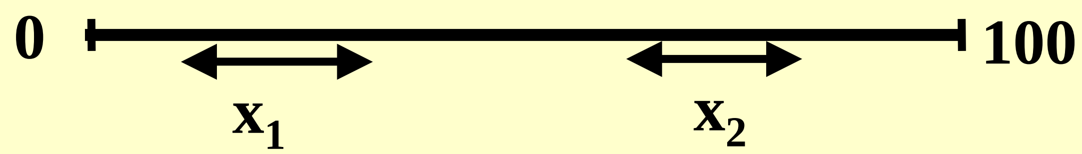
			Case 1	Case 2
I	$X_1 < X_2$	$X_1 - X_1 > 0$		
II	$0 \ll X_1$	$h_{II}(X_1) \geq h$	h_{II} = 0.27	0.33
III	$X_1 \ll 100$	$h_{III}(X_2) \geq h$	h_{III} = 0.44	0.33
IV	$0 \ll (X_1 - X_1)$	$h_{IV}(X_2 - X_1) \geq h$	h_{IV} = 0.28	0.33
V	$X_1 \approx 20$	$h_V(X_1) \geq h$	h_V = 0.27	0.33
VI	$X_2 \approx 70$	$h_{VI}(X_2) \geq h$	h_{VI} = 0.27	0.83







Example 4



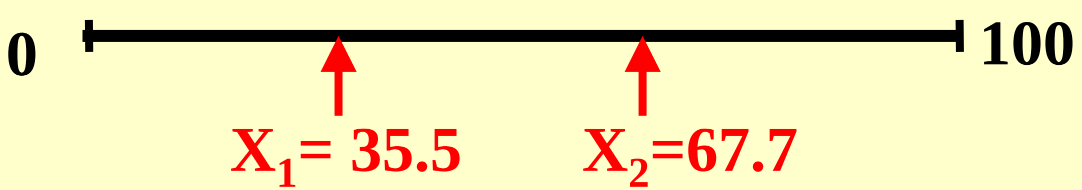
I: x_2 must be to the right of x1.

II: x_1 should be much greater than 0.

III: x_2 should be much less than 100.

IV: x_1 and x_2 should be as far away as possible.

V: $2\,x_1 - x_2 \approx 10$.

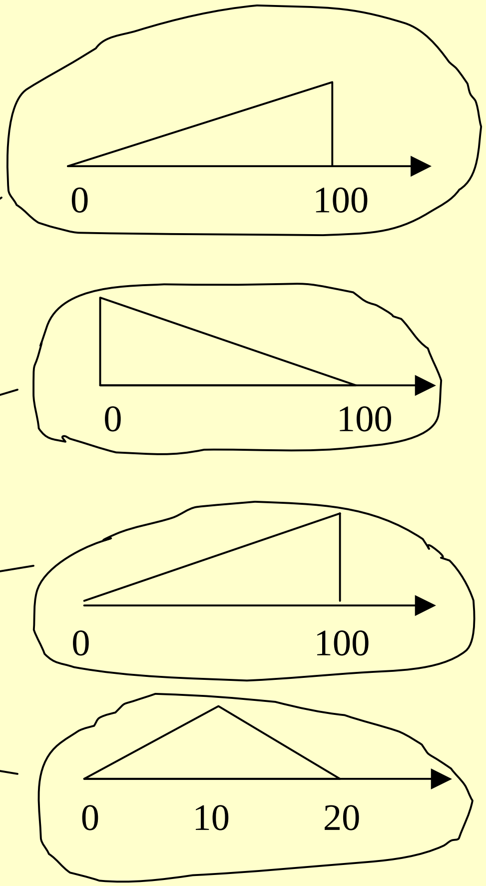


Example 4

Objective : max h

Constraints:

I	$X_1 < X_2$	$X_1 - X_2 > 0$
II	$0 \ll X_1$	$h_{II}(X_1) \geq h$
III	$X_2 \ll 100$	$h_{III}(X_2) \geq h$
IV	$0 \ll (X_2 - X_1)$	$h_{IV}(X_2 - X_1) \geq h$
V	$2X_1 - X_2 \approx 10$	



