CHAPTER 9 SHEAR STRENGTH THEORY

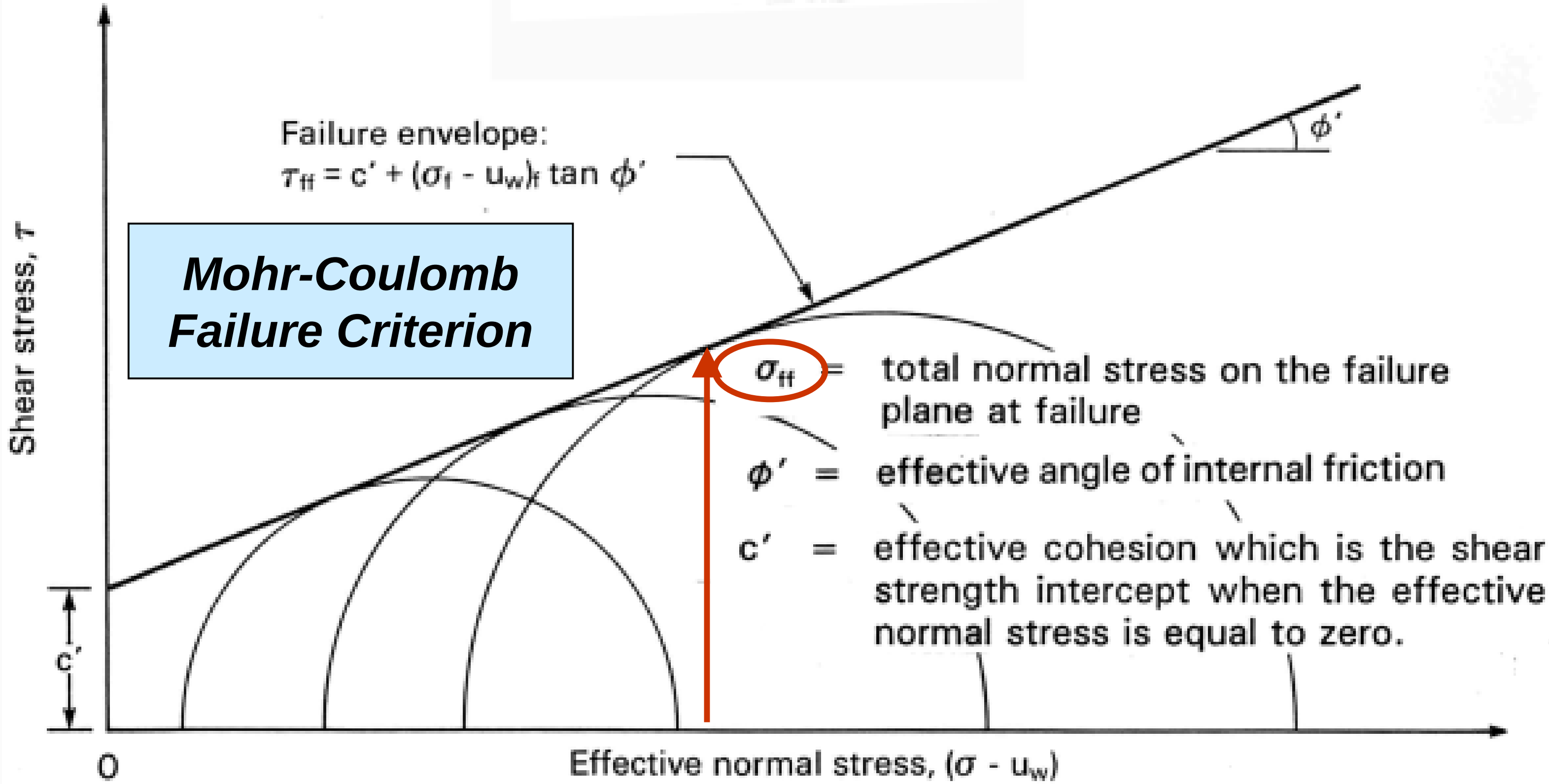
Must know u_a & u_w

Mohr-Coulomb Failure Criterion

Types of Triaxial Tests

CHAPTER 9

SHEAR STRENGTH THEORY

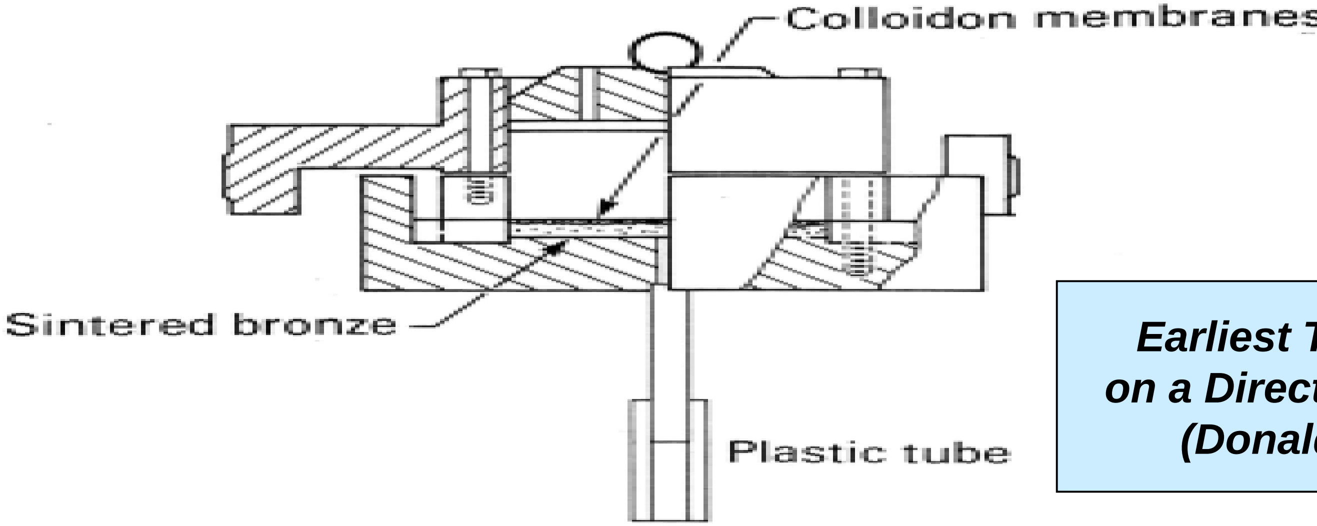


Failure envelope for a saturated soil showing the Mohr-Coulomb failure criterion

Shear strength designation for a saturated soil

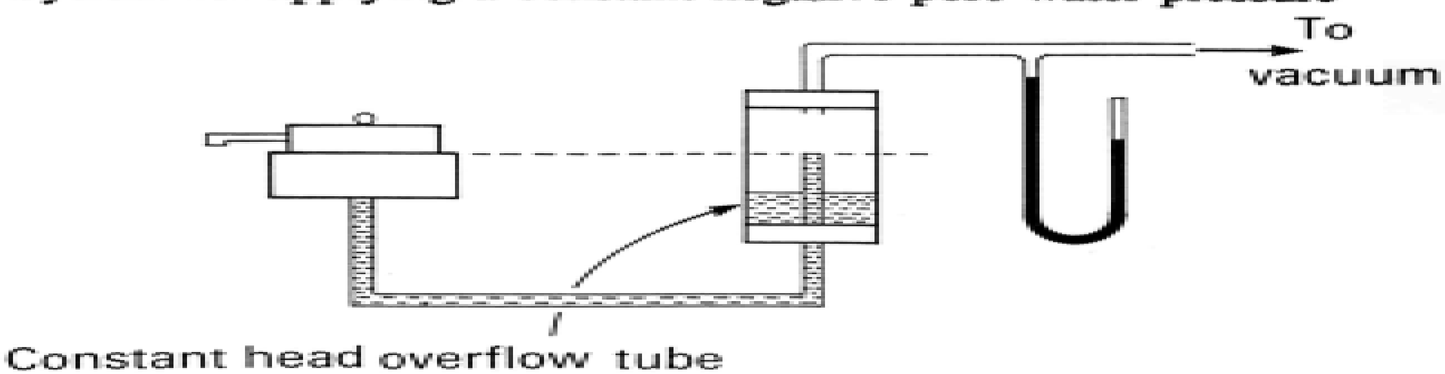
HISTORY OF SHEAR STRENGTH

Modified direct shear box with a colloidon membrane

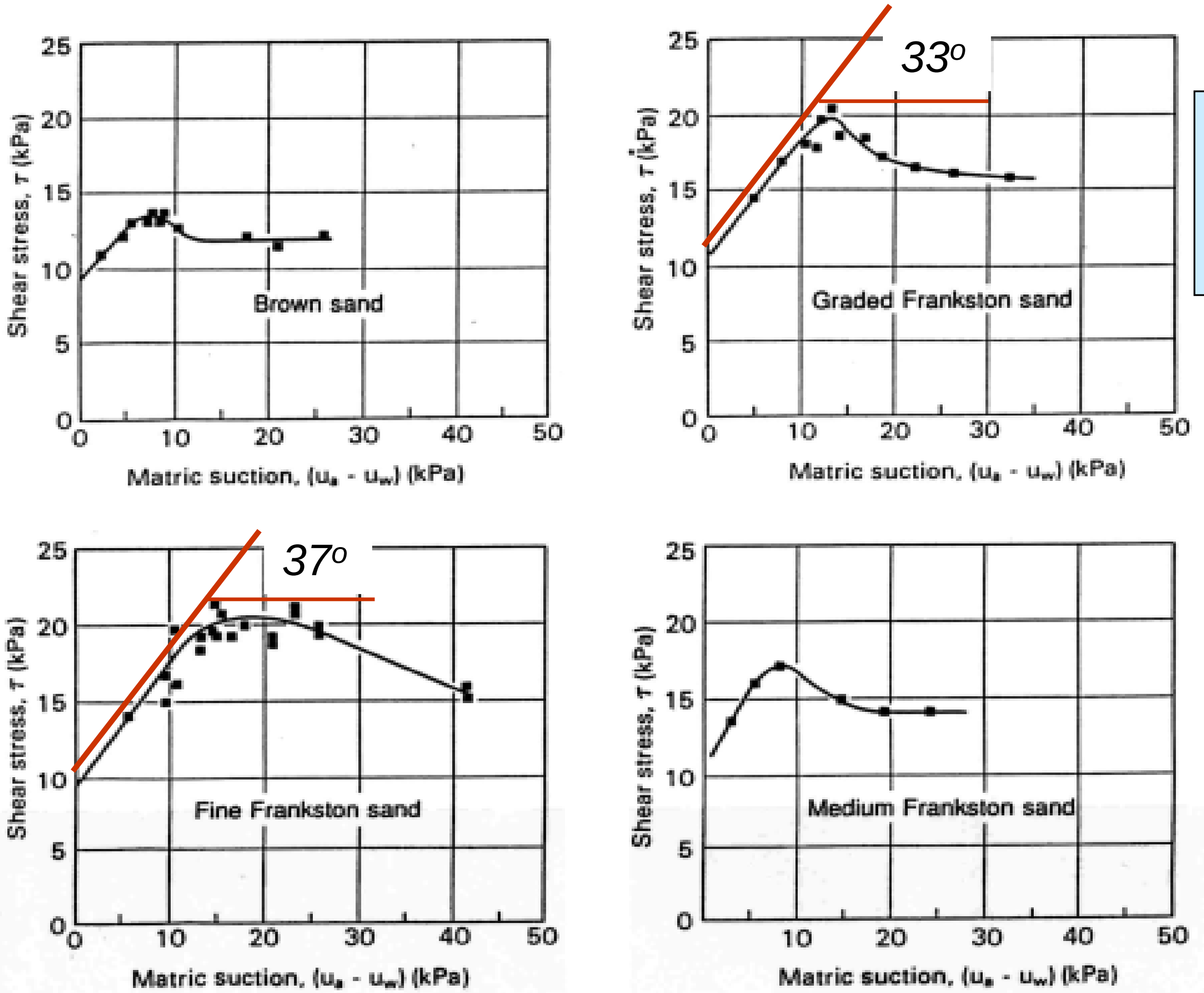


Earliest Test done on a Direct Shear Box (Donald, 1956)

System for applying a constant negative pore-water pressure



Modified direct shear equipment for testing soils under low matric suction (from Donald, 1956)



