Uncertainty in Transportation Analysis and Its Treatment

Shinya Kikuchi, Ph.D., P.E.
Professor
University of Delaware
Newark, DE 19716 USA

August 2004

Nature of Transportation Problems and Solutions

The transportation phenomena are the manifestation of the outcome of the complex social economic and political interactions over time and space.

Causality is complex and mostly unknown.

Goals of improving transportation infrastructure not clearly defined.

Outcome of an action is not predictable.

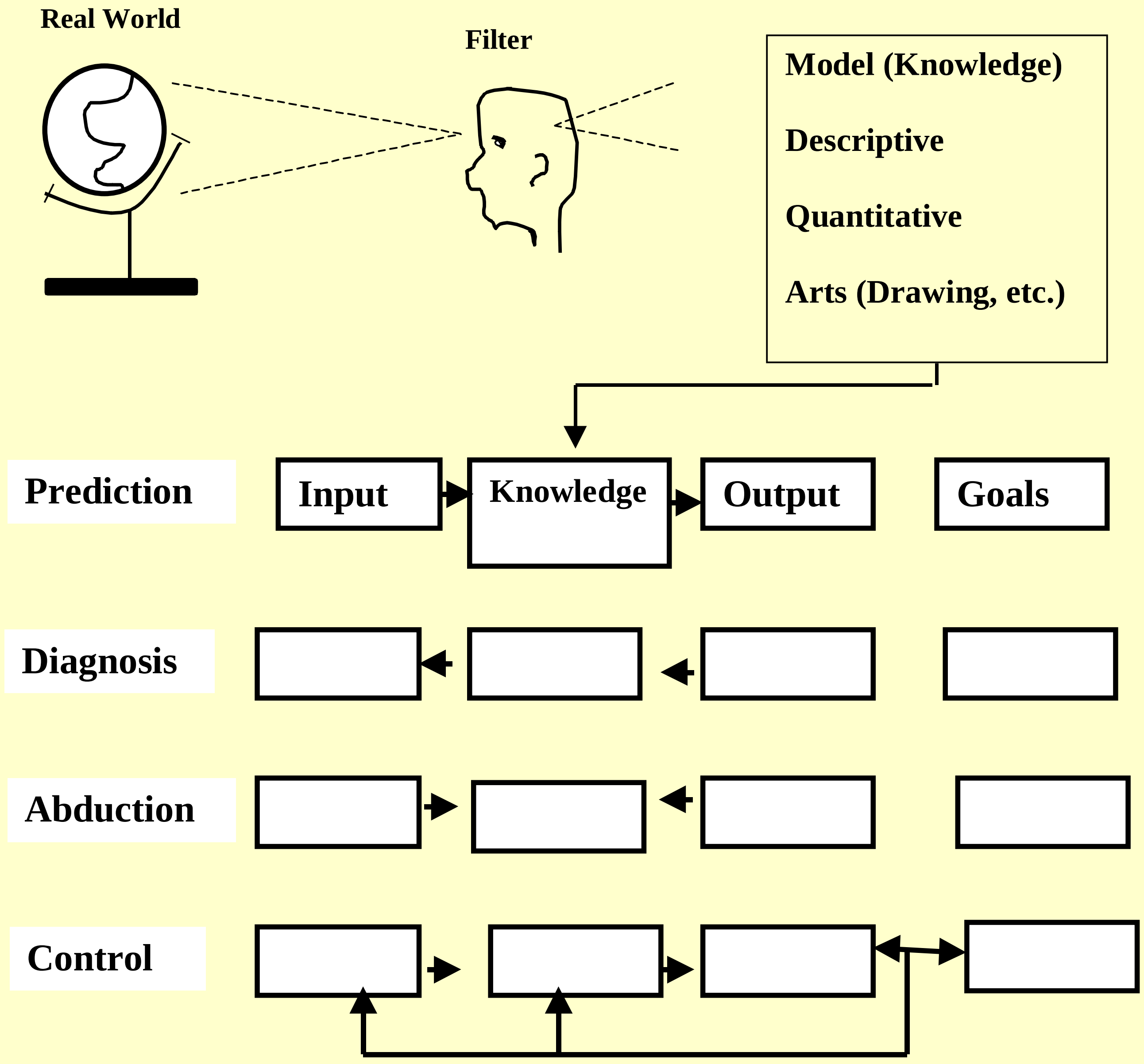
The solution is not optimum rather it is better than other solutions.

Decisions can be influenced by politics; hence, reasoning and logic is important.

What are the implications of complexity?

- Many Players (users, operators, non-users, etc.) must be considered.
- Multi-objective and conflicting objectives.
- Uncertainty.
  - data
  - knowledge base
  - objectives, constraints
- Analysis process handles large amount of information.
- Massive computing and simulation effort involved.
- Outcomes changes in time and space - constantly changing initial conditions.
- Difficult to control outcome.

System Modeling and Analysis



Nature of Analysis Elements

Complex problem
Multi-objective
Multi-constraint

.Data (Input information)
- **Noise**
- **Incomplete**
- **Qualitative**

A lot of approximation

Knowledge base Incomplete
- **Applicable to local area**
- **Qualitative**
- **Pattern (not well defined)**

Output
- **Required accuracy not known, at best approximate**

Target (objectives) **Not clear**
- **Multi-dimensional**
- **Priorities among the objectives not clear**

Uncertainty

We do not know exactly

- What will happen? (prediction)
- What is the problem? (abduction)
- What should be done? (control)
- What is responsible? (diagnosis)

Issues Important to Civil Systems Analysis with regard to uncertainty

How to present analyst’s uncertainty honestly

How to separate what we know and what we do not know

How to represent the process of propagation of uncertainty

Mathematical treatment of uncertain quantities and human perception

How to show the effects of added information, value of information

How to deal with multi-objective and multi-constraint subjective problems

WHY DO WE NEED TO STUDY UNCERTAINTY?

To make an honest presentation on proposition and reasons (to understand how much is known and how much unknown)

To preserve uncertainty in the analysis process

To measure information and its value

Information is measured by the reduction of uncertainty.

State change from A to B, then $I(A,B) = U(A) - U(B)$

What is Uncertainty???????:

Questions (What? Who? Why? Whom? How? Where? Whose?)
State of information deficiency

INDEFINITE, INDETERMINATE

NOT CERTAIN TO OCCUR, PROBLEMATICAL

NOT RELIABLE, UNTRUSTWORTHY

NOT KNOWN BEYOND DOUBT, DUBIOUS

NOT HAVING CERTAIN KNOWLEDGE, DOUBTFUL

NOT CLEARLY IDENTIFIED OR DEFINED
NOT CONSTANT VARIABLE, FITFUL

(Ref. Webster's New Collegiate Dictionary)

Traditional Treatment of Uncertainty

Use of probability theory

Use of a range (optimistic and pessimistic scenarios)

Sensitivity analysis

Traditional Approach for Handling Uncertainty

About the Truth of "X is A"

Probability theory framework

Information about "X" must be available in statistical evidence

"A" must be defined exactly.

Causal relationship should be random.

Scenario and Sensitivity analysis considers the **variation of input**

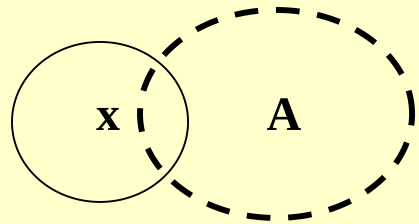
About vagueness of language

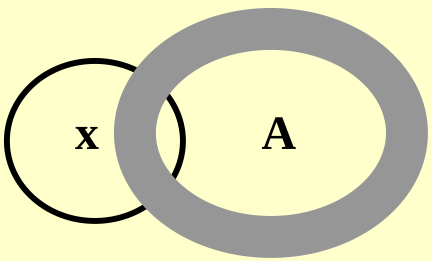
Eliminate vagueness

Making assumptions

Two Types of Uncertainty

"X is A"

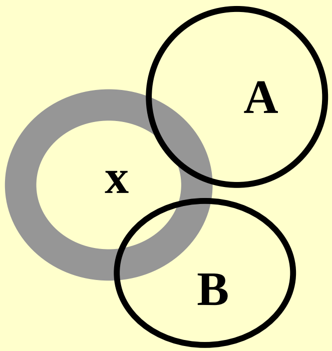




When A is not well defined.

Fuzziness

Traffic volume is large.



When information about x is not clear.

Ambiguity

The volume is between 1000 veh/hr.