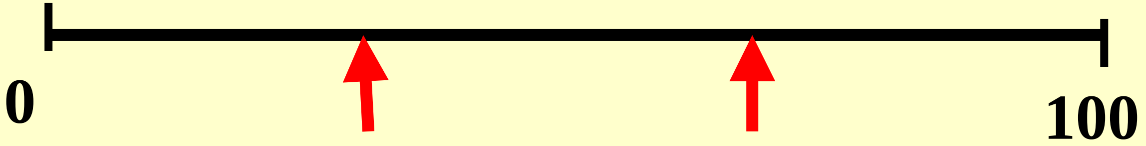
$h_{II} = 0.35$

$h_{III} = 0.32$

$h_{IV} = 0.\ 32$

$h_V = 0.43$

Solution: $2X_1 - X_2 = 4.3$



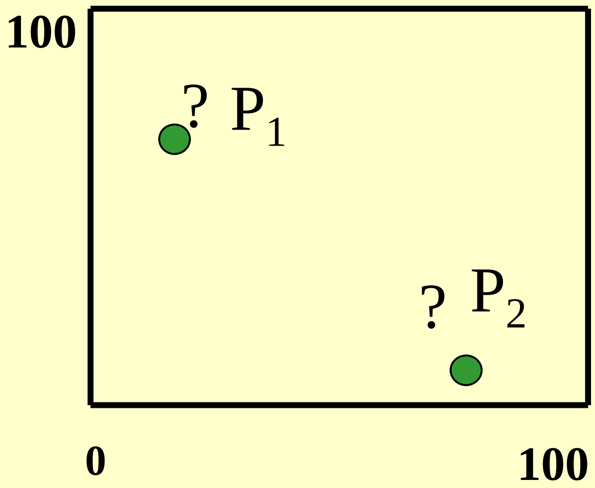
$X_1 = 35.5$

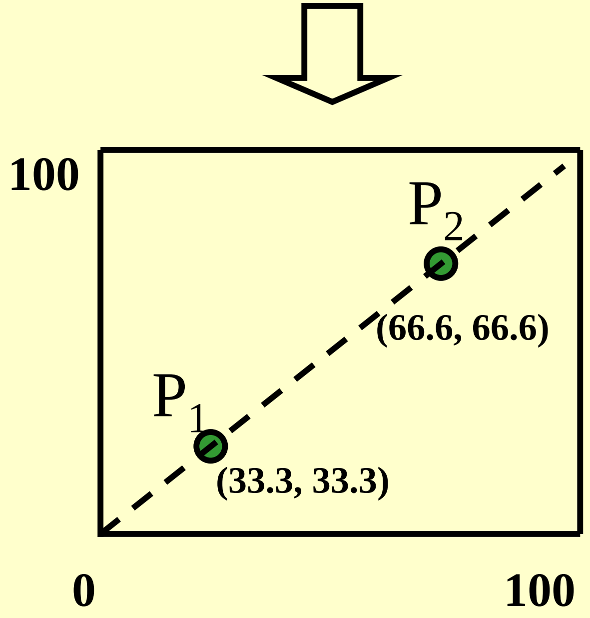
$X_2 = 67.7$

Example 5

Requirements

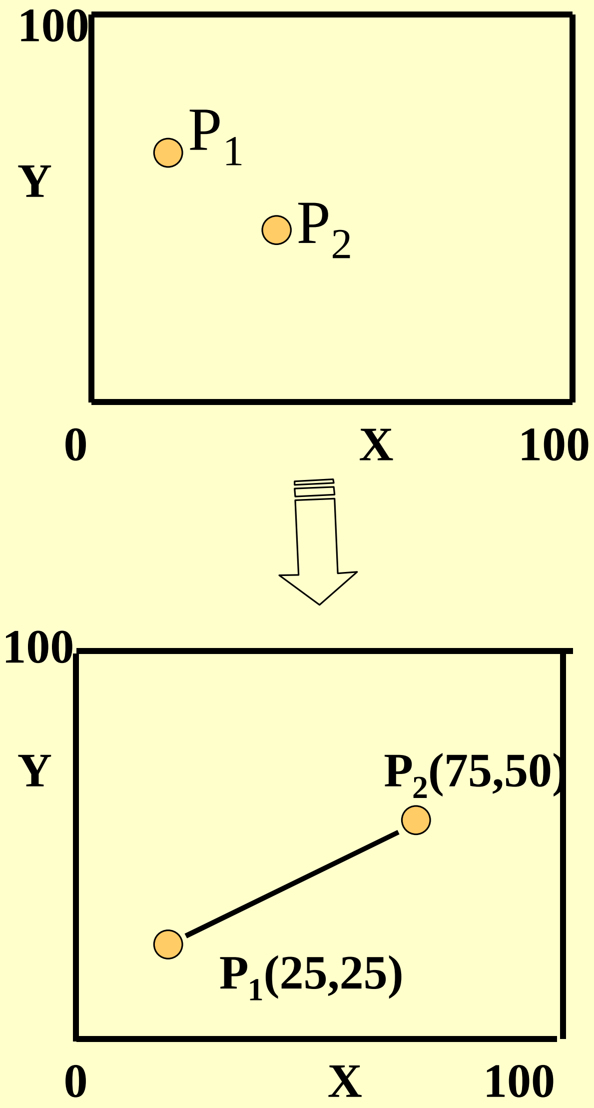
- **P_1 and P_2 must be far away from each other**
- **P_1 and P_2 must be far away from the walls.**
- **P_2 must be to the right of P_1.**





Example 5

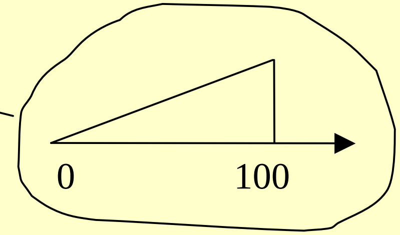
- P_1 and P_2 far away from each other.
- P_1 and P_2 far away from the X axis.
- P_1 and P_2 far away from the Y axis.
- Slope of line connecting P_1 and P_2 = 0.5.



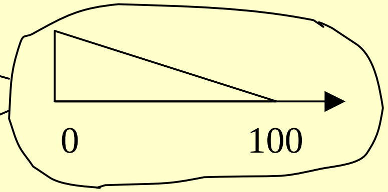
Objective : max h

Constraints: $X_2 - X_1 > 0$ and $Y_2 - Y_1 > 0$

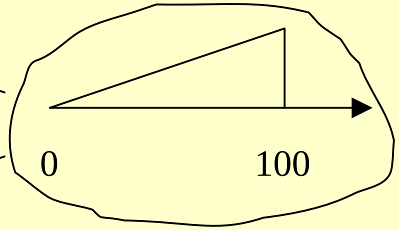
I	$0 \ll Y_1$	$h_I(Y_1) \geq h$	$h_I = 0.25$
II	$0 \ll X_1$	$h_{II}(X_1) \geq h$	$h_{II} = 0.25$



III	$X_2 \ll 100$	$h_{III}(X_2) \geq h$	$h_{III} = 0.25$
IV	$Y_2 \ll 100$	$h_{IV}(Y_2) \geq h$	$h_{IV} = 0.5$



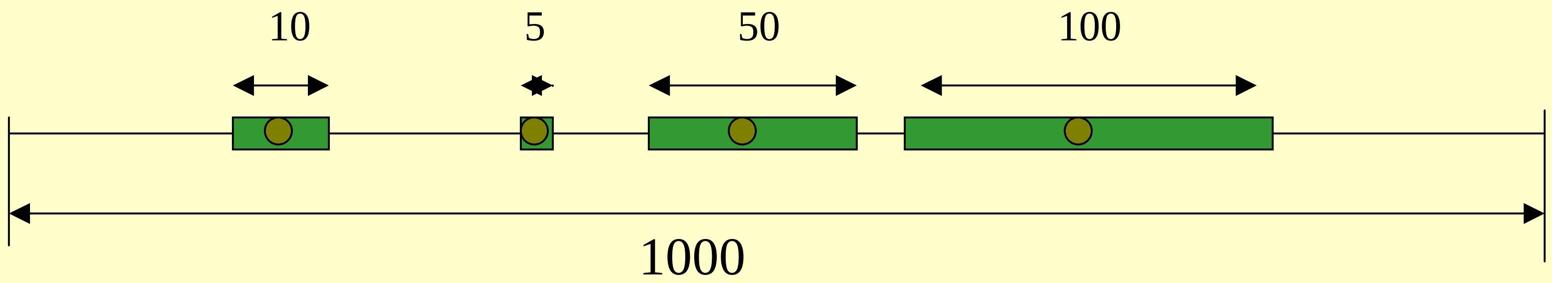
V	$0 \ll (X_2 - X_1)$	$h_V(X_2 - X_1) \geq h$	$h_V = 0.5$
VI	$0 \ll (Y_2 - Y_1)$	$h_{VI}(Y_2 - Y_1) \geq h$	$h_{VI} = 0.25$



VII Slope of the line

$$(Y_2 - Y_1) = 0.5(X_2 - X_1)$$

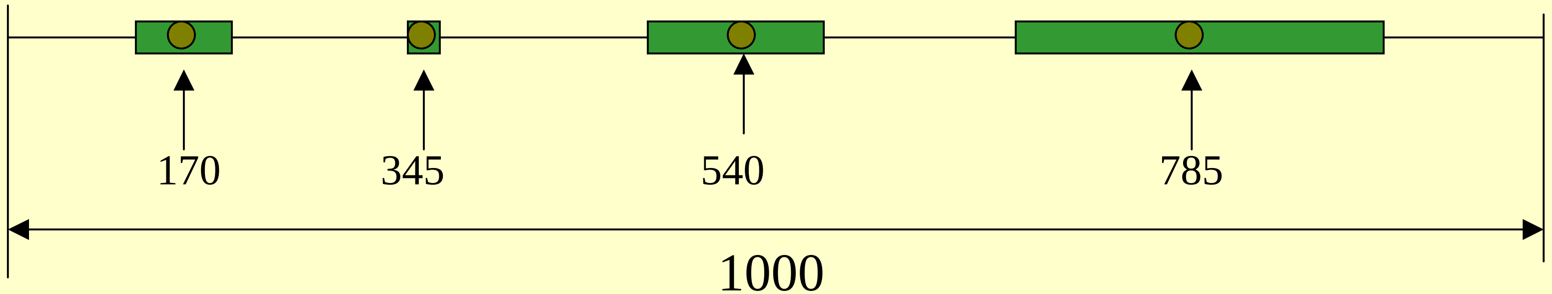
Solution points are (25, 25), (75, 50) and h* = 0.25



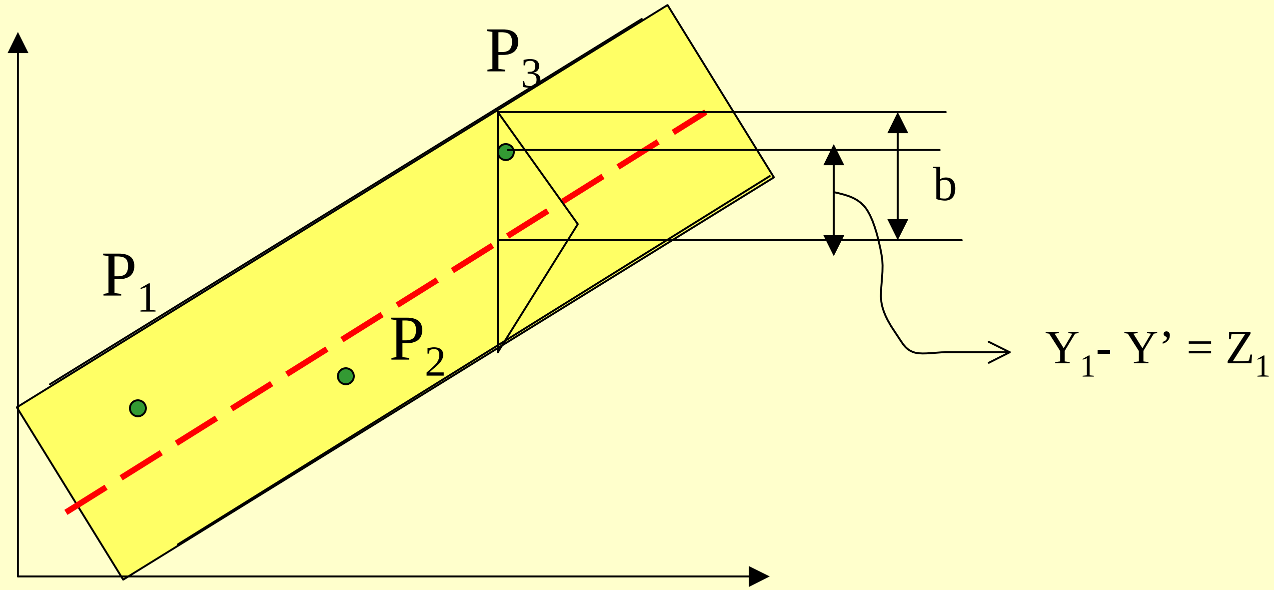
Rules:

Stay Far away from the walls.