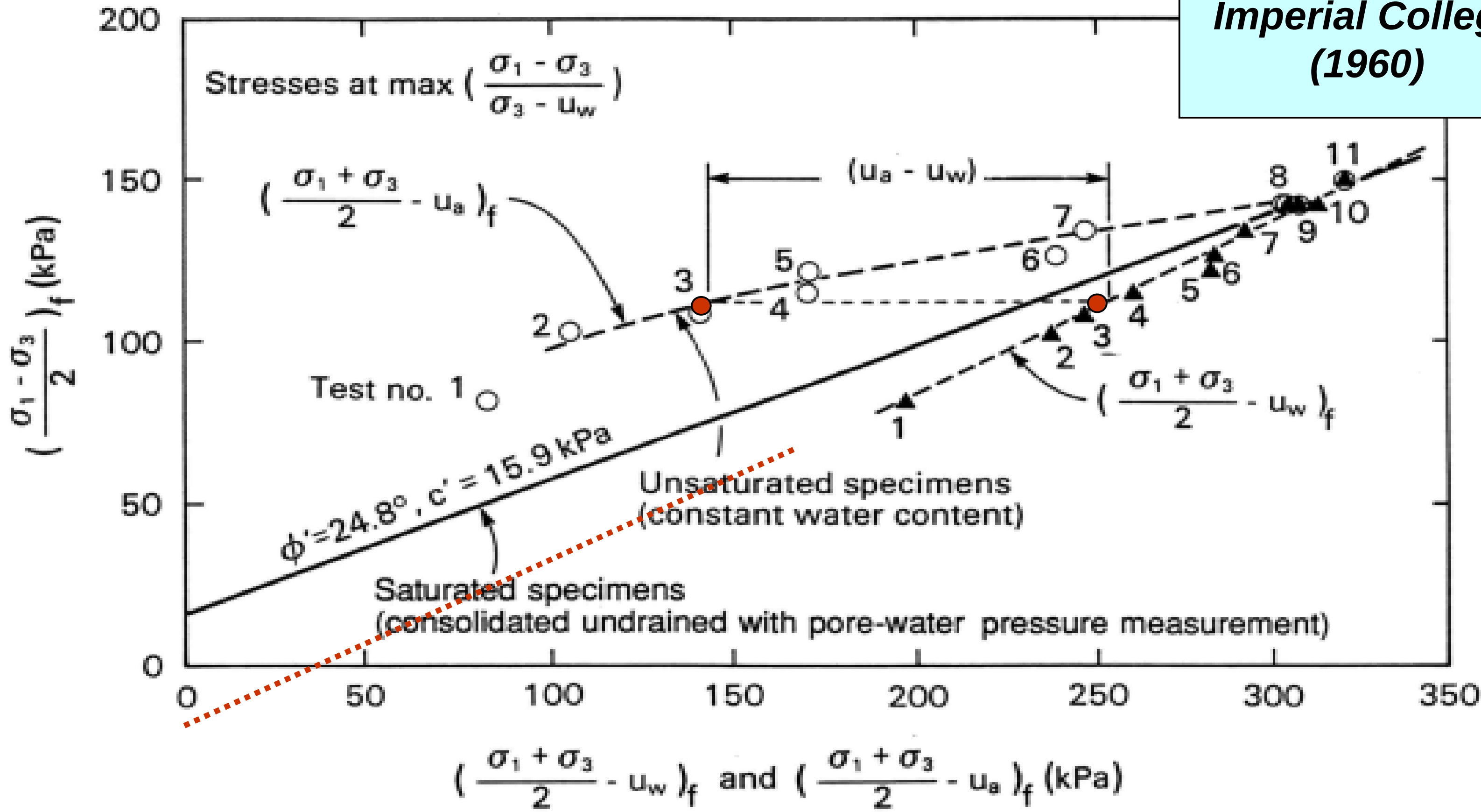
Direct Shear Test Results (Donald, 1956)

$\sigma = $ ***17 kPa***

Results of direct shear tests on sands under low matric suctions (modified from Donald, 1956)

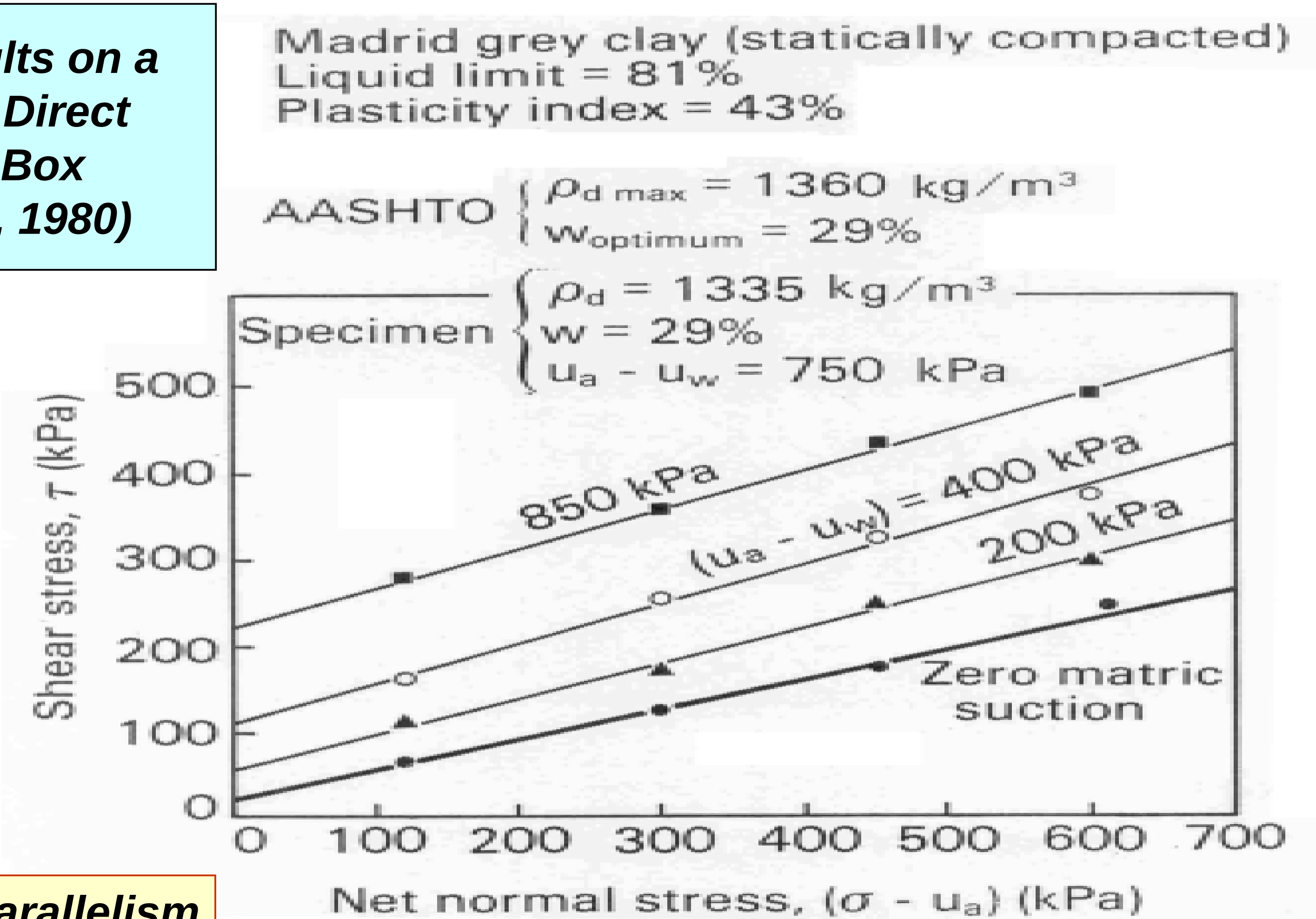
Shear Strength of Unsaturated Soils

Triaxial Tests Imperial College (1960)



Results of constant water content triaxial tests on a shale (clay fraction 22%) compacted at a water content of 18.6% (from Bishop, Alpan, Blight and Donald, 1960)

Test Results on a Modified Direct Shear Box (Escario, 1980)



Note the parallelism in the lines

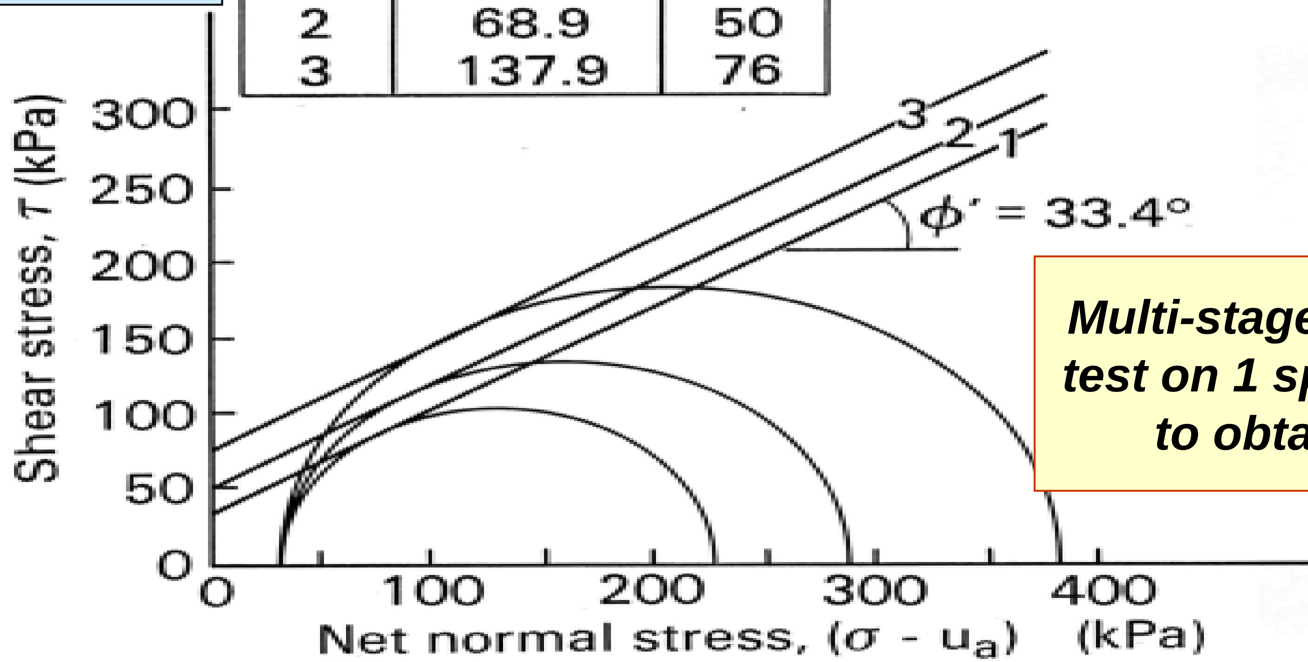
Increase in shear strength for Madrid clay due to an increase in matric suction, obtained from direct shear tests (from Escario, 1980)

Failure envelope projection onto the τ versus $(\sigma - u_a)$ plane

Test Results on a Modified Triaxial Apparatus (Ho & Fredlund, 1982)