Uncertainty as to

Meaning of words	**vagueness**
Truth of the proposition (X is A)	**ambiguity**

Modeling of Language-based Expressions for

Inference and Control

Fuzzy Set Theory

Captures linguistic expressions and measurements

- **Approximate values and measurements**
  - **Estimates, e.g. highway capacity**
  - **Desires, goals, targets,**
- **Natural language expressions and vague expressions**
  - **Distance, Time, Size, e.g. “long”, “late”, “ small”**
  - **Feeling, e.g. “good”, “acceptable”**

Facilitates mathematical framework to preserve the information quality throughout the analysis processes of

- **Arithmetic operations for prediction, Diagnosis**
- **Inferences, Optimization, Control**

Examples of Fuzziness

Notion of Desire and Target

Desired departure time and desired arrival time

Desired cost

Objective, goal, target

Desired compensation

Design values

Notion of Satisfaction and Acceptability: unclear demarcation point

Satisfactory level of achievement, e.g., acceptable air pollution level

Acceptable cost, acceptable delay and travel time, acceptable error

Willingness to pay

Perception and quantities based on memory

Time spent for an activity, e.g., travel time between two point.

Distance traveled

Appearance and condition on the past

Description of condition, performance, or quality - linguistic expression

Many attributes are involved

Traffic congestion – “bad” traffic and “good” traffic condition

Comfort level, Safety level, Level of service

High speed - fast or slow, large or small

Imprecise Values - hard to measure or hard to summarize

Sight distance, Reaction time

Value of time, Capacity of roadway

Memory - answers to the revealed preference surveys.

When one is asked about the distance to a particular location, on the street. One answers it is 200m. 200m is not a random value. Because the distance is a single value and no probability distribution. But 200m is an approximate value.

Similarity

Match between two similar objects

(Yet it is interesting to see that probability that one is A can be looked at similarity between x and A) But when term similarity is used most people think it is fuzzy.

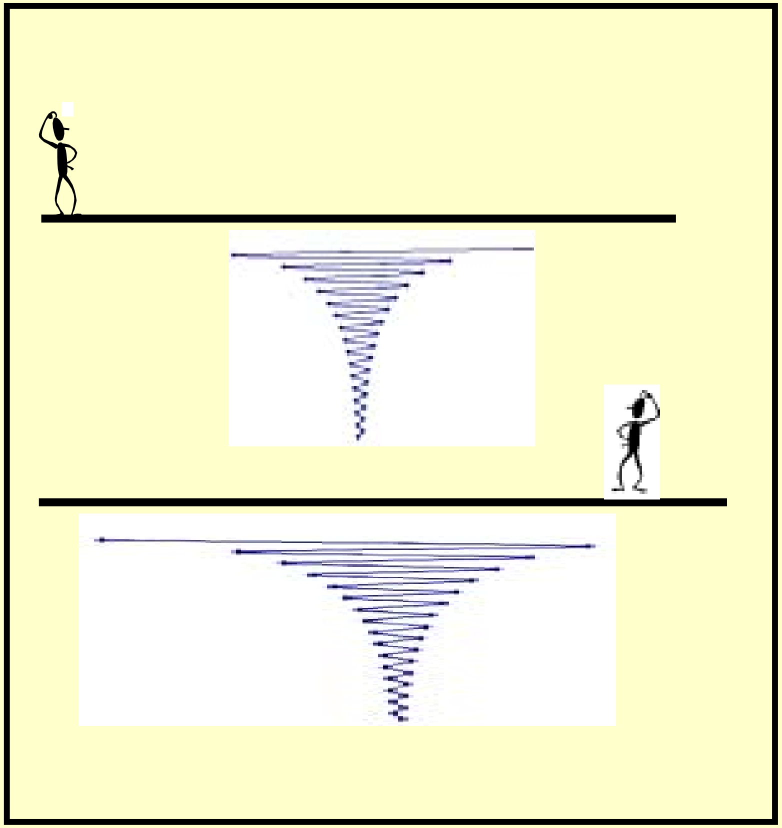
Cushion value or Safety factor

A padded value to consider or absorb fluctuation. Interpretation of PHF.

Ranking

Many things cannot be assigned an absolute value. The values are relative. For example, tasting food. This is better than this. Ordering relationship.

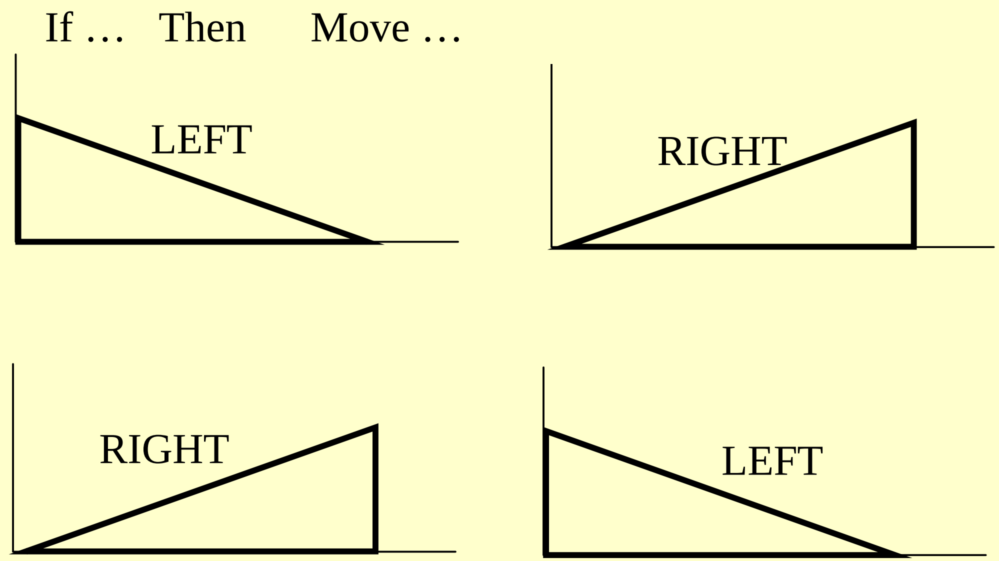
Simple Example of Fuzzy Control (1)



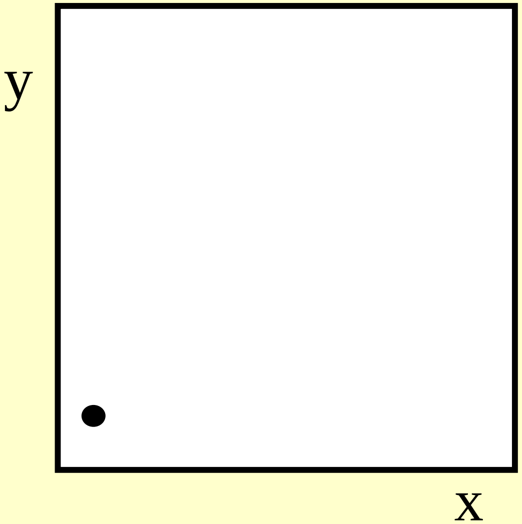
Rule:

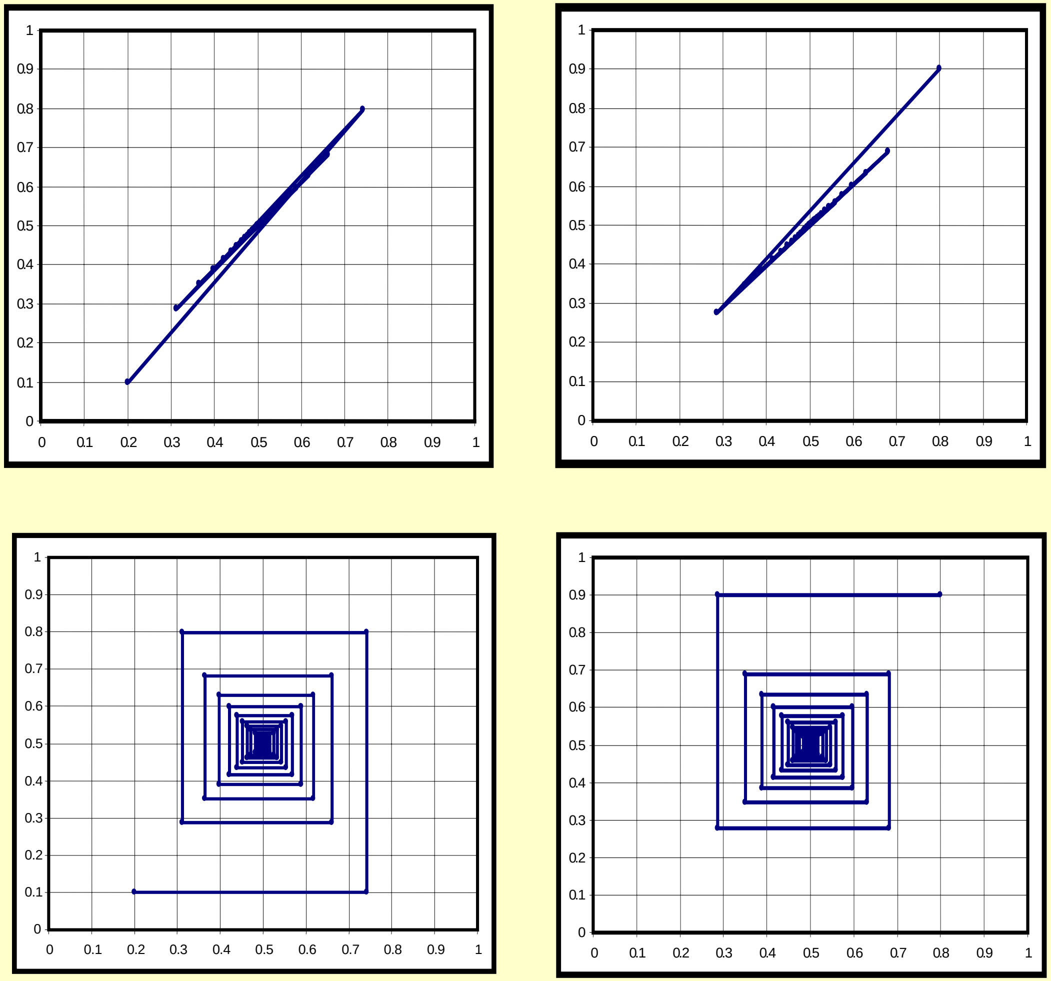
If *LEFT*,**Then** move to the *RIGHT*

If *RIGHT*,**Then** move to the *LEFT*



Simple Example of Fuzzy Control (2)





Rule:

If *LEFT*, **Then** move to the *RIGHT*

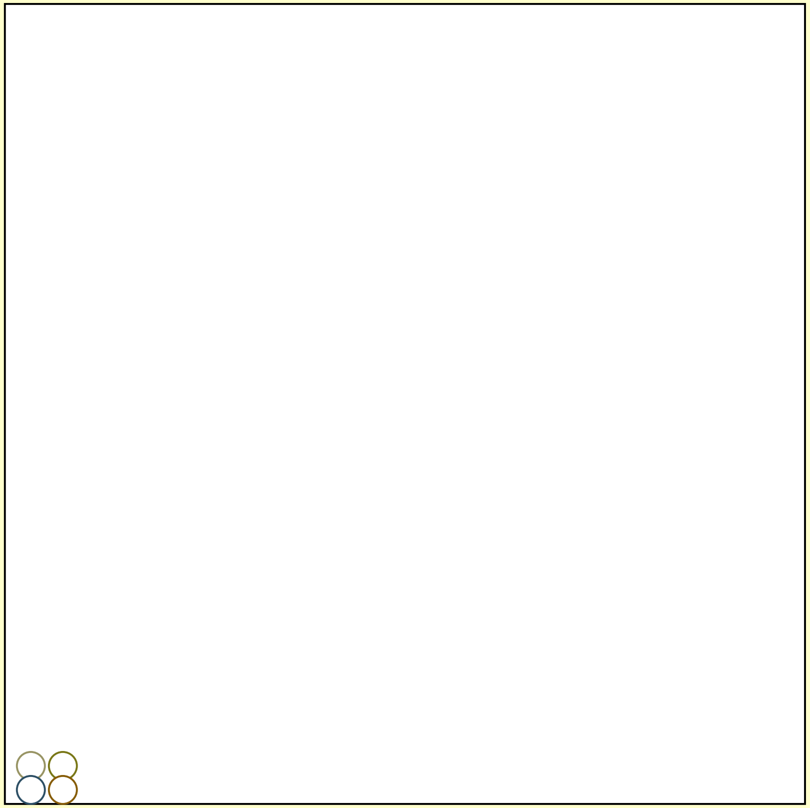
If *RIGHT* **Then** move to the *LEFT*

If *LOW* **Then** move to *HIGH*

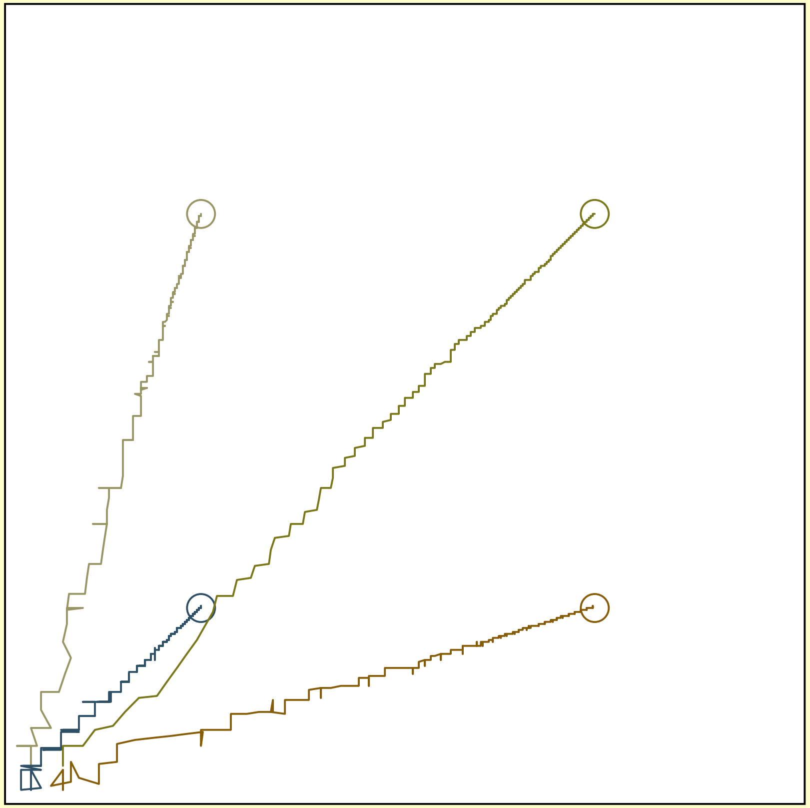
If *HIGH* **Then** move to *LOW*

Results for Move Away From Closest Object:
4 Dancers Start in Corner of 300 x 300 Dance Floor