Stay Far away from each other.



b = 15



Solution is

$$Y = 3.06X + 14.5$$

$$h_1 = 0.811$$
$$h_2 = 0.811$$
$$h_3 = 0.811$$

Conditions:

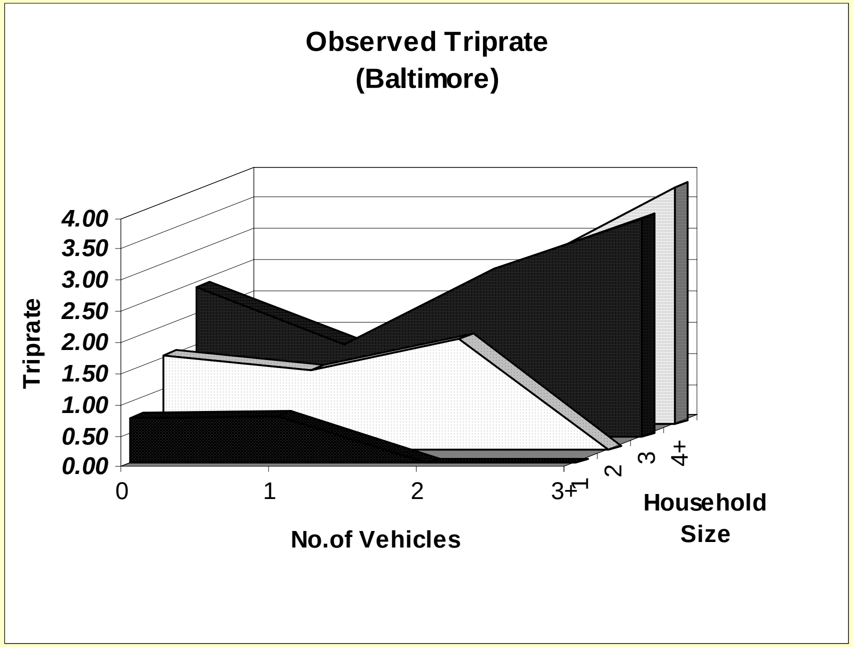
$$h_1 = (b-Z_1)/b \geq h$$

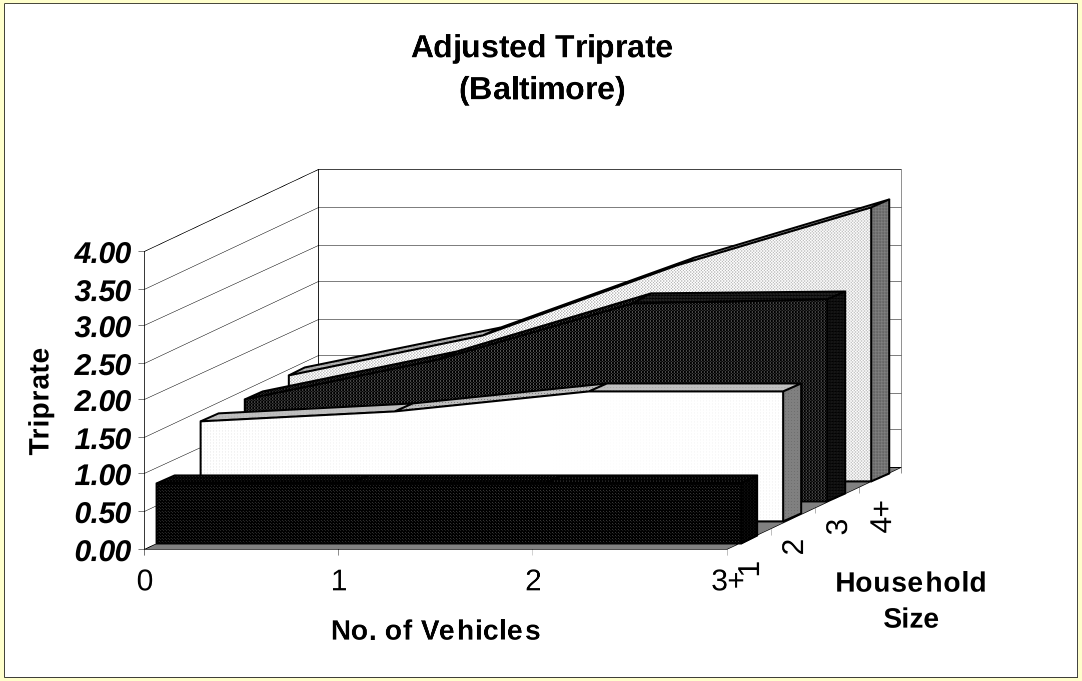
$$h_2 = (b-Z_2)/b \geq h$$

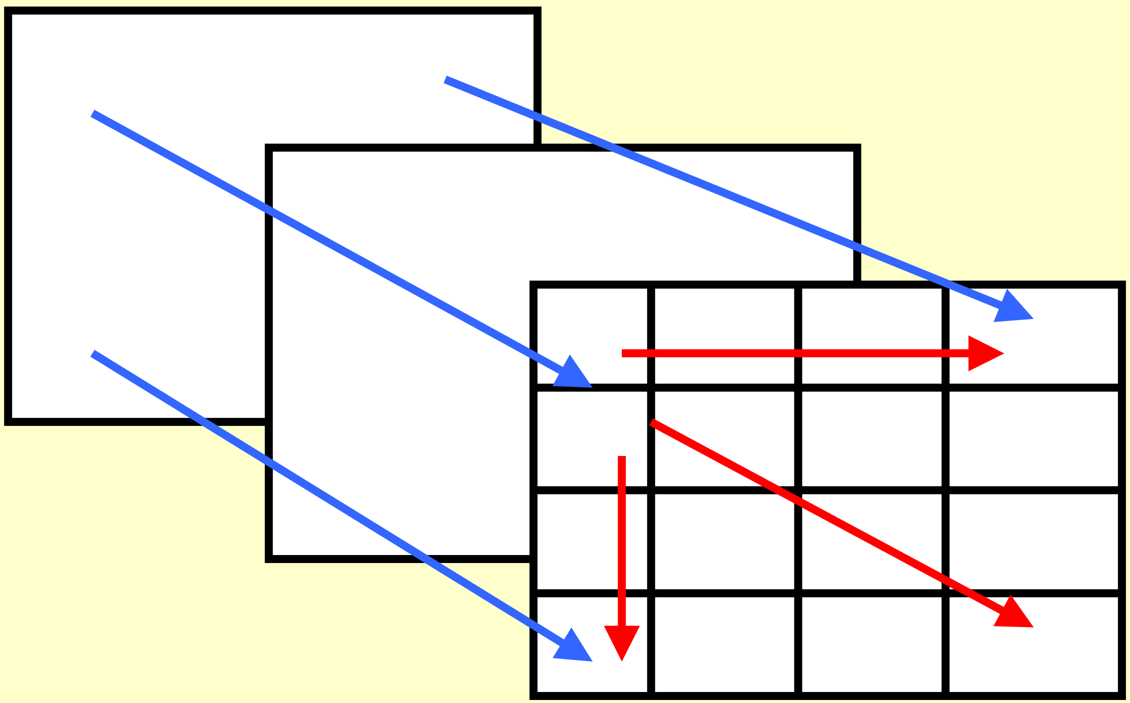
$$h_3 = (b-Z_3)/b \geq h$$

		Auto-ownership			