No.	$(u_a - u_w)_f$ (kPa)	C (kPa)
1	34.5	32.5
2	68.9	50
3	137.9	76

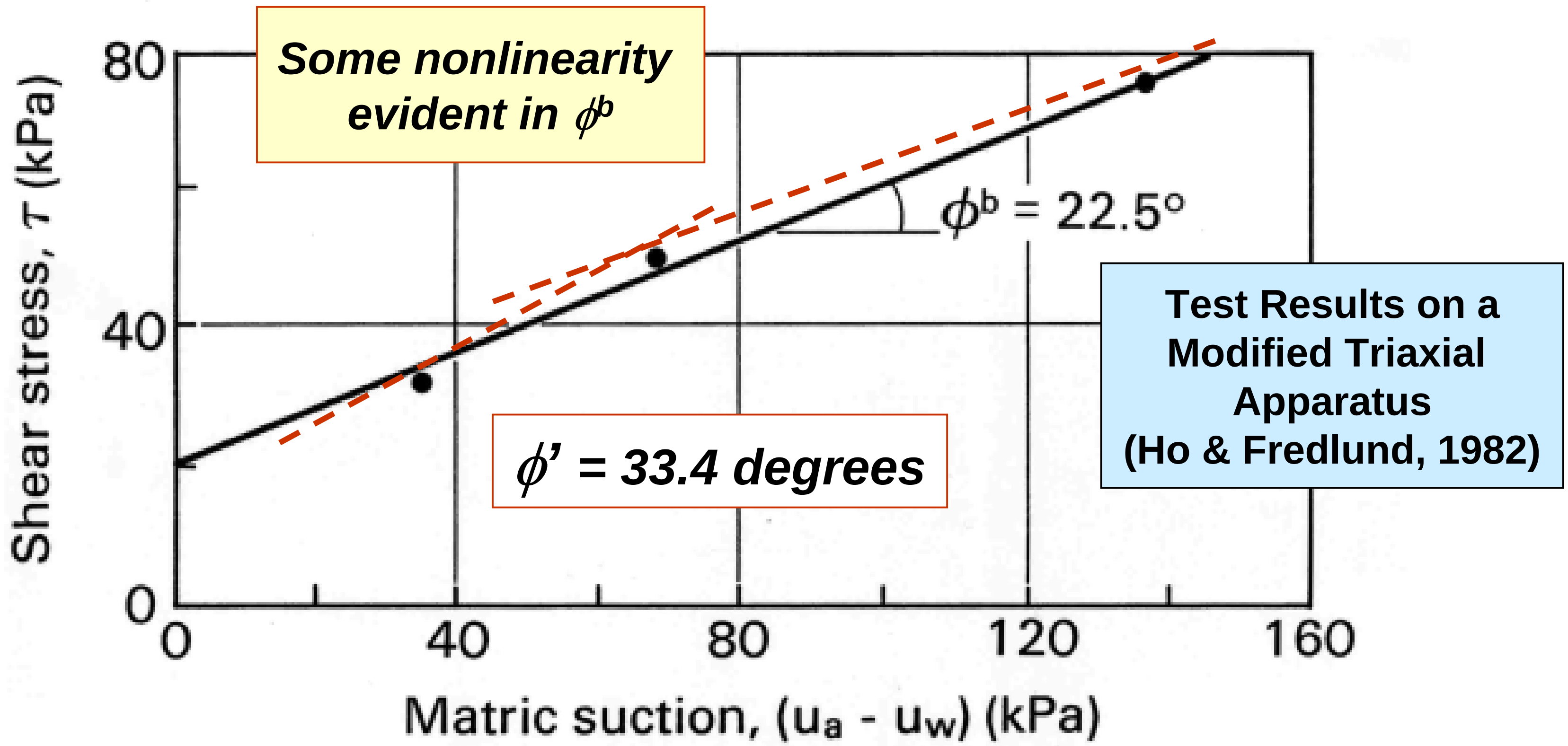
Parallelism assumed in the interpretation



Multi-stage triaxial test on 1 specimen to obtain ϕ^b

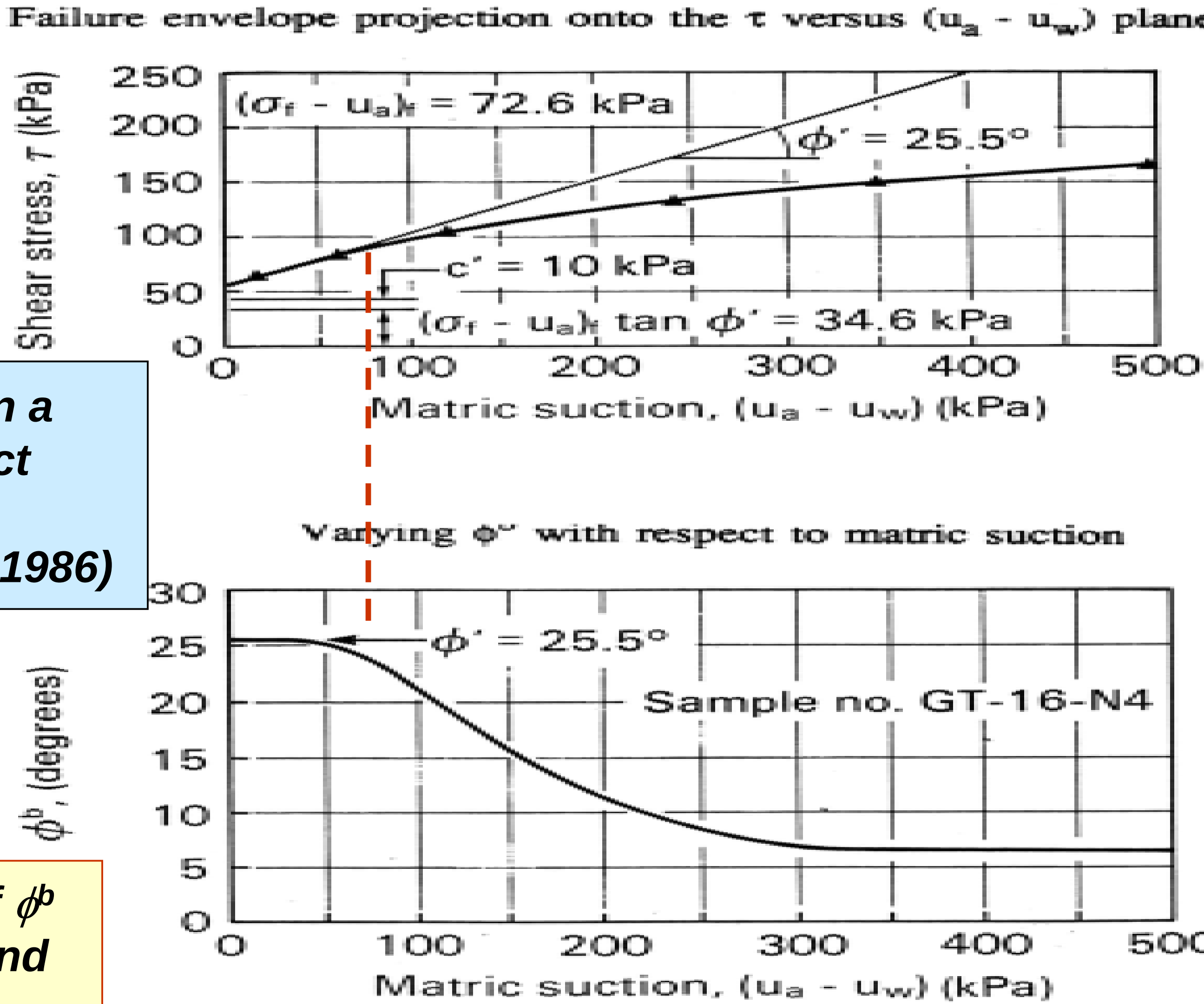
Two-dimensional presentation of failure envelope for decomposed granite specimen No. 22 (from Ho and Fredlund, 1982a)

Intersection line between the failure envelope and the τ versus $(u_a - u_w)$ plane



Two-dimensional presentation of failure envelope for decomposed granite specimen No. 22 (from Ho and Fredlund, 1982a)

Unsaturated Soil Technology

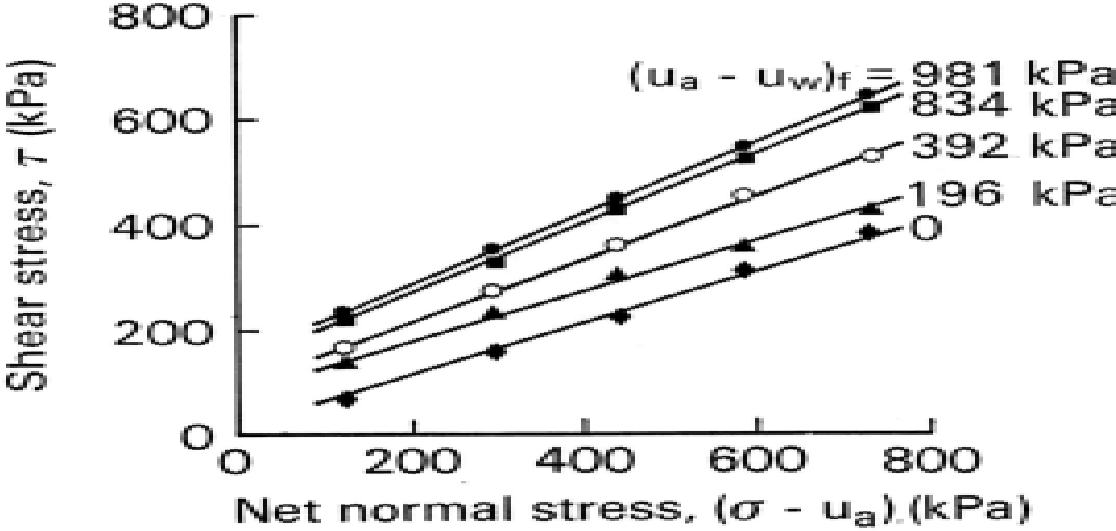


Test Results on a Modified Direct Shear Box (Gan & Fredlund, 1986)

Departure of ϕ^b from ϕ' around 75 kPa

Direct shear test results exhibiting a nonlinear relationship between τ versus (u_a - u_w) (from Gan, 1986)

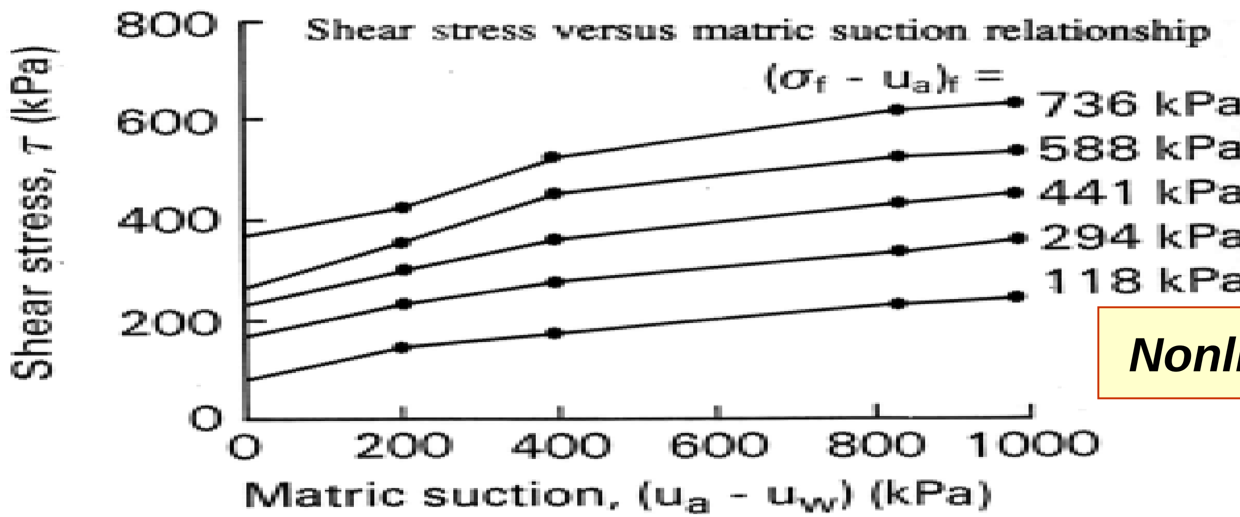
Unsaturated Soil Technology



Shear stress versus net confining pressure relationship for various matric suctions

Linearity of ϕ'

Test Results on a Modified Direct Shear Box (Escario & Saez, 1986)