Initial Positions

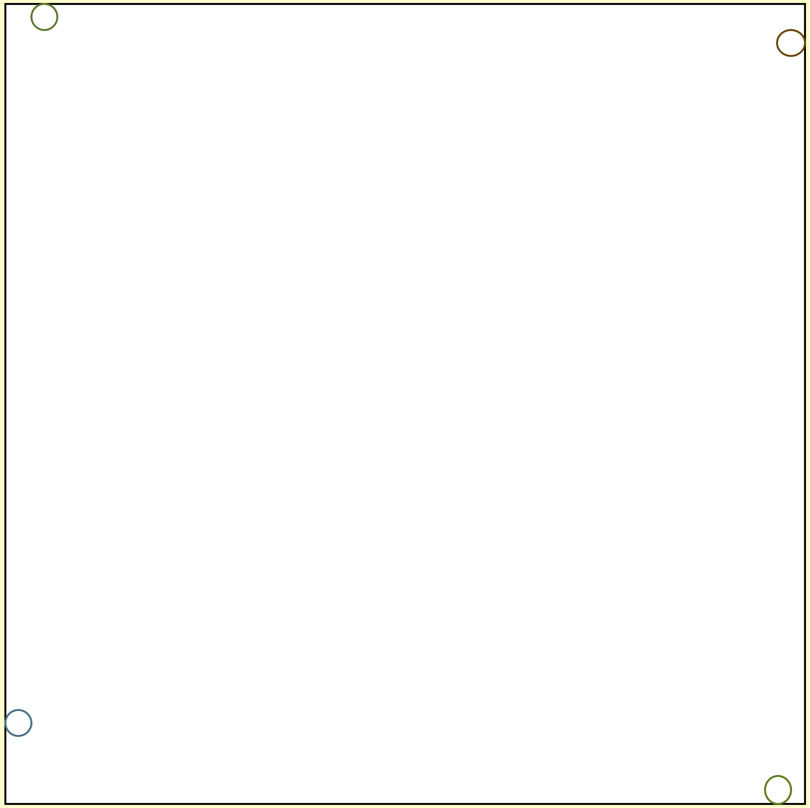


Final Positions

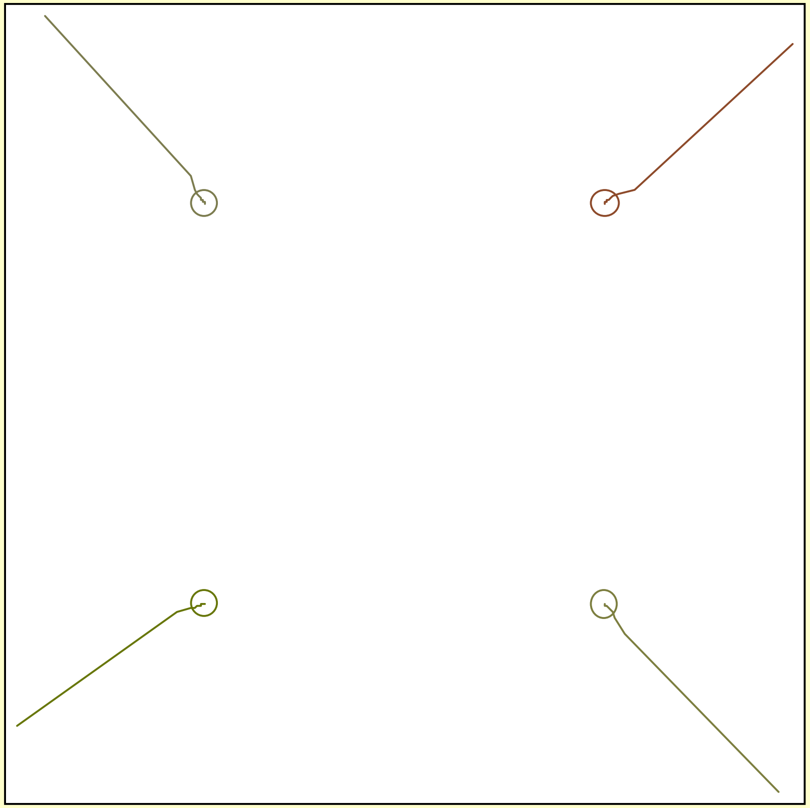


Results for Move Away from All:
4 Dancers start along the outside of a 300 x 300 Dance Floor

Initial Positions

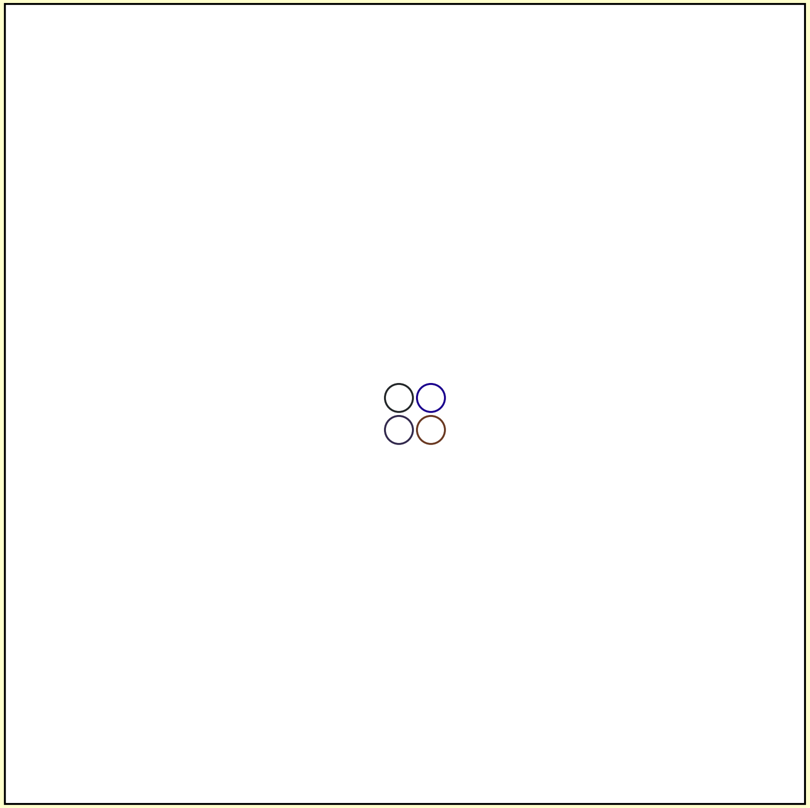


Final Positions

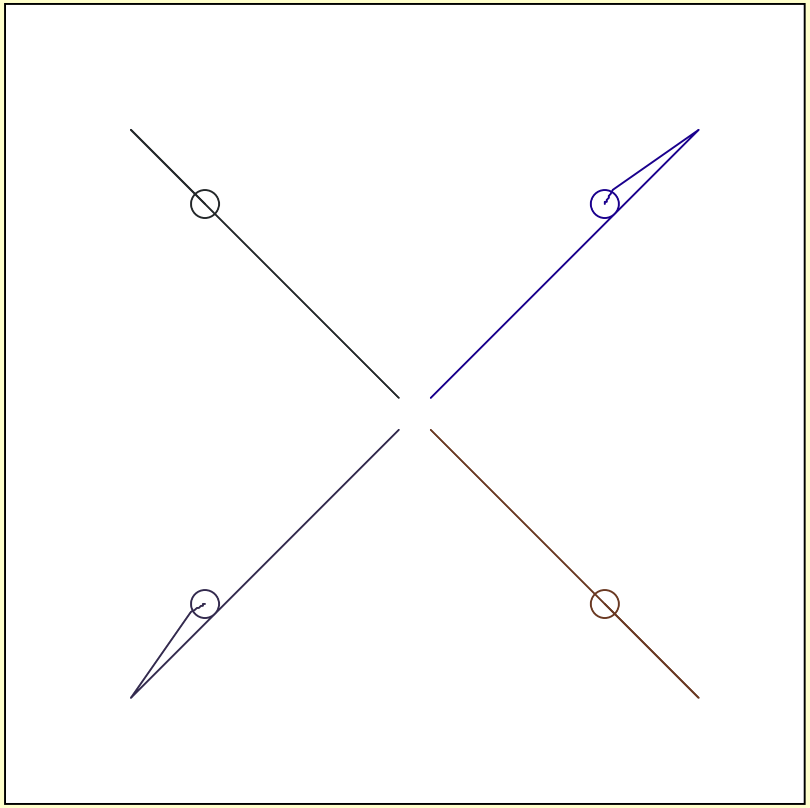


Results for Move Away from All: 4 Dancers start in the center of a 300 x 300 Dance Floor