Household size		0	1	2	3+
	1				
	2				
	3				
	4+				

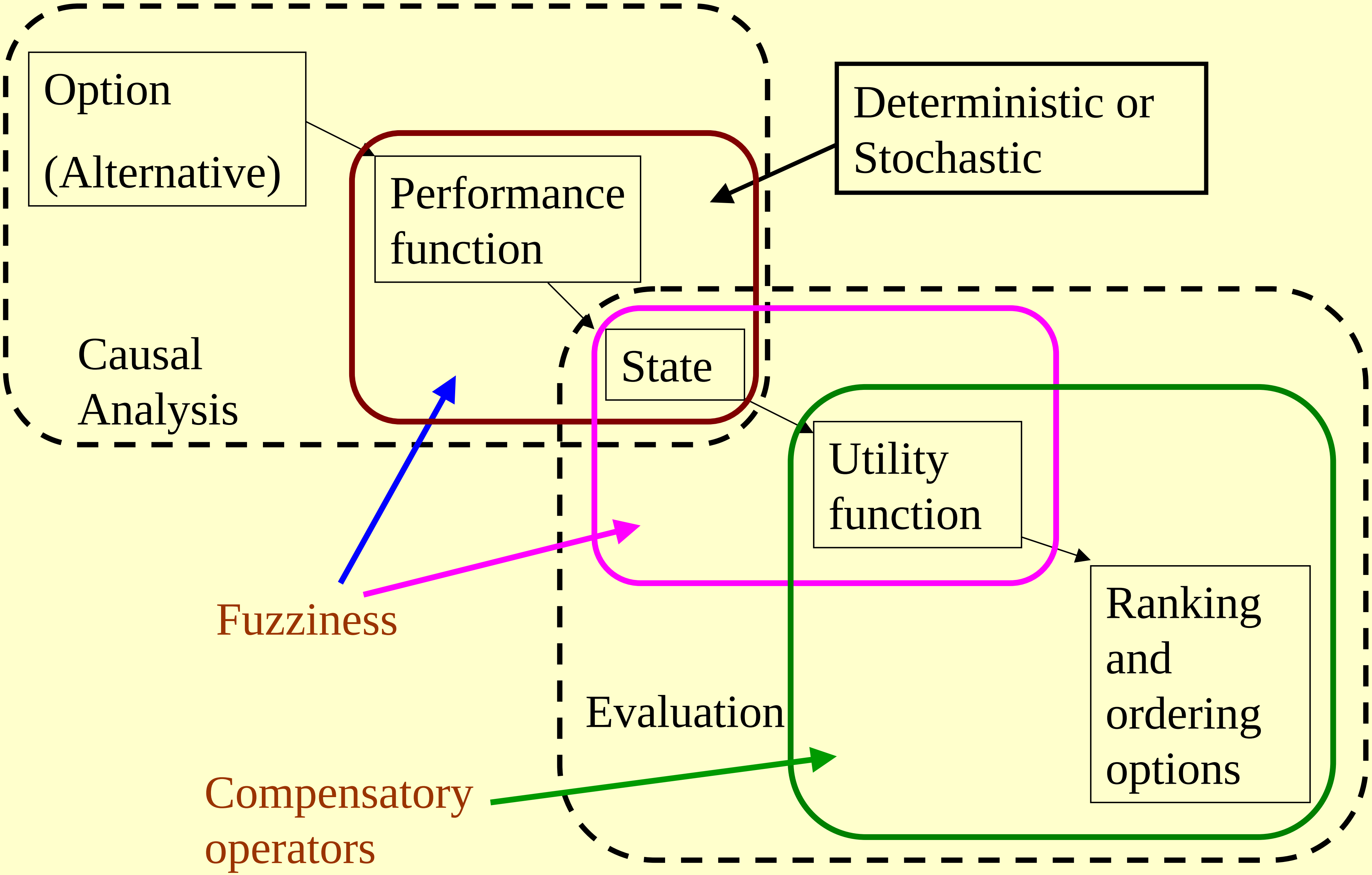
		Auto-ownership			
Household size		0	1	2	3+
	1				
	2				
	3				
	4+				

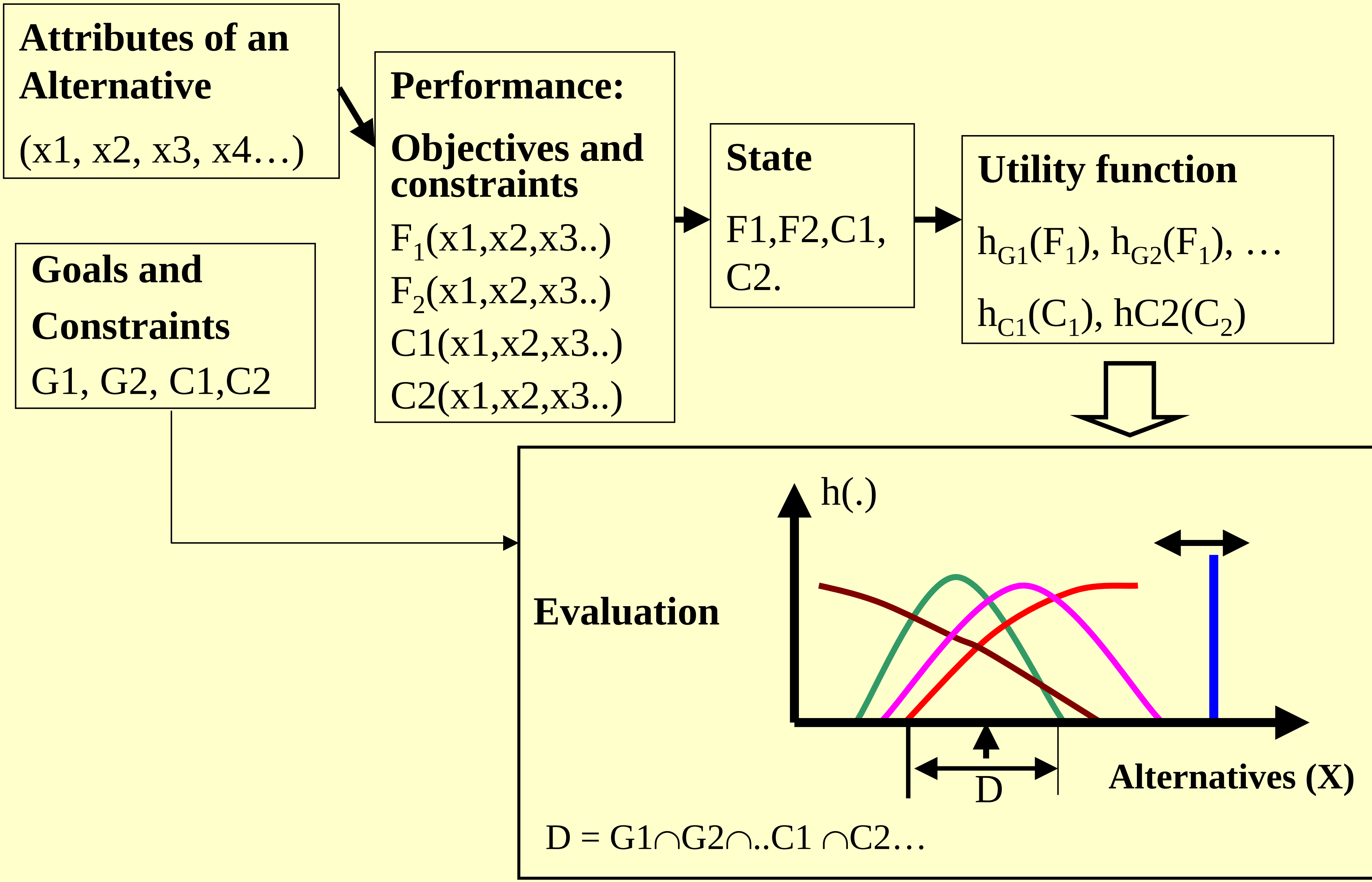

Observed Triprate
(Baltimore)
Triprate
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
1
2
3+
1
2
3
4+
No.of Vehicles
Household Size