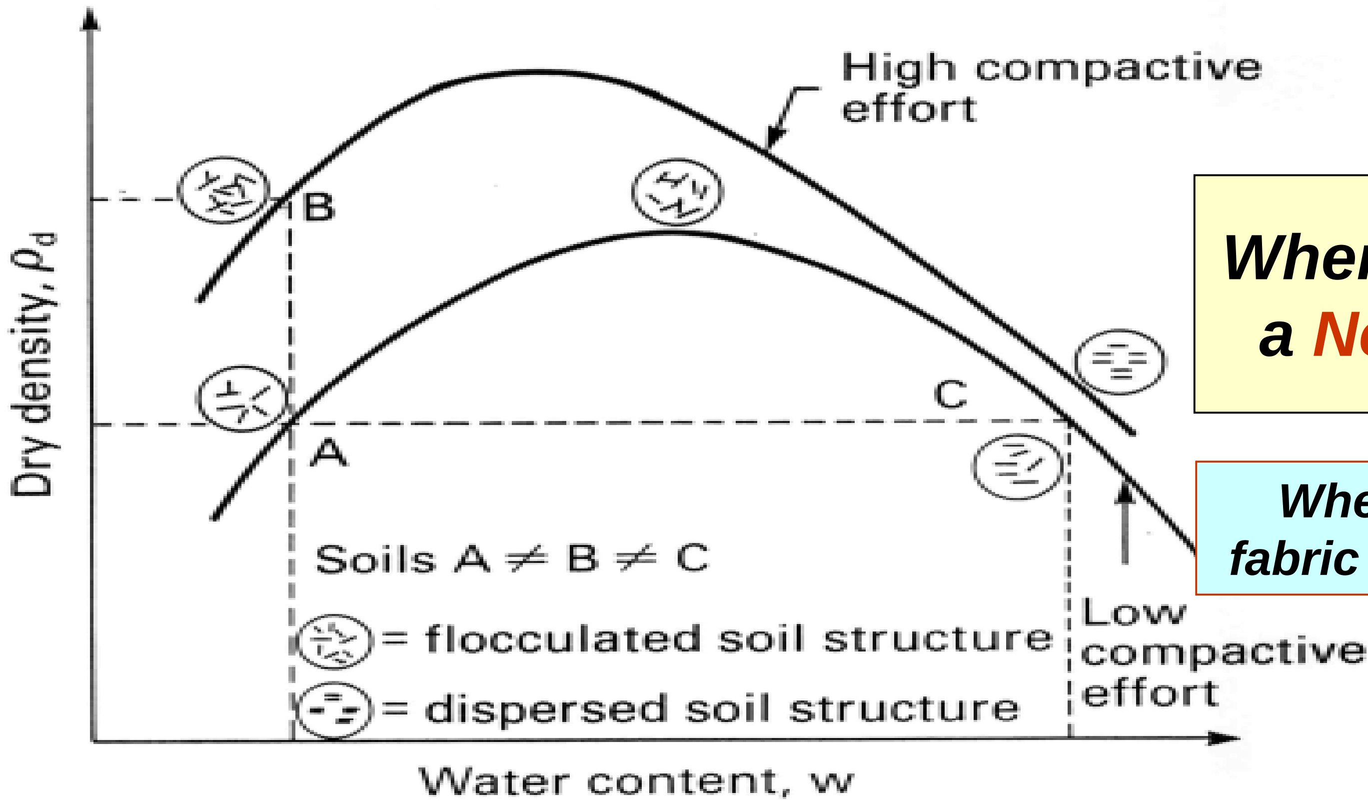


25 tests

Nonlinearity of ϕ^b

Direct shear test results for Madrid grey clay, under controlled matric suctions (from Escario and Sáez, 1986)

- a Change in Compaction Density Changes the Shear Strength Parameters.

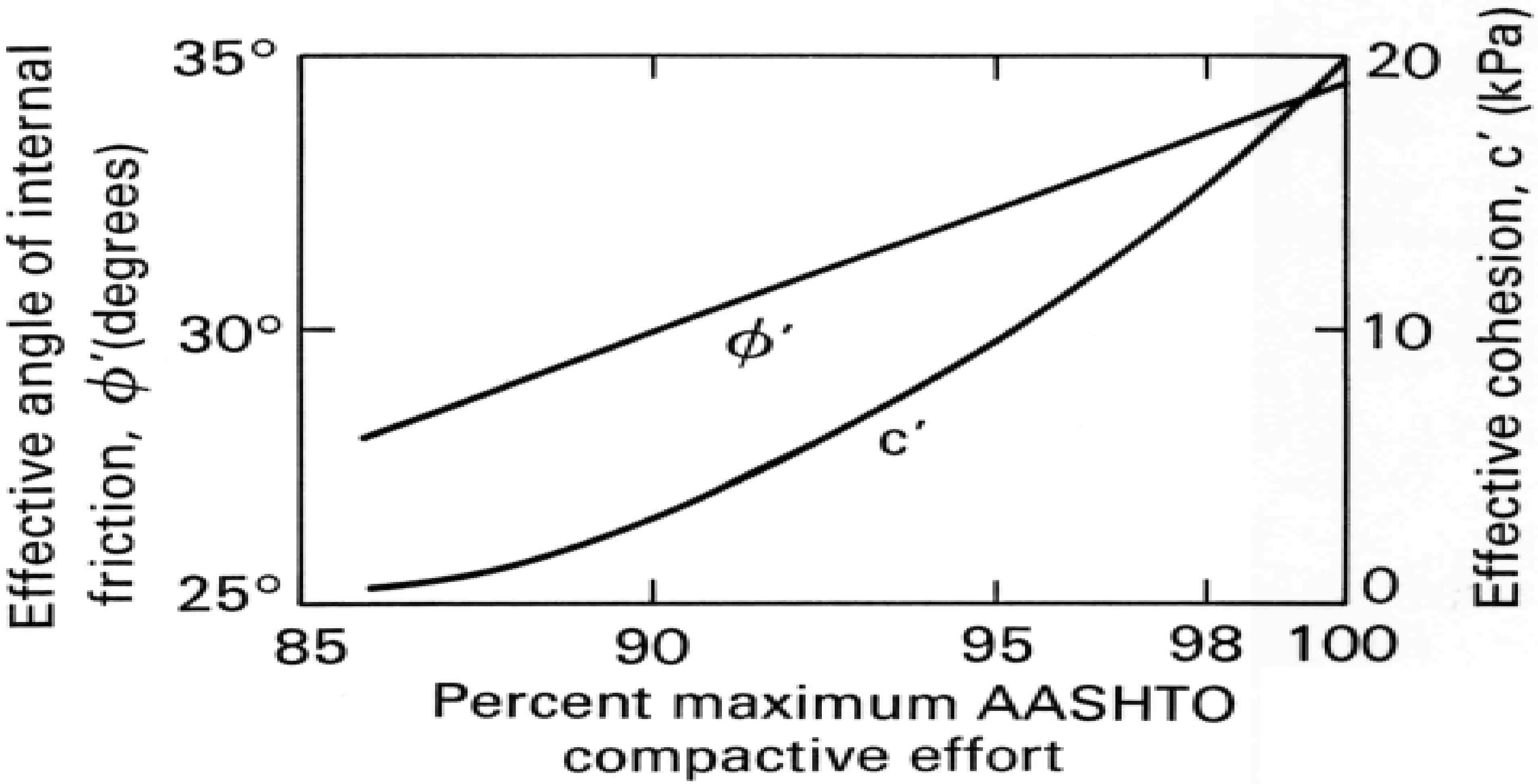


When is a Soil a New Soil?

When the soil fabric is changed?

The particle structure of clay specimens compacted at various dry densities and water contents (from Lambe, 1958)

When is a Soil a "NEW SOIL"?

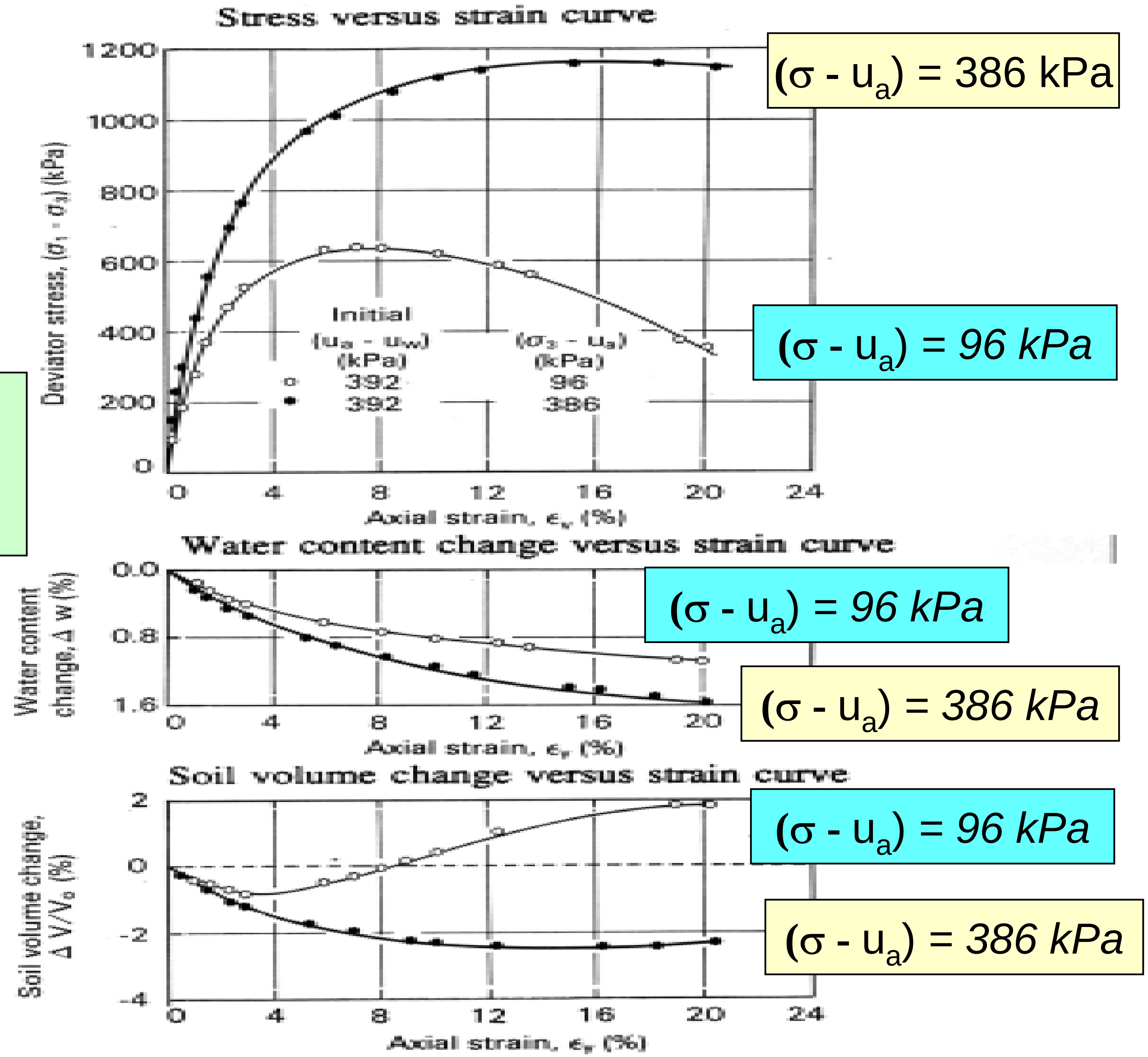


Same soil compacted at different water contents can have different soil parameters due to different soil fabrics

Effect of compactive effort on φ′ and c′ for a clayey sand (from Moretto, Bolognesi, Lopez and Nunez, 1963)

How should "Failure" be defined?

Consolidated Drained Triaxial



Consolidated drained triaxial test results on Dhanauri clay (from Satija, 1978)

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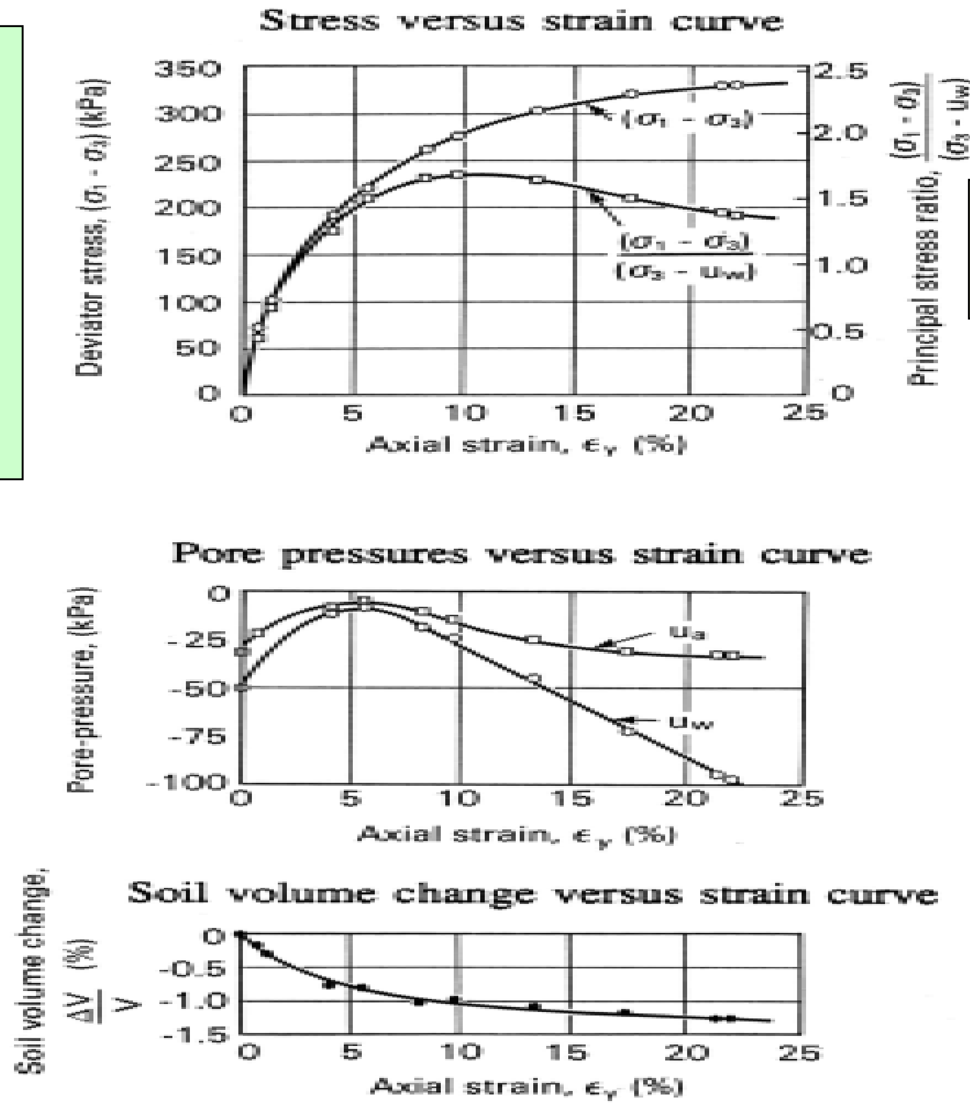
***Consolidated Undrained* triaxial test with pore pressure measurements**