Initial Positions

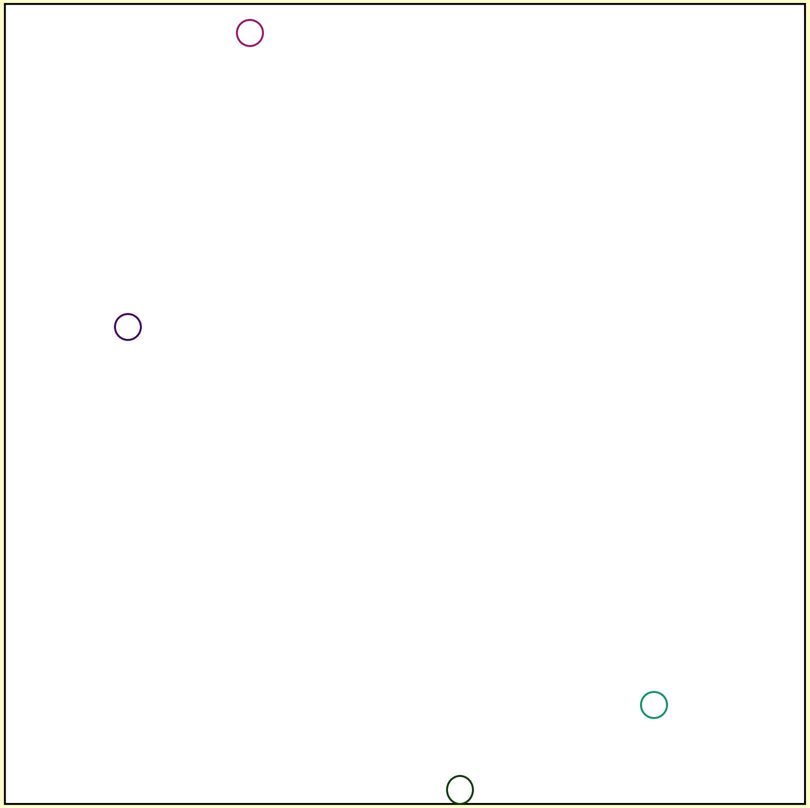


Final Positions

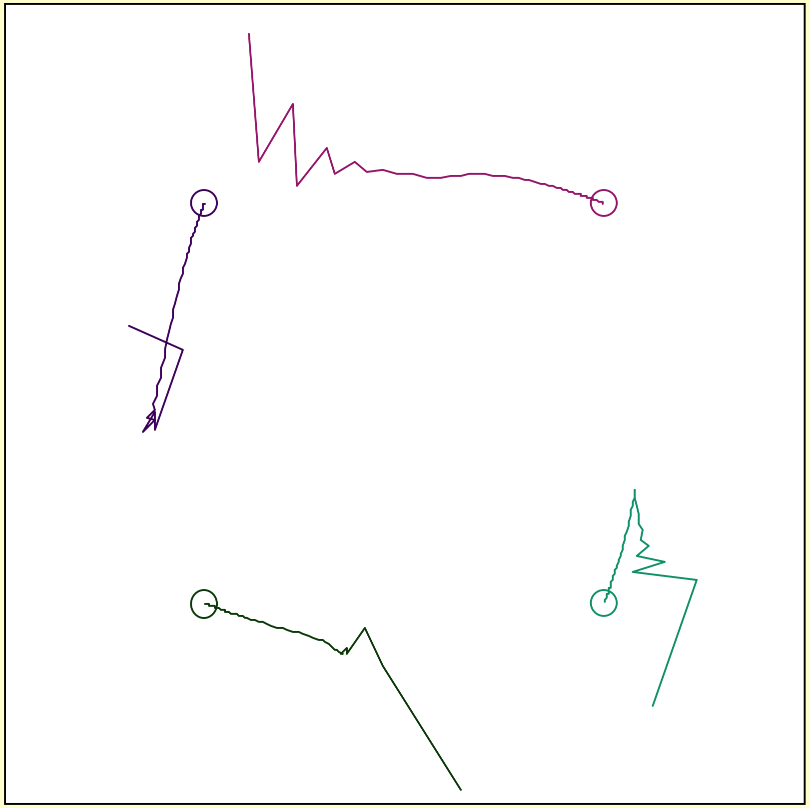


Results for Move Away from All: 4 Dancers start at Random Positions of a 300 x 300 Dance Floor

Initial Positions

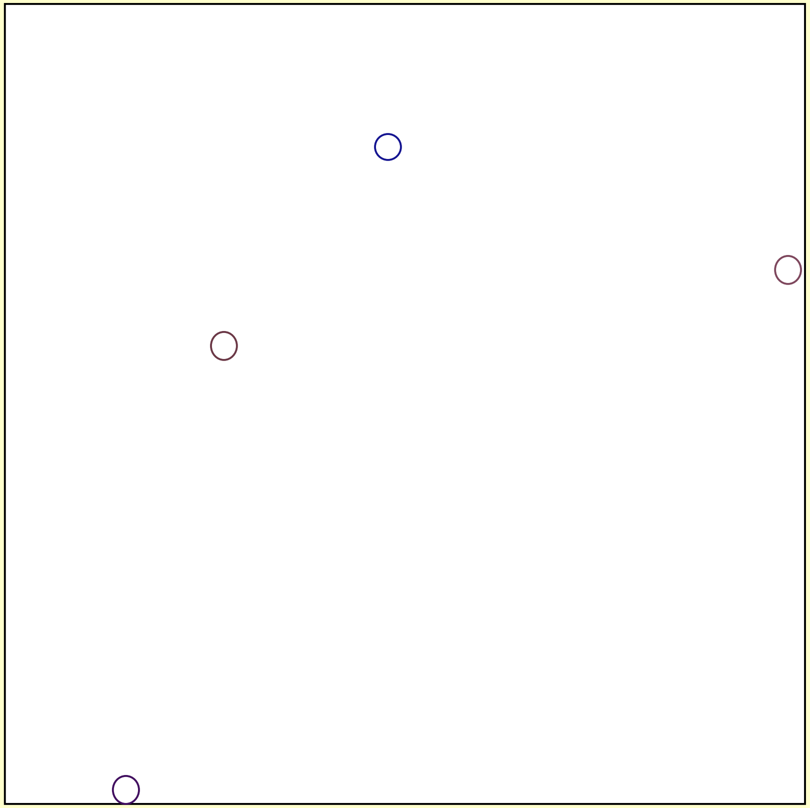


Final Positions

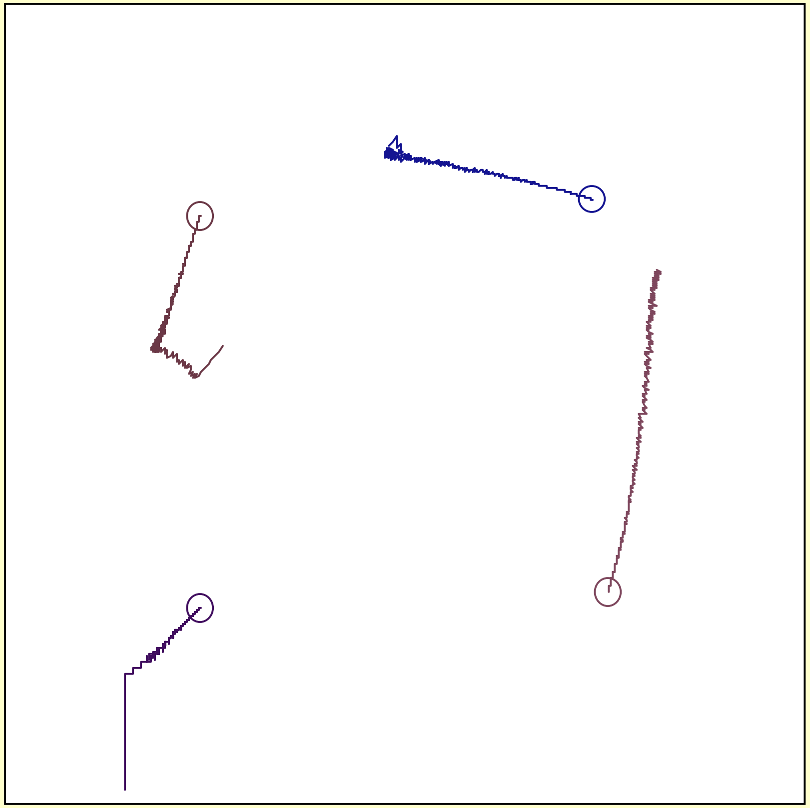


Results for Move Away from Closest Object: 4 Dancers start at Random Positions of a 300 x 300 Dance Floor