Adjusted Triprate
(Baltimore)
Triprate
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
1
2
3+
1
2
3
4+
No. of Vehicles
Household Size



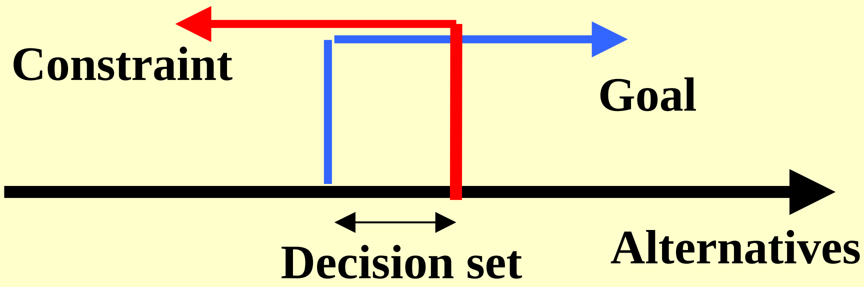
Fuzzy Decision Theory

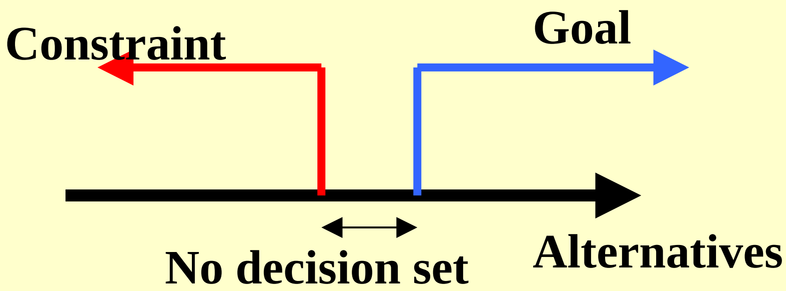


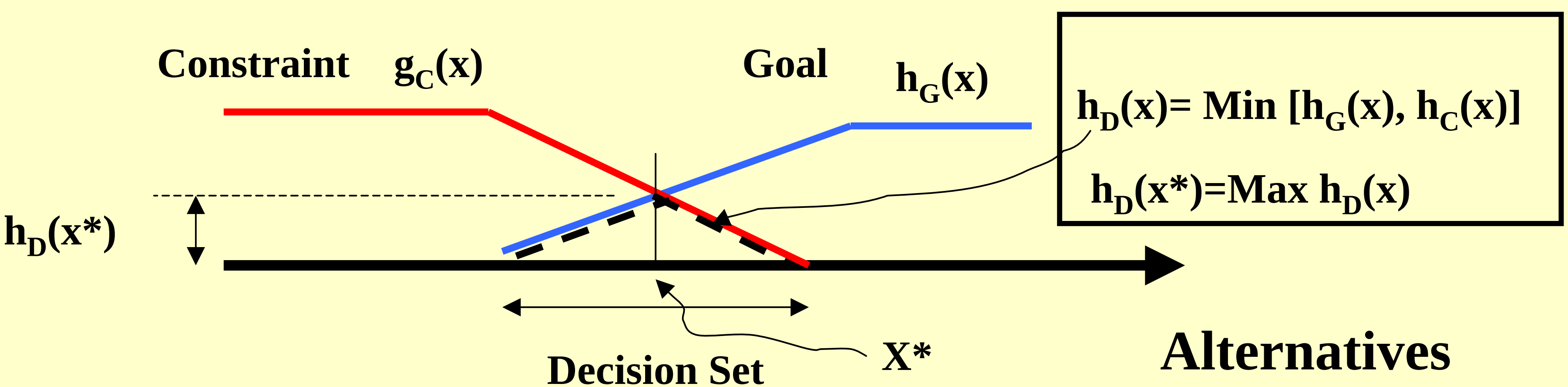

Attributes of an Alternative
(x1, x2, x3, x4…)
Performance:
Objectives and constraints
F1(x1,x2,x3..)
F2(x1,x2,x3..)
C1(x1,x2,x3..)
C2(x1,x2,x3..)
State
F1,F2,C1, C2.
Utility function
hG1(F1), hG2(F1), …
hC1(C1), hC2(C2)
Goals and Constraints
G1, G2, C1,C2
Evaluation
h(.)
D
Alternatives (X)
D = G1∩G2∩..C1 ∩C2…

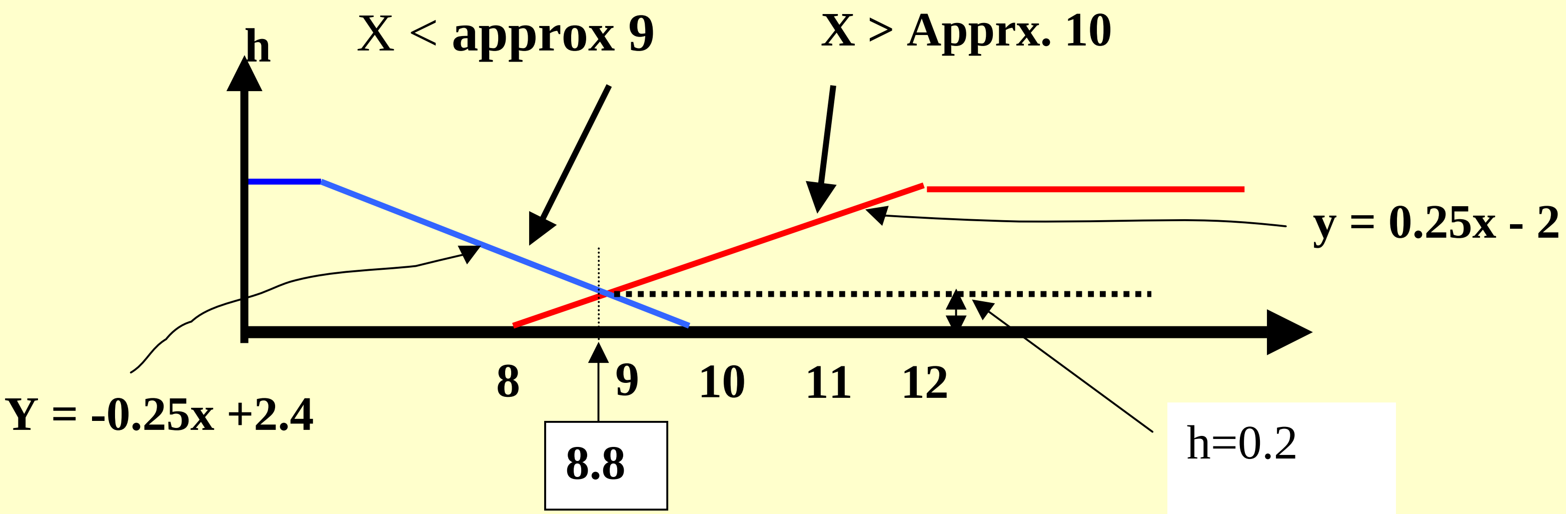
Decision under Fuzzy Environment

Decision : Confluence of Goals and Constraints









LP formulation

Max h

Subject to:

$0.25x - 2 \geq h$

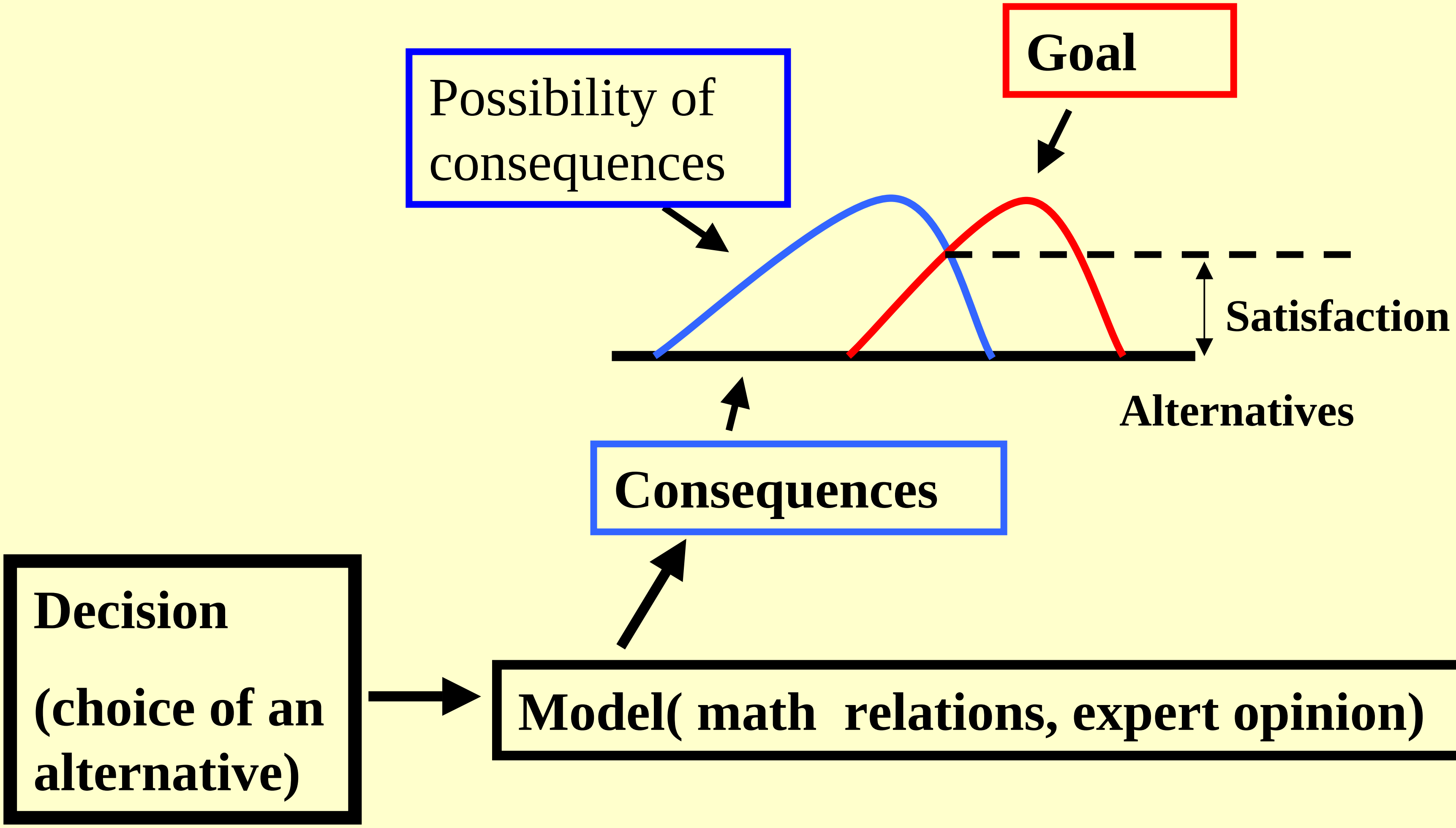
$-0.25x + 2.4 \geq h$

$x > 0, \; h > 0$

Opt. Solution

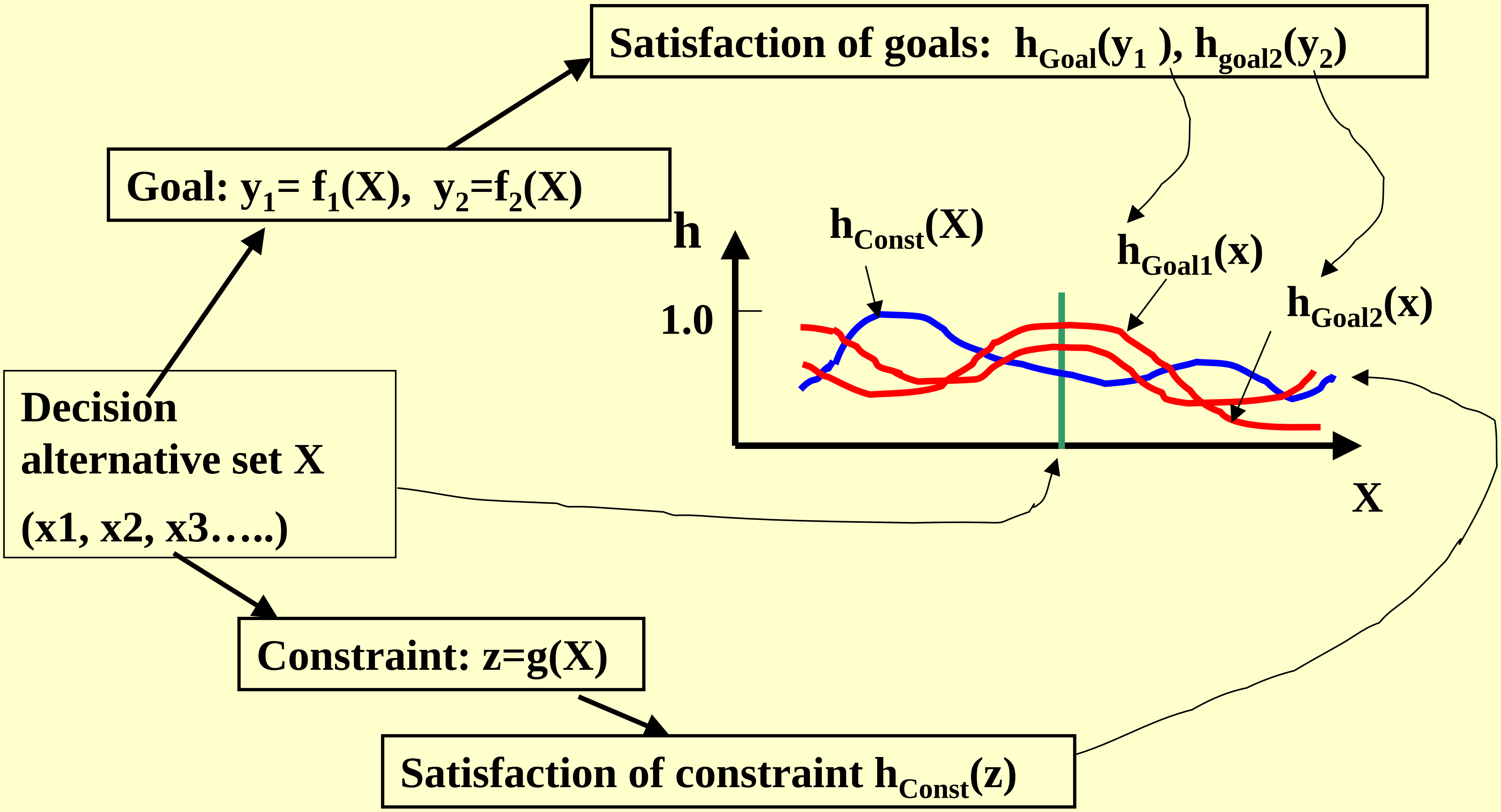
x= 8.8

h=0.2 (satisfaction)