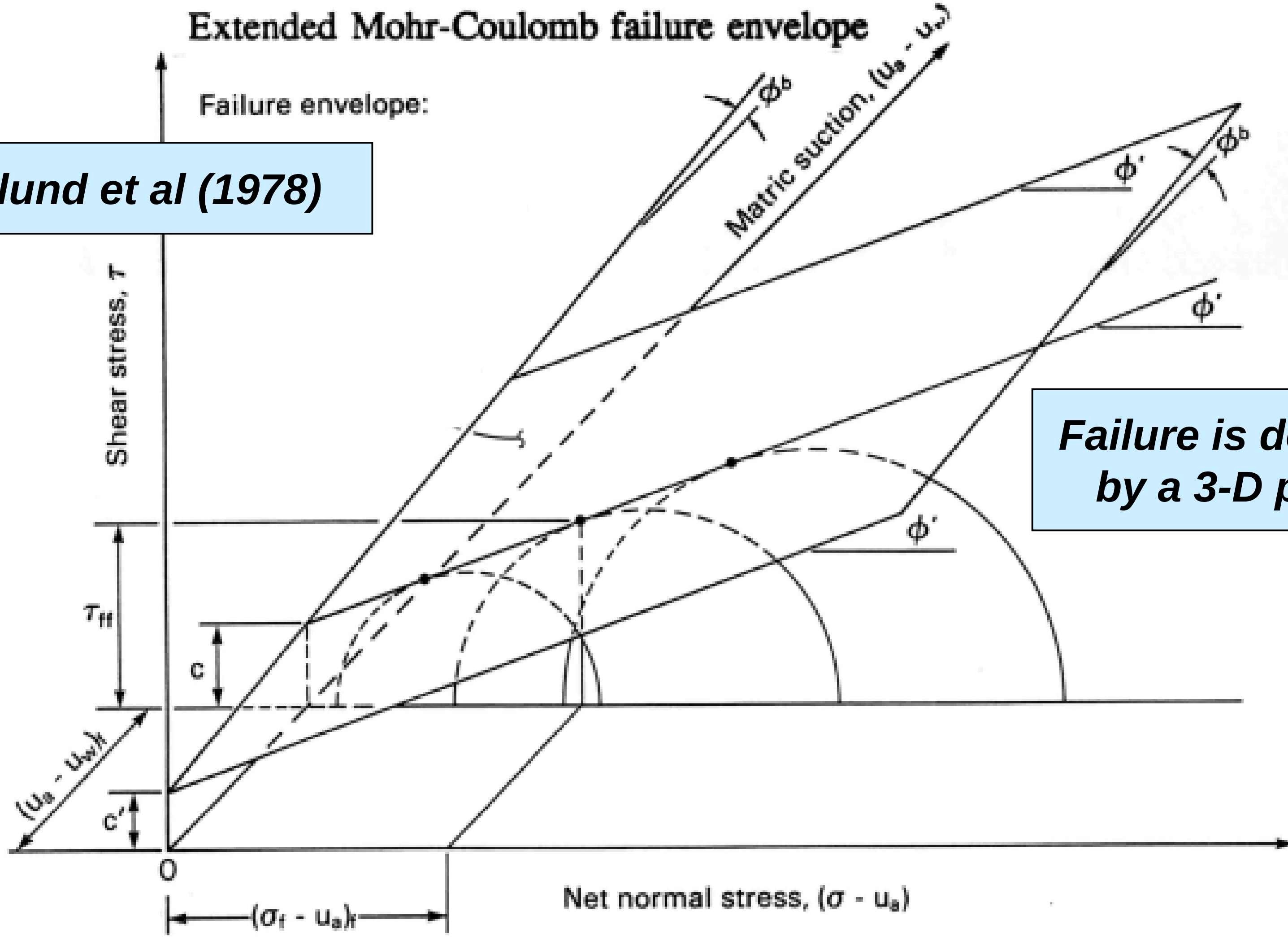
Deviator stress

Principal stress ratio



Undrained triaxial tests on a compacted shale (from Bishop, Alpan, Blight and Donald, 1960)

Unsaturated Soil Technology



Fredlund et al (1978)

Failure is defined by a 3-D plane

In its simplest form, an "Extended Mohr-Coulomb" type failure surface can be drawn

FAILURE ENVELOPE FOR UNSATURATED SOILS

Linear form

Extended Mohr-Coulomb Failure Envelope

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b$$

where:

- c' = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis where the net normal stress and the matric suction at failure are equal to zero. It is also referred to as "effective cohesion".
- $(\sigma_f - u_a)_f$ = net normal stress state variable on the failure plane at failure
- u_{af} = pore-air pressure on the failure plane at failure
- ϕ' = angle of internal friction associated with the net normal stress state variable, $(\sigma_f - u_a)_f$
- $(u_a - u_w)_f$ = matric suction on the failure plane at failure
- ϕ^b = angle indicating the rate of increase in shear strength relative to the matric suction, $(u_a - u_w)_f$

Proposed by Fredlund et al (1978)

FAILURE ENVELOPE FOR UNSATURATED SOILS

Extended Mohr-Coulomb Failure Envelope

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b$$

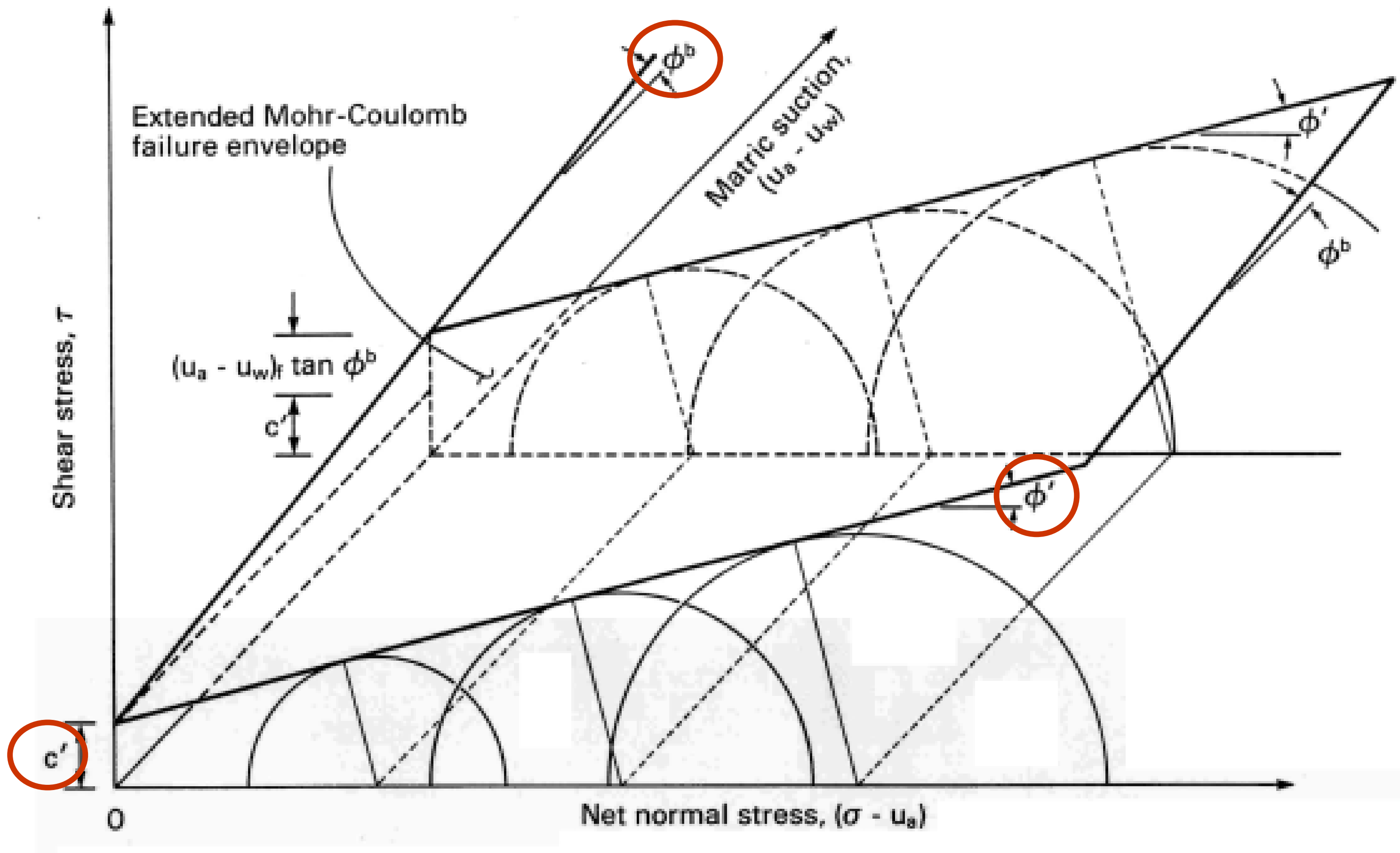
Proposed by Fredlund et al (1978)