Initial Positions



Final Positions

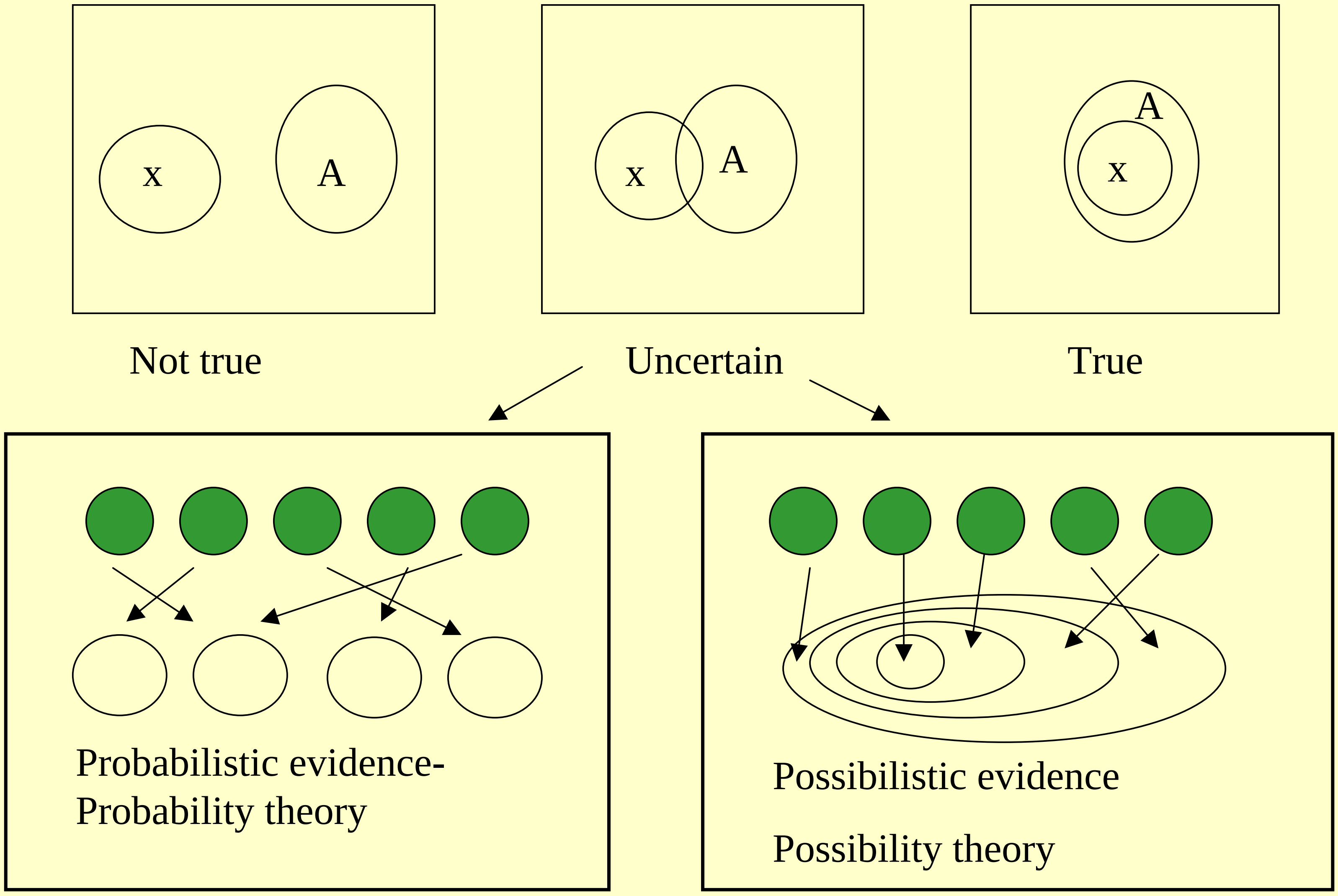


Ambiguity

Determination of Truth

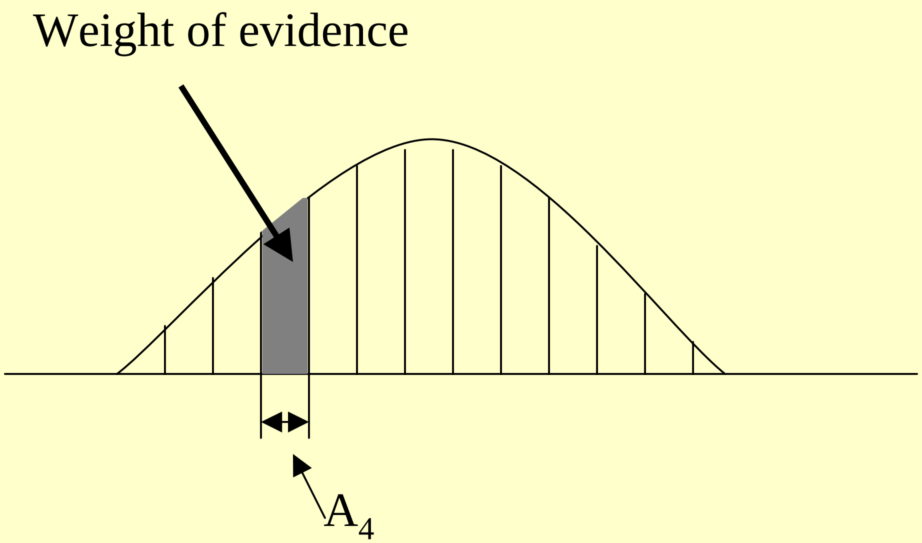
Case of Ambiguity: Truth of “x is A” is not certain

Truth is determined by the availability of evidence



Probability Distribution
(Distribution of evidence)

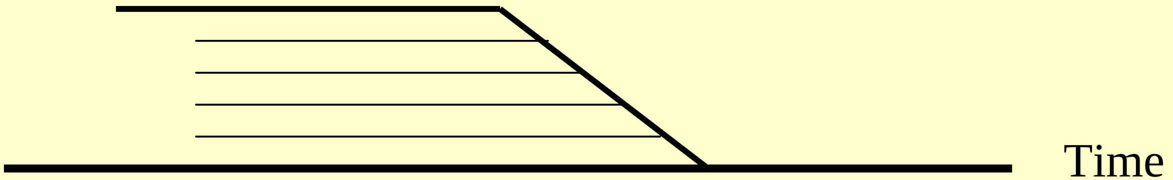
Σ Prob $(A_i) = 1$



Examples of Possibility Distribution

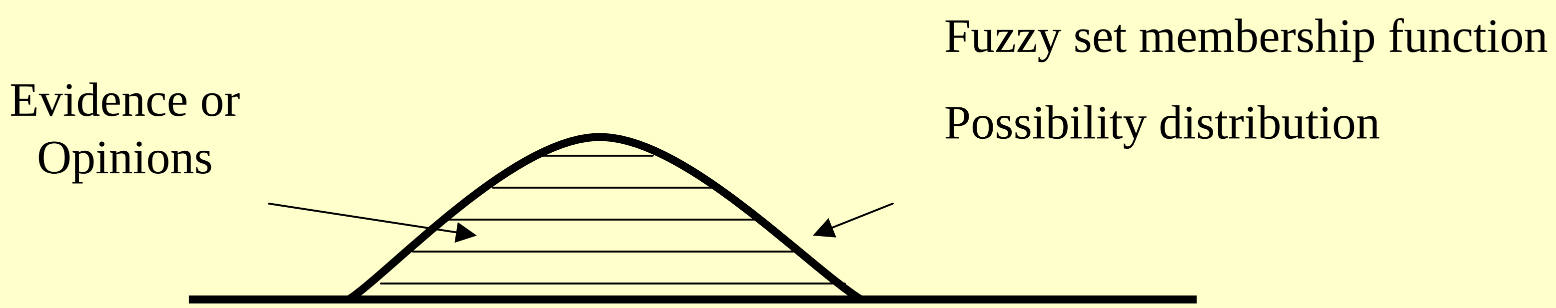
Quantities related to desire, target, and satisfaction

Desired arrival time "before 5pm"



Approximate quantity

(interpretation of capacity, estimated cost, estimated demand)



Possibility and Necessity Measures

Possibility Measure

Poss $(A \cup B) = \text{Max}\{\text{Poss}(A), \text{Poss}(B)\}$ $\quad$ $\text{Poss}(A \cap B) \leq \text{Min}\{\text{Poss}(A), \text{Poss}(B)\}$

Necessity Measure

Nec $(A \cap B) = \text{Min}\{\text{Nec}(A), \text{Nec}(B)\}$ $\quad$ $\text{Nec}(A \cup B) \geq \text{Max}\{\text{Nec}(A), \text{Nec}(B)\}$

Poss (A) and Nec (A) : the measures of evidence toward A

Poss (A) = Any non - negative evidence counts to assert A

Nec (A) = Only positive evidence counts to assert A

Poss(A) = 1- Nec (Not A), $\qquad$ **Nec (A) = 1 – Poss (Not A)**

Poss(A) = Anything other than Necessity of (Not A)

Nec(A) = <u>Im</u>possibility of (Not A)

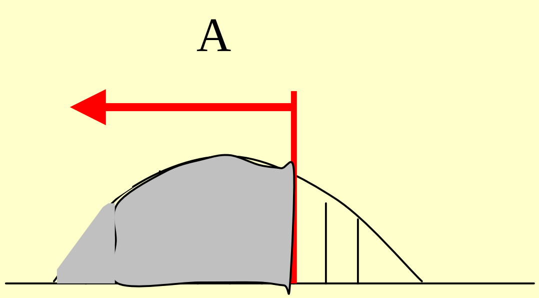
Possibility vs. Probability Measures

Truth of “ x is less than A.” It is measured by the (weight of) evidence.

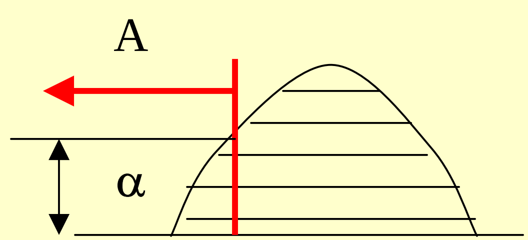
Probability: Weights given to distinct alternatives.