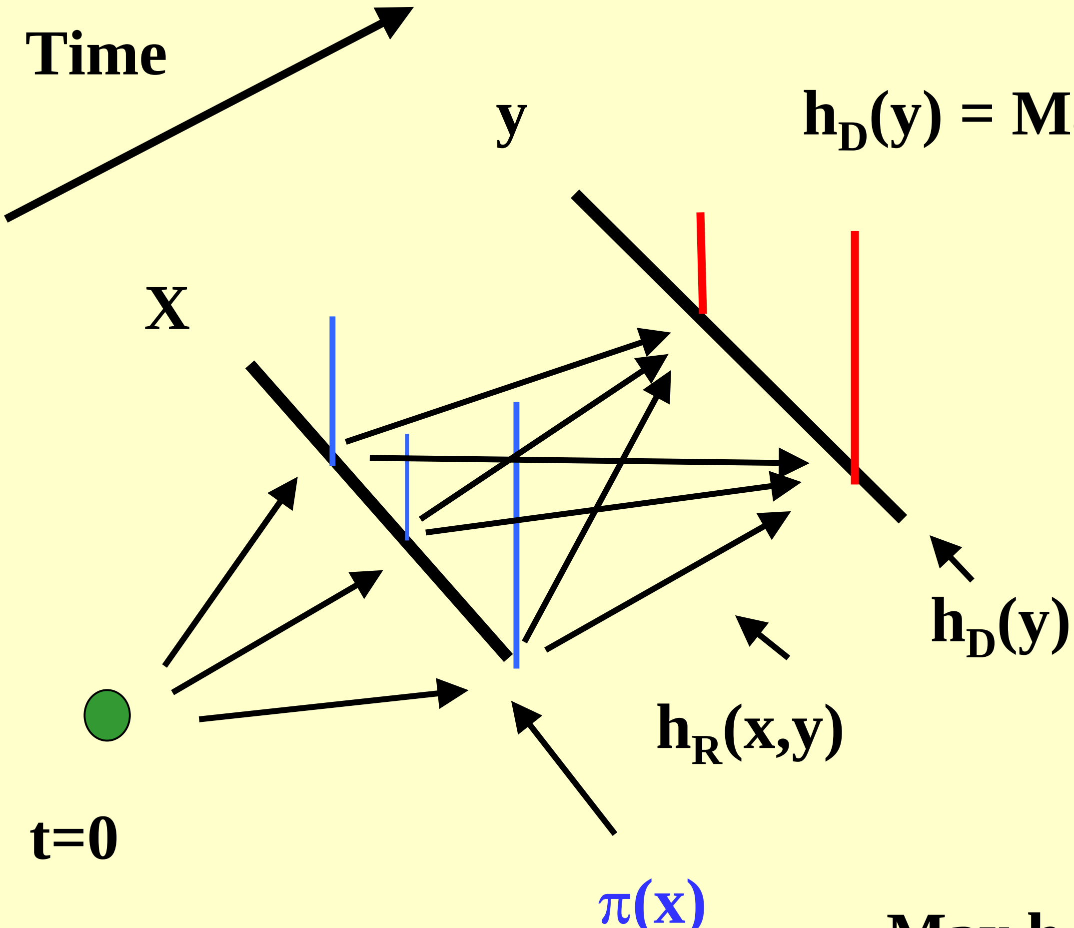
Goal
Possibility of consequences
Satisfaction
Alternatives
Consequences
Decision
(choice of an alternative)
Model(math relations, expert opinion)

Framework of Fuzzy Decision Model



Expansion to Fuzzy Dynamic Programming

$$h_D(y) = \text{Max Min}\{\pi(x), h_R(x.y)\}$$



Max $h_D(y)$

= Pos. of satisfaction at time 0

Applications

Adjustment of data for consistency

Traffic volumes, transit ridership data in a network

Travel forecast data (e.g., trip generation, travel time).

Reconstruction of data based on memory

Trip diary consistency.

Selection of design values

Space allocation, budget allocation by communication.

Parameter specification.

Scheduling activities under multiple constraints

Transit schedule coordination (timed transfer problem).

Decisions in Transportation Problem

Objectives and Constraints:Vague and elastic

Objectives (desires) and constraints are many and not well-defined.

Each constraint and goal represents the interest of different constituent.

The edge of the constraint and goal is usually fuzzy; a notion of "tolerance" exist.

Solution: usually a compromise

Different parties' *satisfaction levels* need to be considered.

The process of reaching the decision: Agreement

Solution is achieved through iterations - communication (Explaining the process is important).

The essence of ultimate decisions remains impenetrable to the observer - often indeed the decider himself… There will always be the dark and tangled stretches in the decision making process - mysterious even to those who may be most intimately involved.

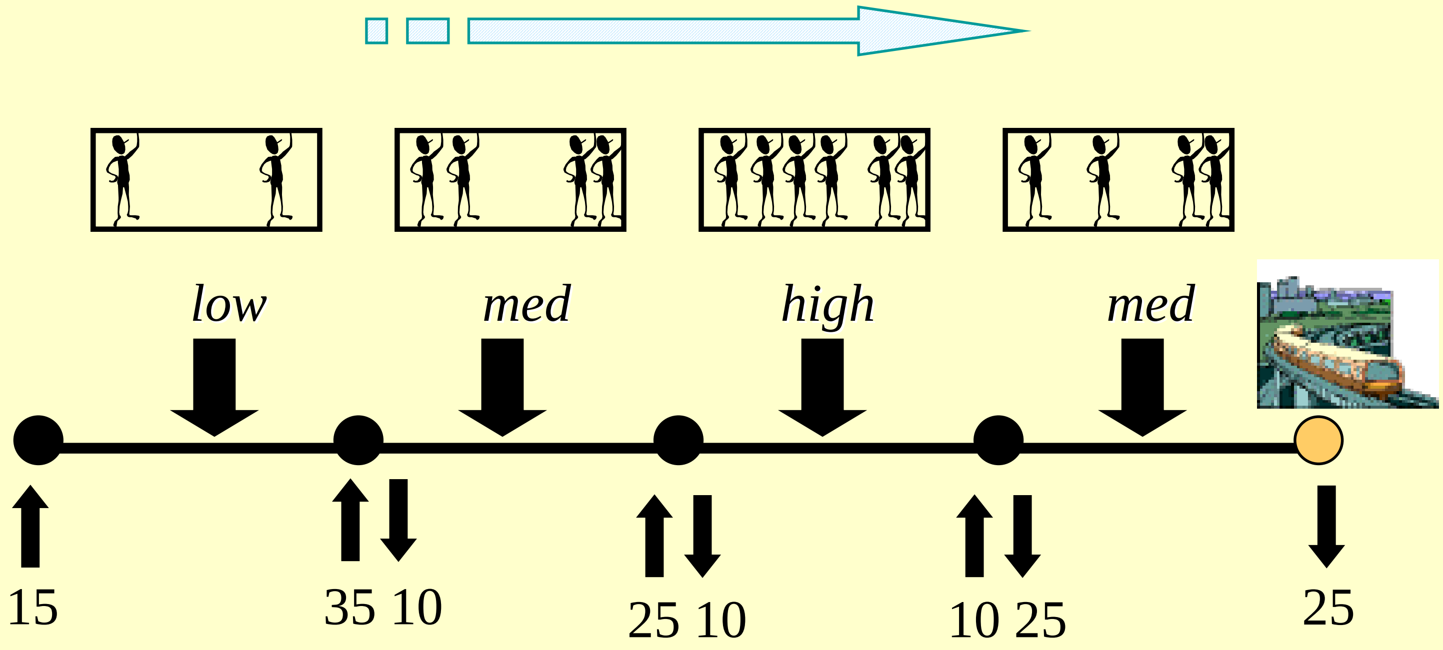
(from *Kennedy*, by Sorenson Theodore, New York, Bantam 1966.)