Let us assume that cohesion has two components; namely effective cohesion and cohesion due to matric suction

$$c = c' + (u_a - u_w)_f \tan\phi^b$$

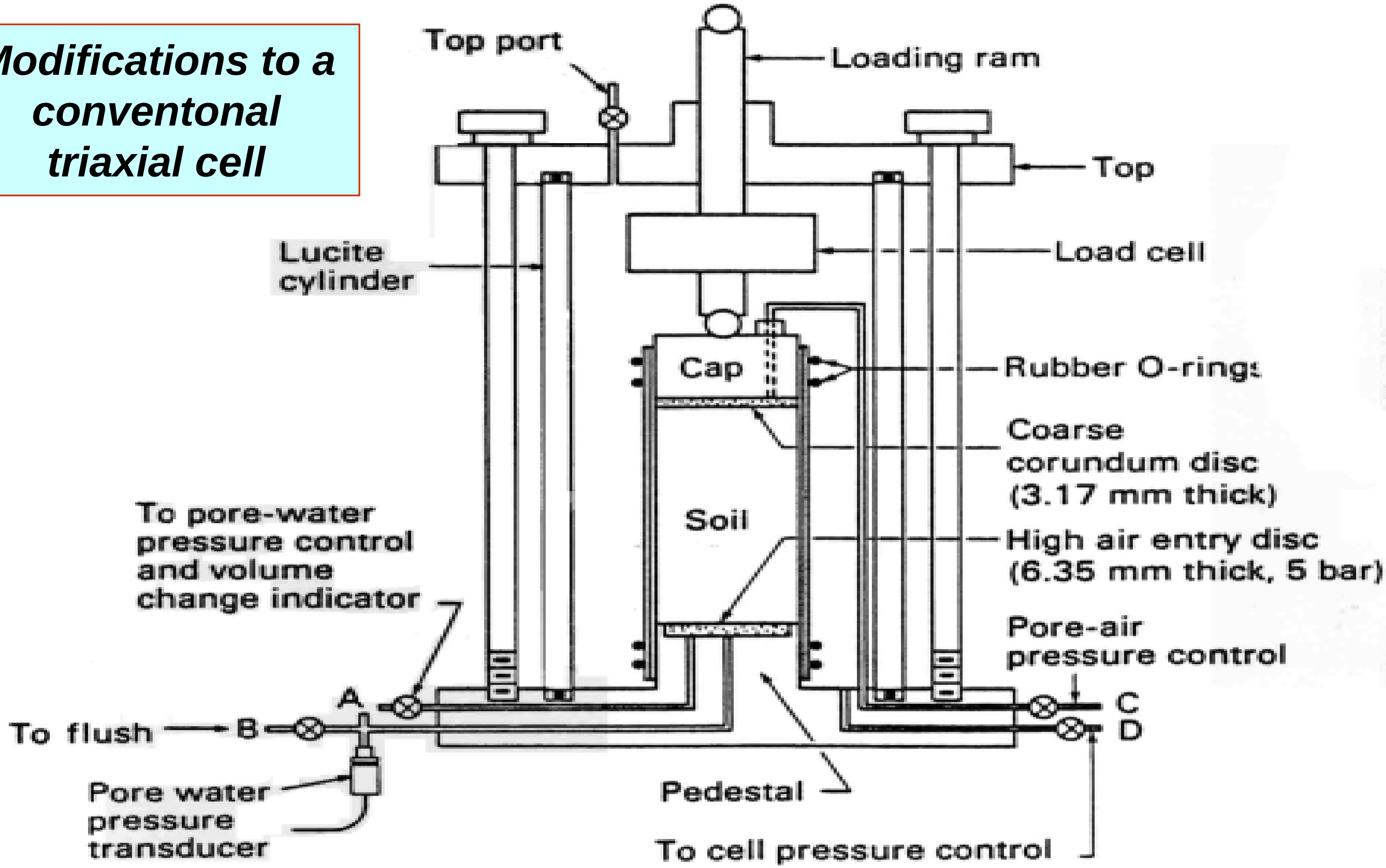
where:

c = intercept of the extended Mohr-Coulomb failure envelope with the shear stress axis at a specific matric suction, $(u_a - u_w)_f$, and zero net normal stress. It can be referred t "total cohesion intercept".



Linear, Extended Mohr-Coulomb Failure Envelope (Fredlund et al, 1978)

Modifications to a conventonal triaxial cell



Primary modification is the high air entry disk sealed into the base pedestal

Modified triaxial cell for testing unsaturated soils

Experimental Values of ϕ^b

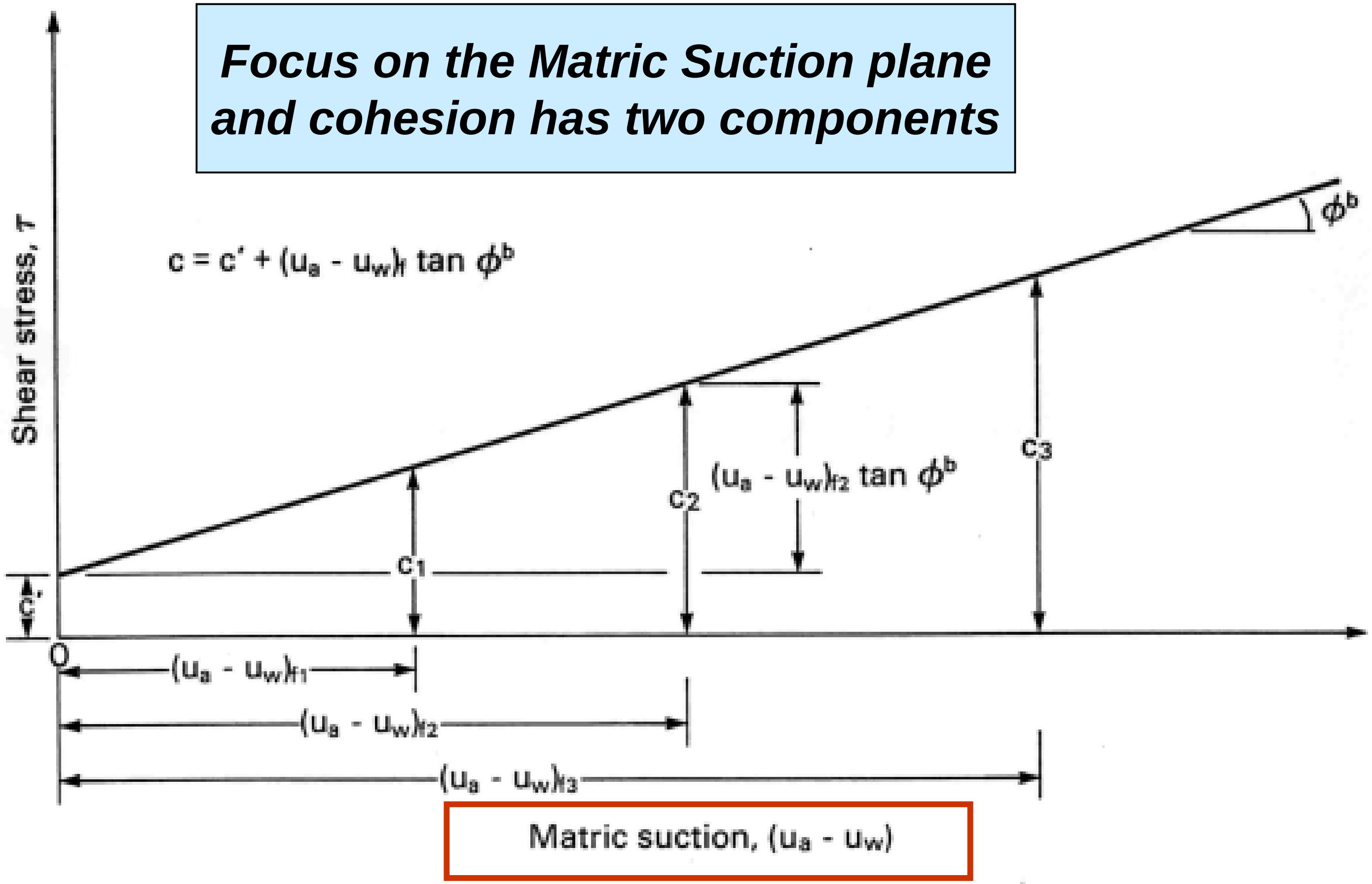
Soil Type	c' (kPa)	ϕ' (degrees)	ϕ^b (degrees)	Test Procedure	Reference
Compacted shale; $w = 18.6\%$	15.8	24.8	18.1	Constant water content triaxial	Bishop *et al.* (1960)
Boulder clay; $w = 11.6\%$	9.6	27.3	21.7	Constant water content triaxial	Bishop *et al.* (1960)
Dhanauri clay; $w = 22.2\%$, $\rho_d = 1580\ kg/m^3$	37.3	28.5	16.2	Consolidated drained triaxial	Satija, (1978)
Dhanauri clay; $w = 22.2\%$, $\rho_d = 1478\ kg/m^3$	20.3	29.0	12.6	Constant drained triaxial	Satija, (1978)
Dhanauri clay; $w = 22.2\%$, $\rho_d = 1580\ kg/m^3$	15.5	28.5	22.6	Consolidated water content triaxial	Satija, (1978)
Dhanauri clay; $w = 22.2\%$, $\rho_d = 1478\ kg/m^3$	11.3	29.0	16.5	Constant water content triaxial	Satija, (1978)
Madrid grey clay; $w = 29\%$,	23.7	22.5[a]	16.1	Consolidated drained direct shear	Escario (1980)
Undisturbed decomposed granite; Hong Kong	28.9	33.4	15.3	Consolidated drain multistage triaxial	Ho and Fredlund (1982a)
Undisturbed decomposed rhyolite; Hong Kong	7.4	35.3	13.8	Consolidated drained multistage triaxial	Ho and Fredlund (1982a)
Tappen–Notch Hill silt; $w = 21.5\%$, $\rho_d = 1590\ kg/m^3$	0.0	35.0	16.0	Consolidated drained multistage triaxial	Krahn *et al.* (1989)
Compacted glacial till; $w = 12.2\%$, $\rho_d = 1810\ kg/m^3$	10	25.3	7–25.5	Consolidated drained multistage direct shear	Gan *et al.* (1988)

[a]Average value.

Average = 17°

Unsaturated Soil Technology

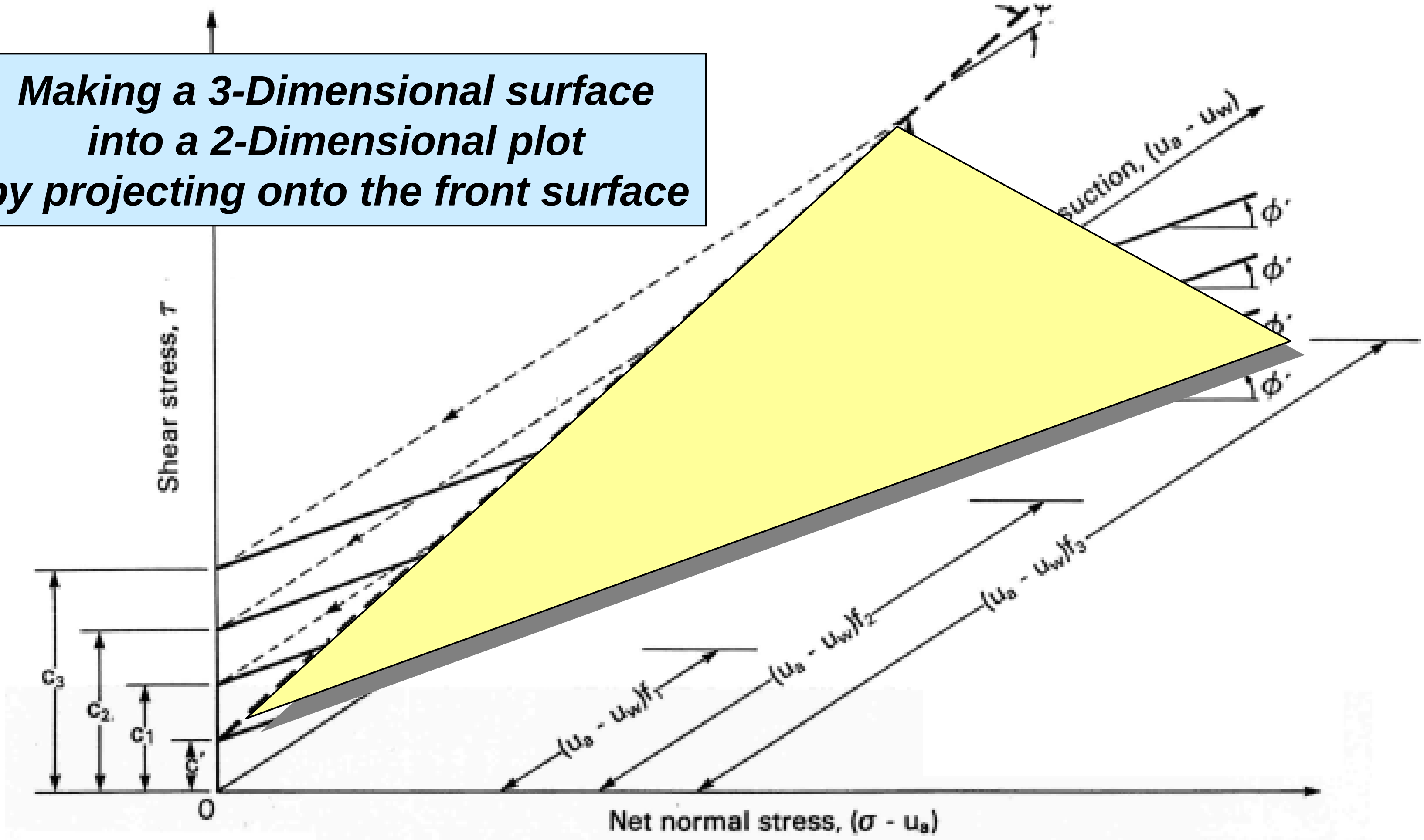
Focus on the Matric Suction plane and cohesion has two components