Possibility: Weights are given in a nested sets

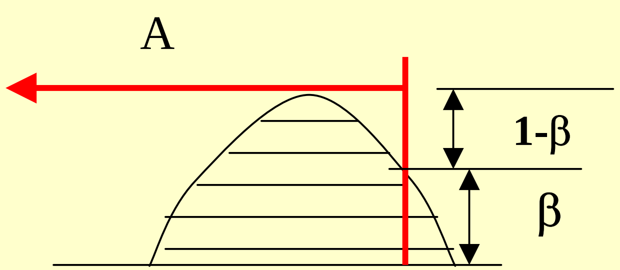
Possibility and Necessity measures depending on how to weigh evidence



Prob (A)=
Shaded area



Poss(A)=α, Nec (A)=0



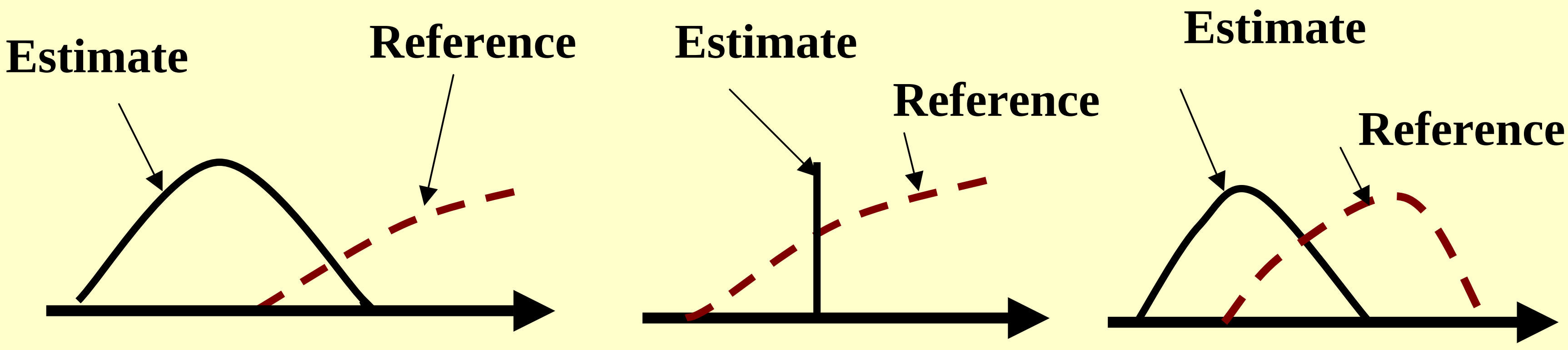
Poss(A)=1, Nec(A)=1-β

Example of Ambiguity

Given the context or reference (evidence) in approximate value,

Determine the truth of a proposition.

Comparison of two numbers.



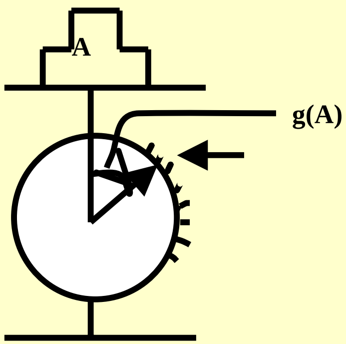
Frameworks for Measuring *Truth*

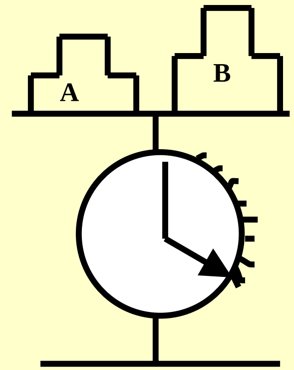
x is A

Information about x | **Nature of the set: crisp or fuzzy**

Type of Information about x		Nature of set A	
		Crisp set	Fuzzy set
	Probabilistic	Probability theory Probability	Probability theory Probability
	Possibilistic	Possibility theory Possibility measure Necessity measure	Possibility theory Possibility measure Necessity measure
	Combined	Dempster-Shafer theory Belief measure Plausibility measure	Dempster - Shafer theory Belief measure Plausibility measure

Measure Theory





Additive Measure
g(A+B)=g(A)+g(B)

Non-Additive Measure
g(A+B) < or > {g(A) + g(B)}

Axioms of Measure

- **Boundary Condition** $g(\emptyset)=0$ $g(X)=1$
- **Monotonicity Condition** $A \subseteq B,$ $g(A) \le g(B)$
- **Continuity Condition**

$$g(A \cup B) \ge \mathrm{Max}\{g(A), g(B)\}$$

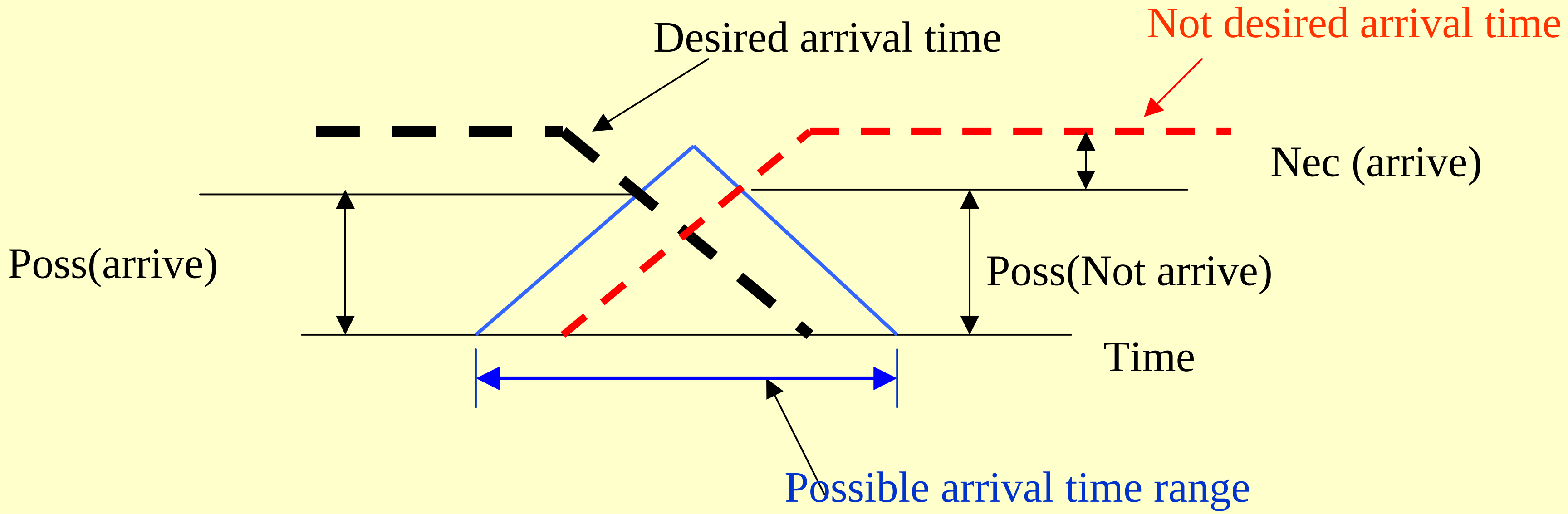
$$g(A \cap B) \le \mathrm{Min}\{ g(A), g(B)\}$$

Attitude Difference Under Uncertainty

Under Uncertainty, one's attitude dictates the decision.

Optimistic Attitude -- based on "We ***can*** do it." " It is ***possible.***"

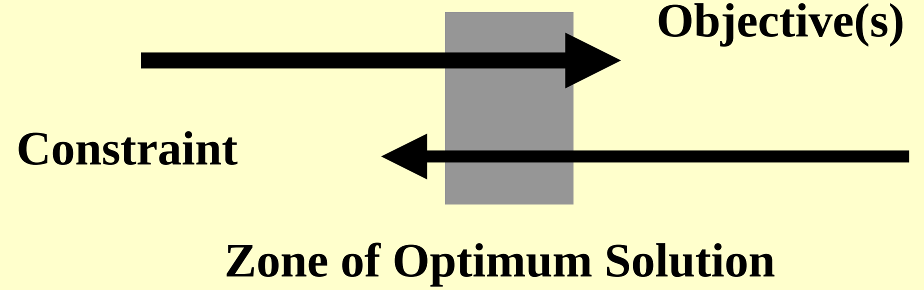
Pessimistic attitude-- based on " 'We ***cannot do it' is impossible***."



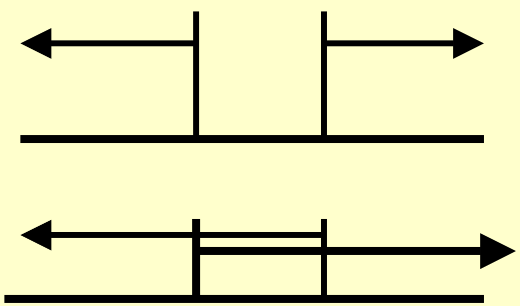
Optimization Model

Objective(s)

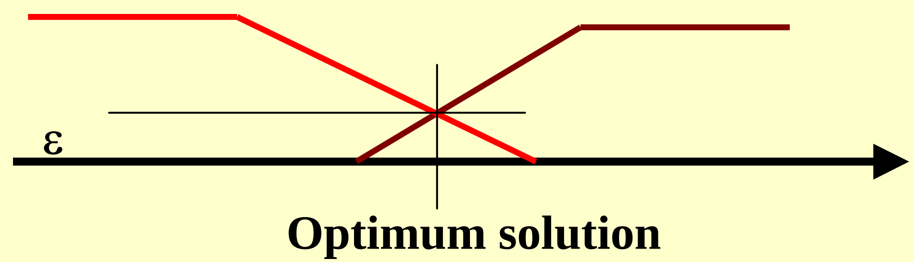
Constraints



Classical Optimization



Fuzzy Optimization



$$\varepsilon = \text{Max Min } h_G(x), h_C(x)$$

The value of the membership function may be considered as the force pushing to accept that value.