Appendix

Consistency of

Bus Boarding and Alighting Data

Problem Example

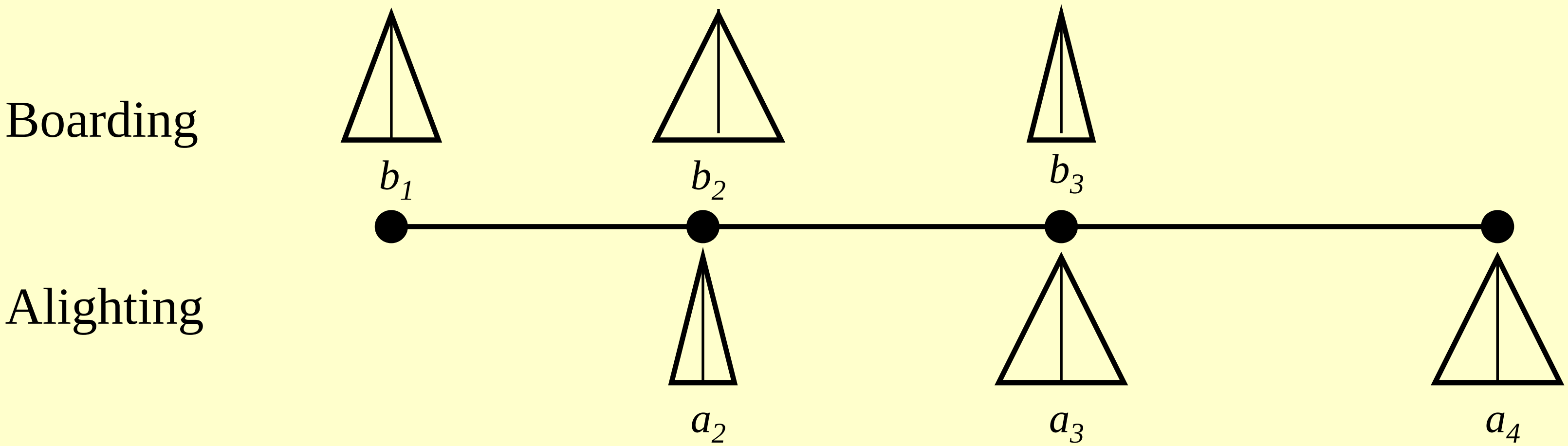


Total Ons = 85

Total Offs = 70

Proposed Method: Fuzzy Optimization Approach

- Consider the observed values “approximate” and assume range.
- Find solution as close to the center value as possible $\mu_{ai}(x)$, $\mu_{bi}(x)$)



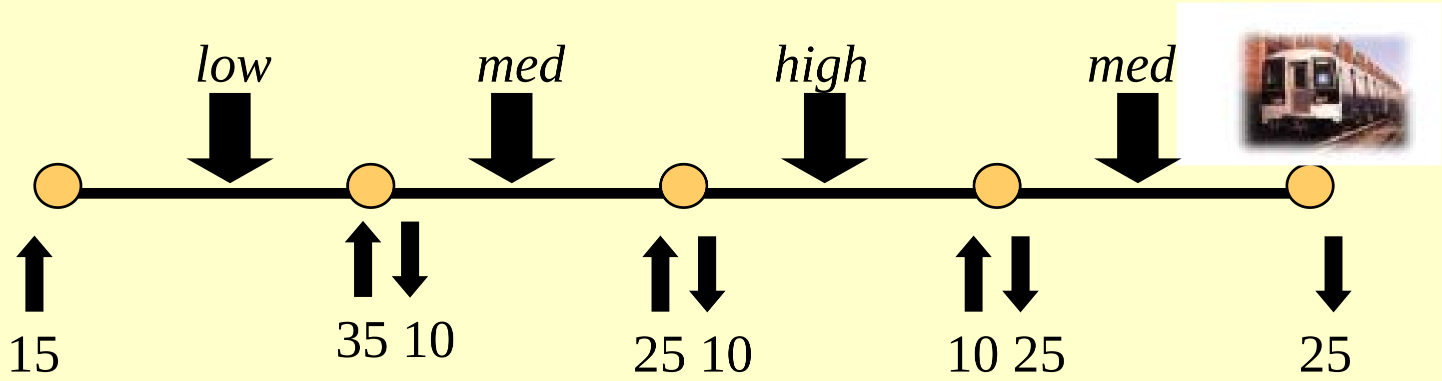
Find A_2 ,A_3 ,A_4 ,B_1 ,B_2 ,B_3 such that:

$$B_1 + B_2 + B_3 = A_2 + A_3 + A_4$$

$$B_1 \geq A_2$$

$$B_1 + B_2 \geq A_2 + A_3$$

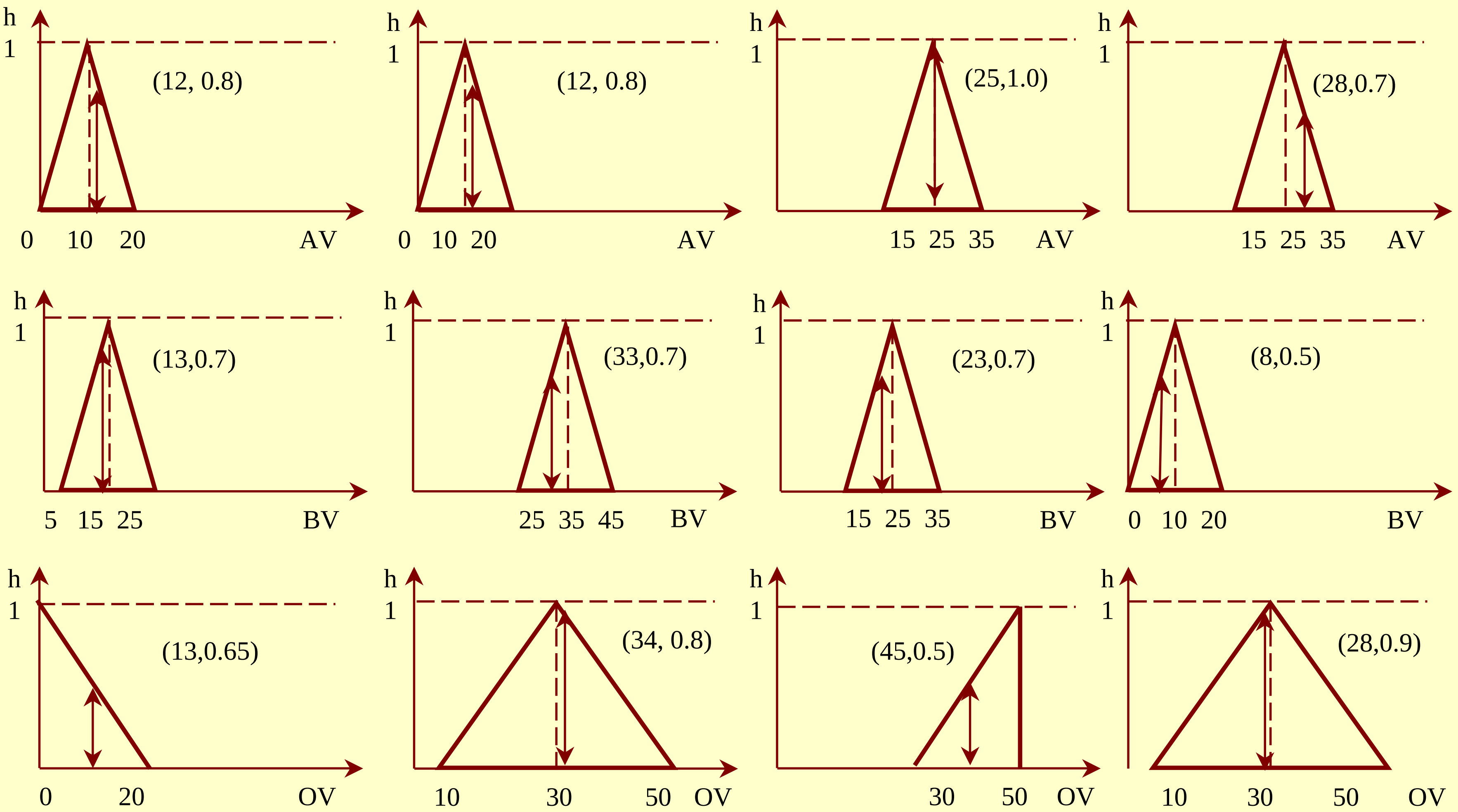
Example 1 (5 Stations)



DATA		
Station Number	Observed Boarding	Observed Alighting
1	15	0
2	35	10
3	25	10
4	10	25
5	0	25
Sum	85	60

RESULTS		
Adjusted Boarding	Adjusted Alighting	Pass. load betwn. Sta
13	0	***
33	12	13
23	12	34
8	25	45
0	28	28
Sum	77	77

Membership Functions



AV - Alighting Volume
BV - Boarding Volume
OV- Observed Volume