Line of intercepts along the failure plane on the τ versus $(u_a - u_w)$ plane

Failure envelope projections onto the τ versus (σ - u_a) plane projections

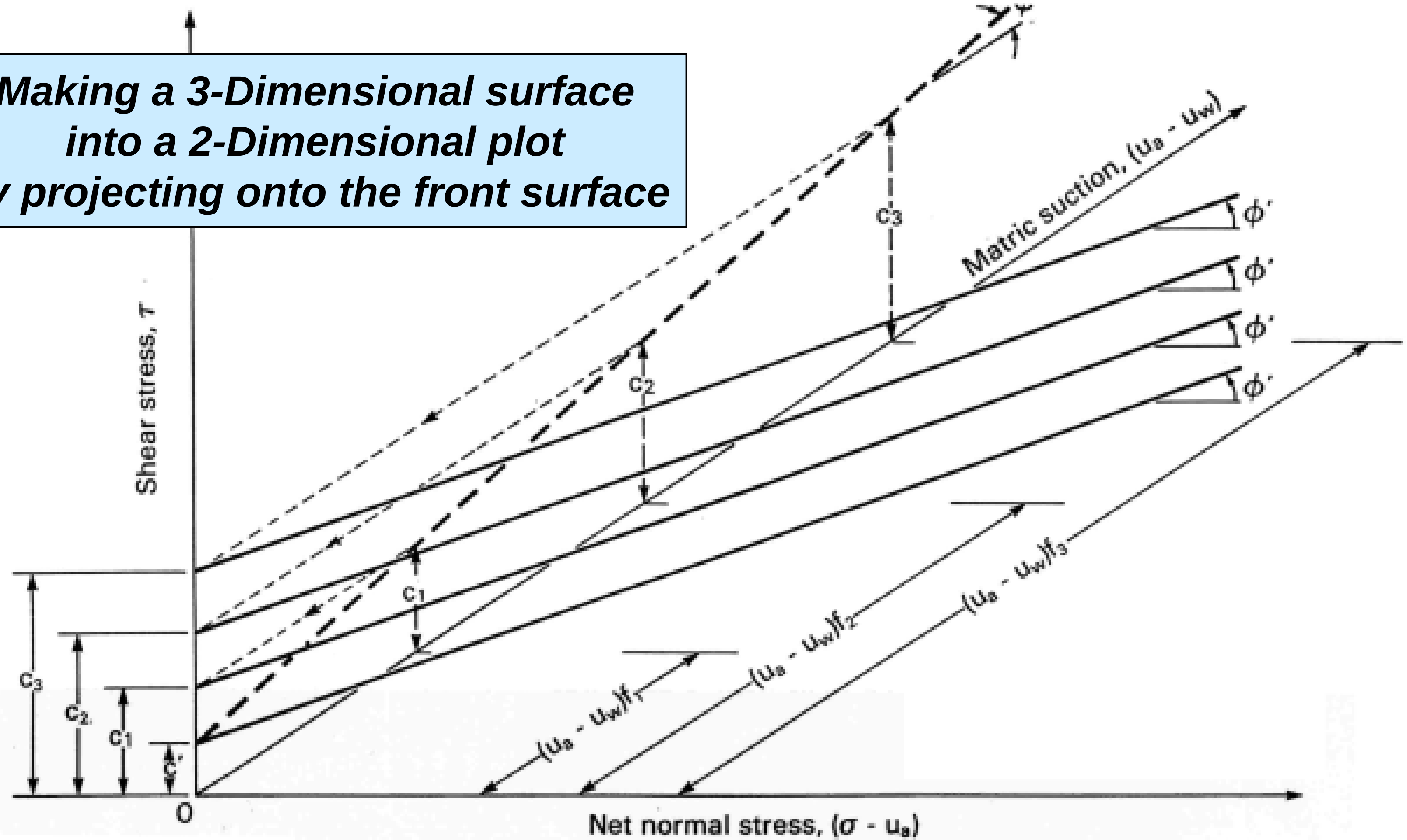
Making a 3-Dimensional surface into a 2-Dimensional plot by projecting onto the front surface



Horizontal projection of the failure envelope onto the τ versus (σ - u_a) plane, viewing parallel to the (u_a - u_w) axis

Failure envelope projections onto the τ versus $(\sigma - u_a)$ plane projections

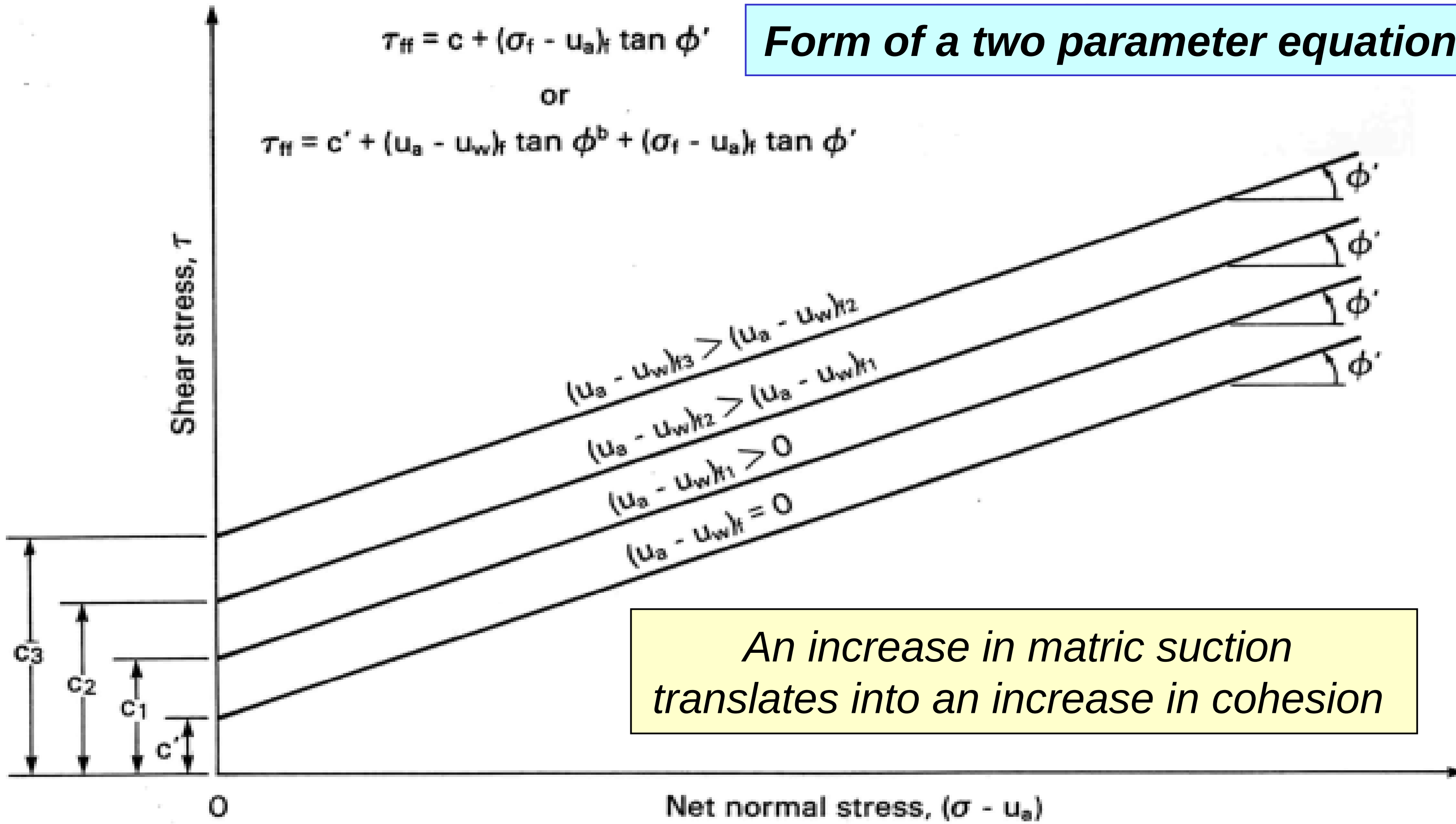
Making a 3-Dimensional surface into a 2-Dimensional plot by projecting onto the front surface



Horizontal projection of the failure envelope onto the τ versus $(\sigma - u_a)$ plane, viewing parallel to the $(u_a - u_w)$ axis

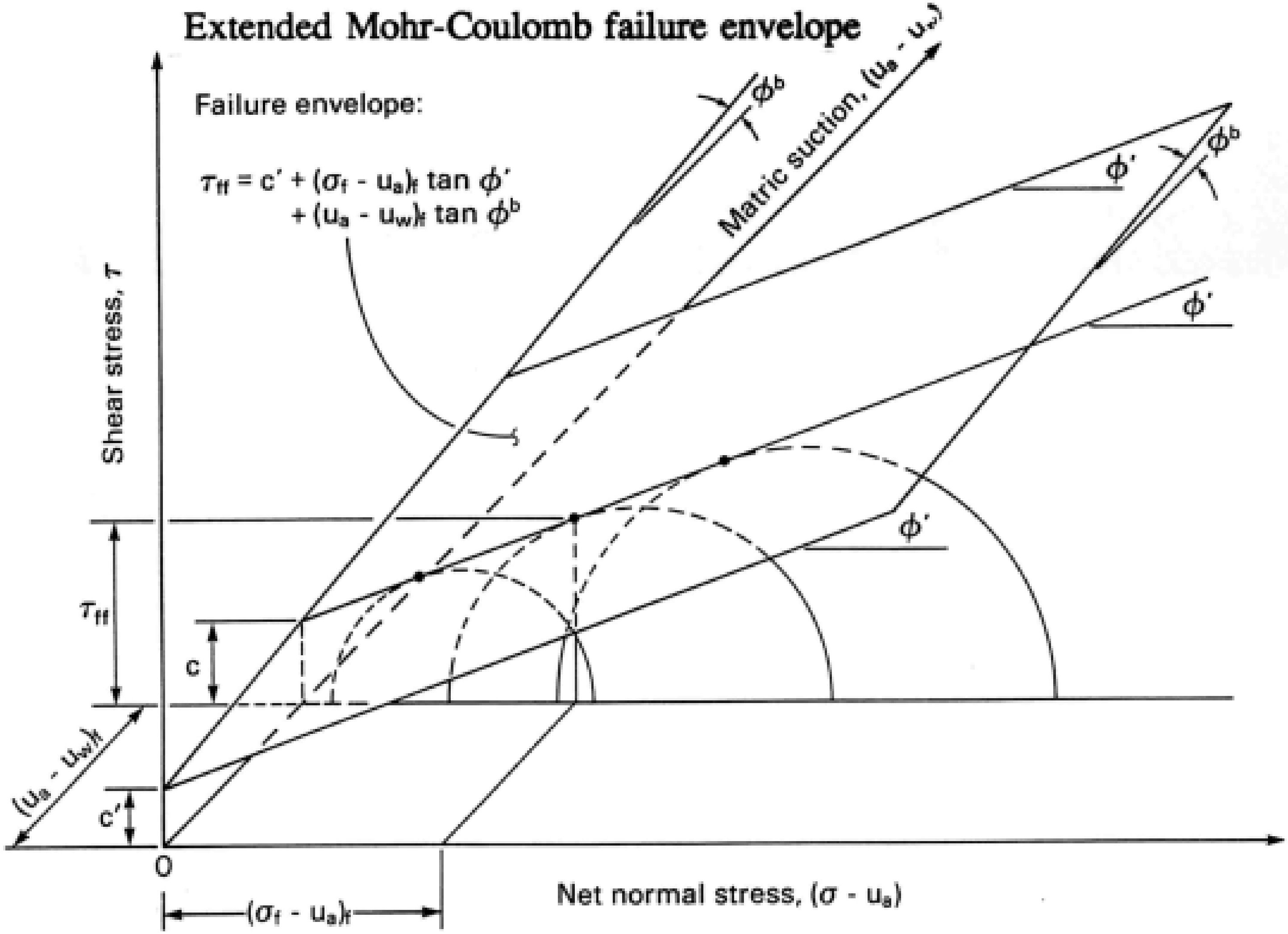
Contour lines of the failure envelope onto the τ versus $(\sigma - u_a)$ plane