Multi-objective multi-constraint problem

Decision parameters : x, y, z

Performance affected by x,y,z: a, b, c

$a = f_1(x,y,z)$

$b = f_2(x,y,z)$

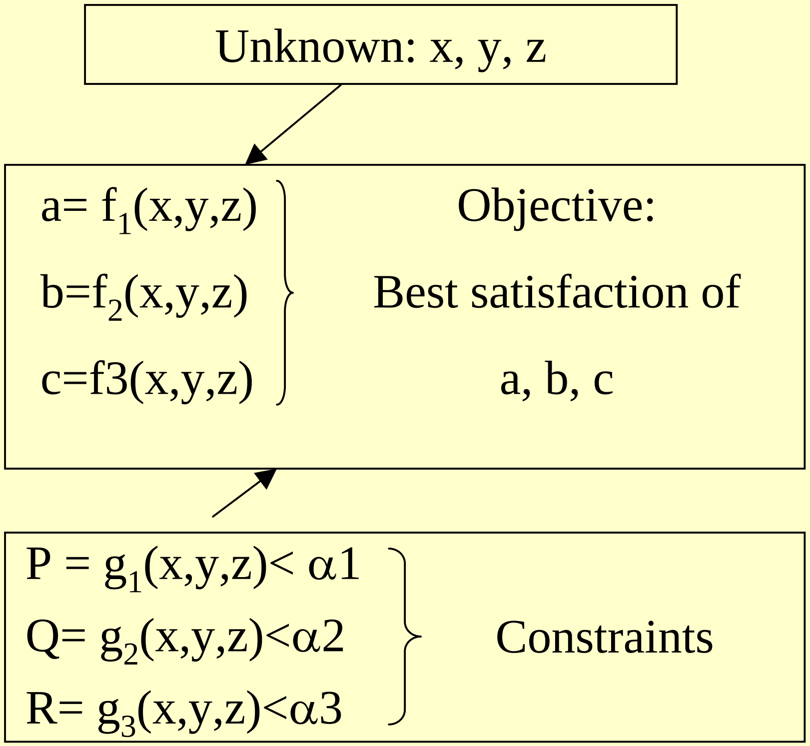
$c = f_3(x,y,z)$

Constraints related to x ,y, z : P, Q, R

$P = g_1(x,y,z) < \alpha_1$

$Q = g_2(x,y,z) < \alpha_2$

$R = g_3(x,y,z) < \alpha_3$



Obejctives: satisfaction of the decision maker

$h_A[f_1(x,y,z)]$ $h_P[g_1(x,y,z)]$

$h_B[f_2(x,y,z)]$ $h_Q[g_2(x,y,z)]$

$h_C[f_3(x,y,z)]$ $h_R[g_3(x,y,z)]$

Max min {

$h_A[f_1(x,y,z)]$ $h_P[g_1(x,y,z)]$

$h_B[f_2(x,y,z)]$ $h_Q[g_2(x,y,z)]$

$h_C[f_3(x,y,z)]$ $h_R[g_3(x,y,z)]$}

Optimization Problem

Max h

$h_A[f_1(x,y,z)]>h$ $h_P[g_1(x,y,z)]>h$

$h_B[f_2(x,y,z)]>h$ $h_Q[g_2(x,y,z)]>h$

$h_C[f_3(x,y,z)]>h$ $h_R[g_3(x,y,z)]>h$

Non-linear optimization problem

Optimum values of x, y, z that satisfy the goals and constraints

Application Areas (1)

Modeling of human behavior and decision process (choice)

Human control: rule based handling of stimulus - response process

- **Driver behavior – stimulus-response process**
- **Air traffic controller's decision process**

Choice modeling:

Comparison of alternatives: comparison of approximate numbers

Comparison of performance and target (or desired state)

Application Areas (2)

Large –scale systems problems

- **Rule based analysis,**
  - **Prediction, diagnosis, control**
- **Optimization**
  - **Multi-objective and multi-constraint problem**
- **Reasoning process**
  - **System justification logic building (e.g. ITS)**

Multi-criteria evaluation

- **Treatment of non-additive weights**
- **Fuzzy measure and fuzzy integral problem**

Application Areas(3)

Data handling

- **Treatment of approximate numbers**

Control problems

- **Traffic signals**
- **Traffic flow control**

Summary

Preserve uncertainty as much as possible during the analysis process.

Information should not be added arbitrarily.

Understand where and when to eliminate uncertainty – cut-off point.

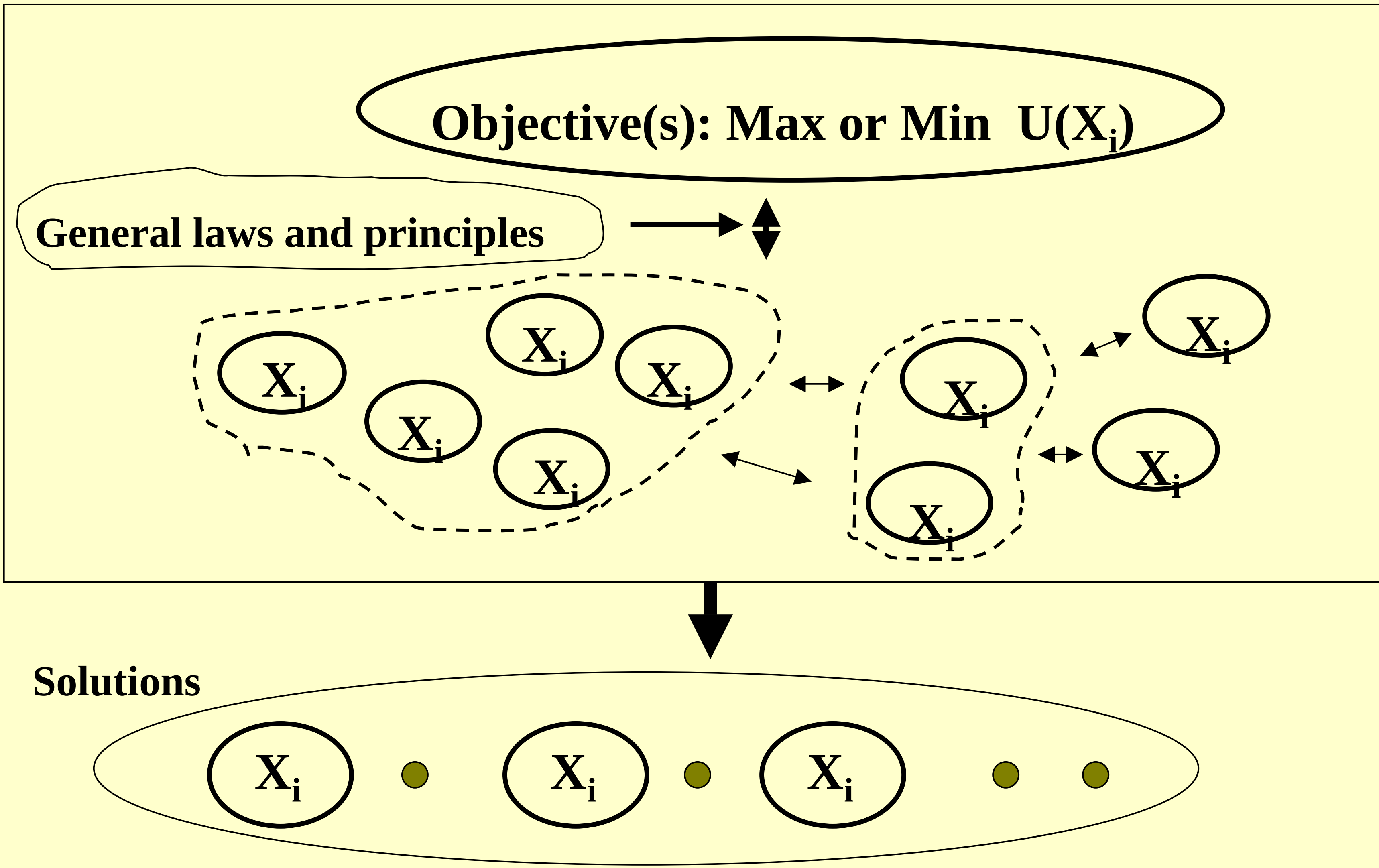
Understand the degree of accuracy required and do not pursue accuracy beyond the requirement.

Do not let mathematical framework control the analysis process – be flexible in the use of mathematical approach. Select the mathematical framework faithful to the type of uncertainty.

Profess uncertainty and ignorance honestly

Comment or questions?

Top-Down Approach:Traditional Approach



Bottom-up Approach: Decentralized Approach