Form of a two parameter equation



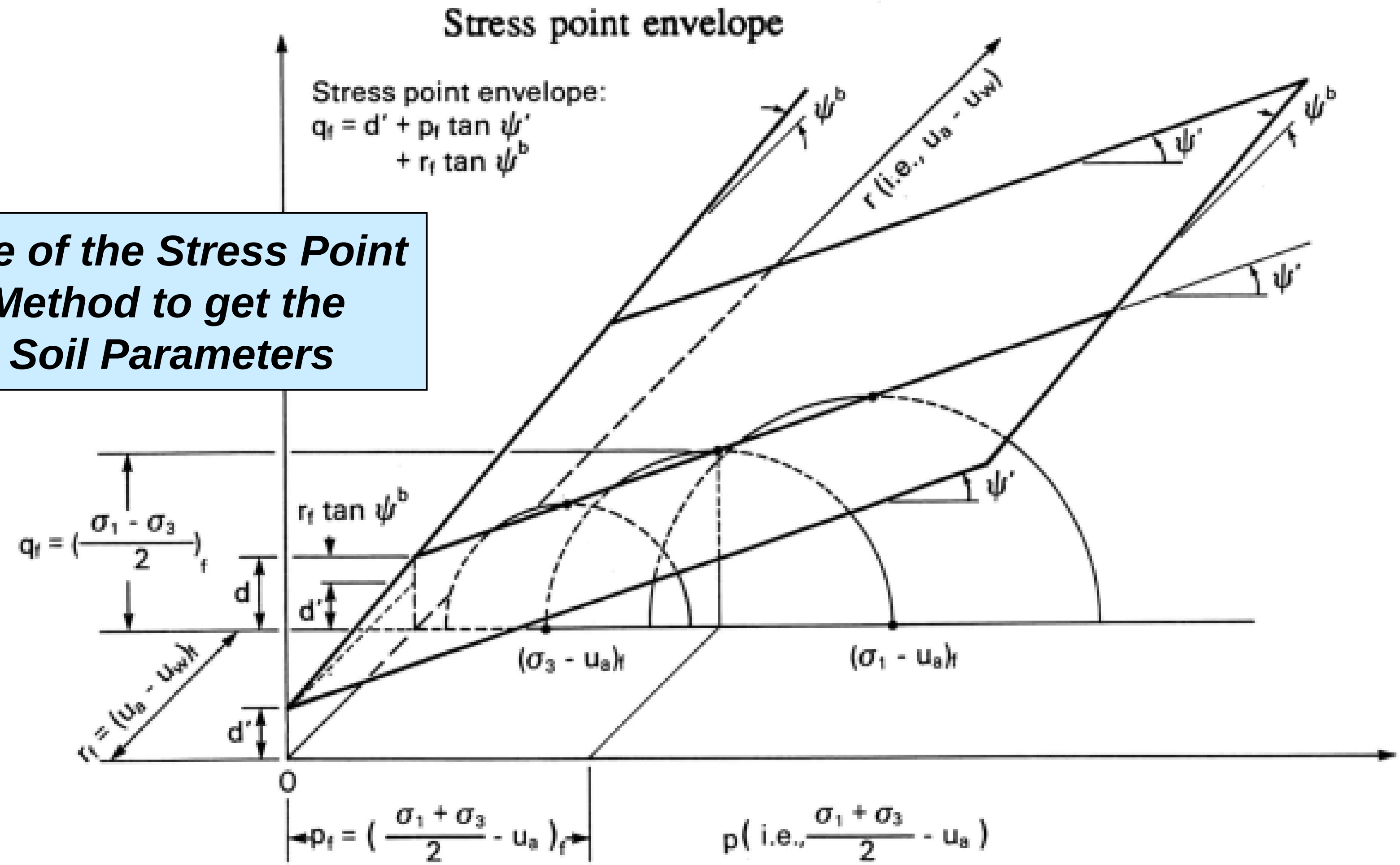
An increase in matric suction translates into an increase in cohesion

Horizontal projection of the failure envelope onto the τ versus $(\sigma - u_a)$ plane, viewing parallel to the $(u_a - u_w)$ axis (continued)



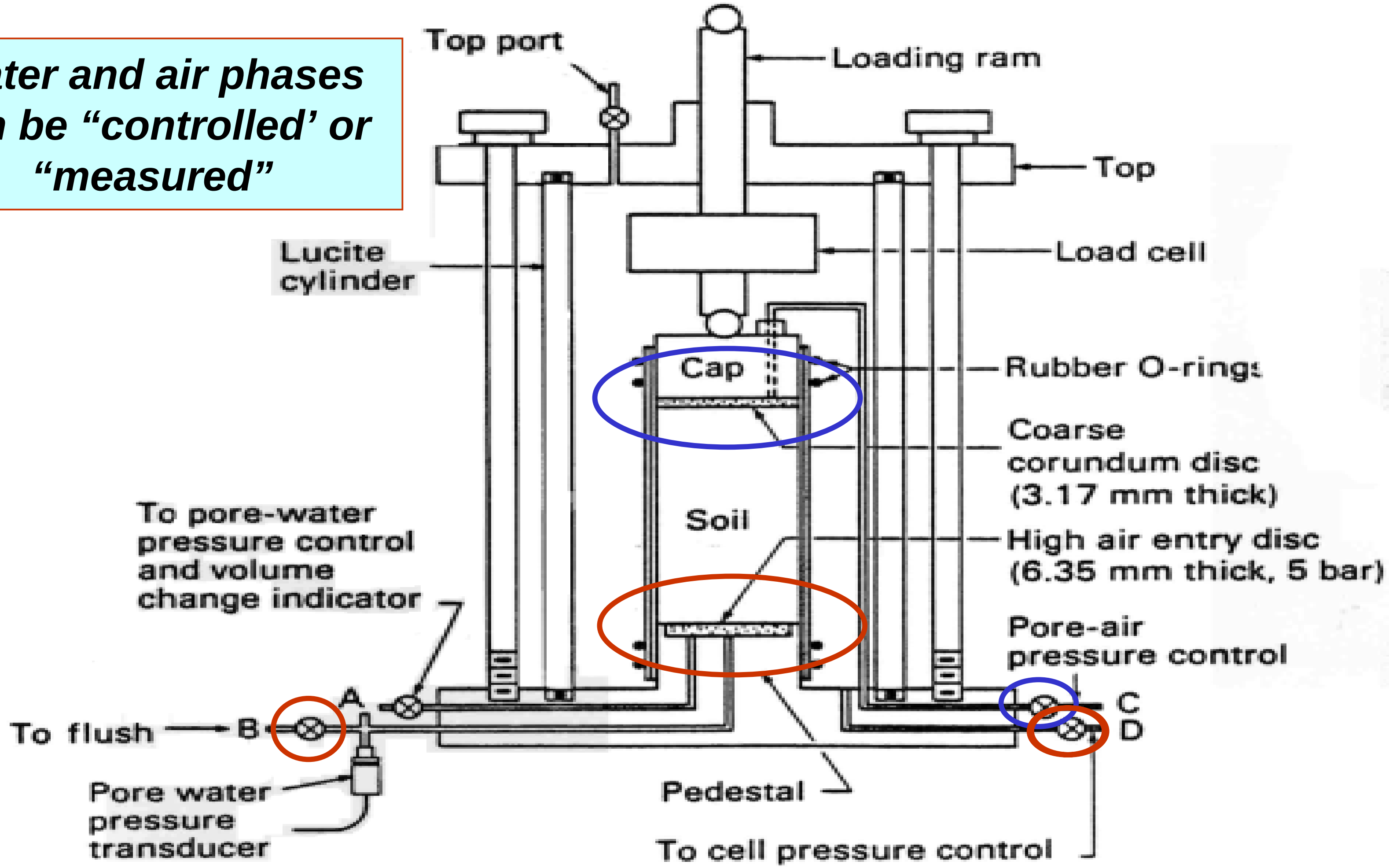
Extended Mohr-Coulomb type failure surface tangent to the Mohr circles at failure

Use of the Stress Point Method to get the Soil Parameters



Comparisons of the failure envelope and the corresponding stress point envelope

Water and air phases can be "controlled' or "measured"



Primary modification is the high air entry disk sealed into the base pedestal

Modified triaxial cell for testing unsaturated soils

Unsaturated Soil Technology

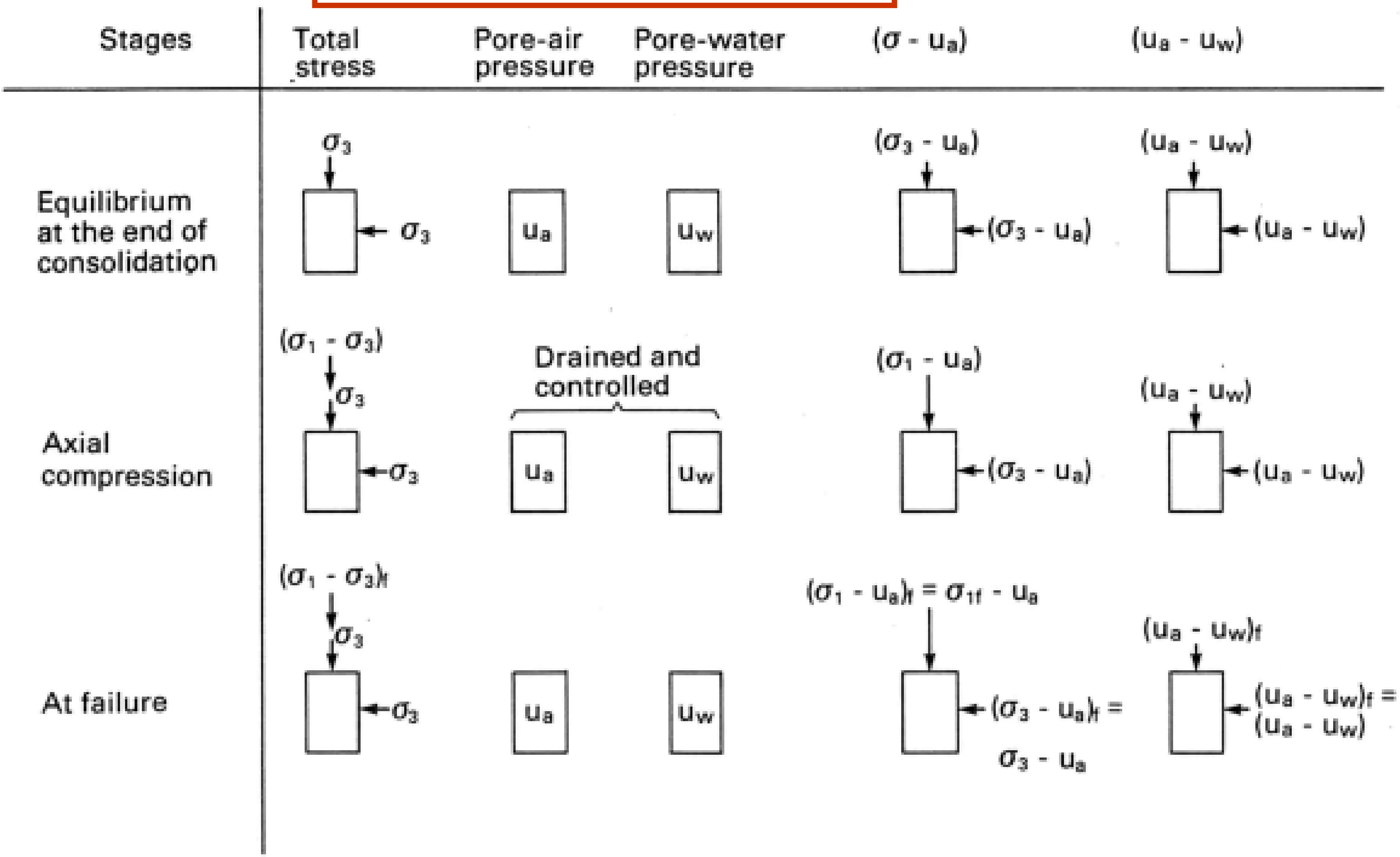
Various Triaxial Tests for Unsaturated Soils

Test Methods	Consolidation Prior to Shearing Process	Drainage		Shearing Process		
		Pore-Air	Pore-Water	Pore-Air Pressure, u_a	Pore-Water Pressure, u_w	Soil Volume Change, ΔV
Consolidated Drained (CD)	yes	yes	yes	*C*	*C*	*M*
Constant water content (CW)	yes	yes	no	*C*	*M*	*M*
Consolidated undrained (CU)	yes	no	no	*M*	*M*	—
Undrained	no	no	no	—	—	—
Unconfined compression (UC)	no	no	no	—	—	—

M = Measurement, *C* = controlled.

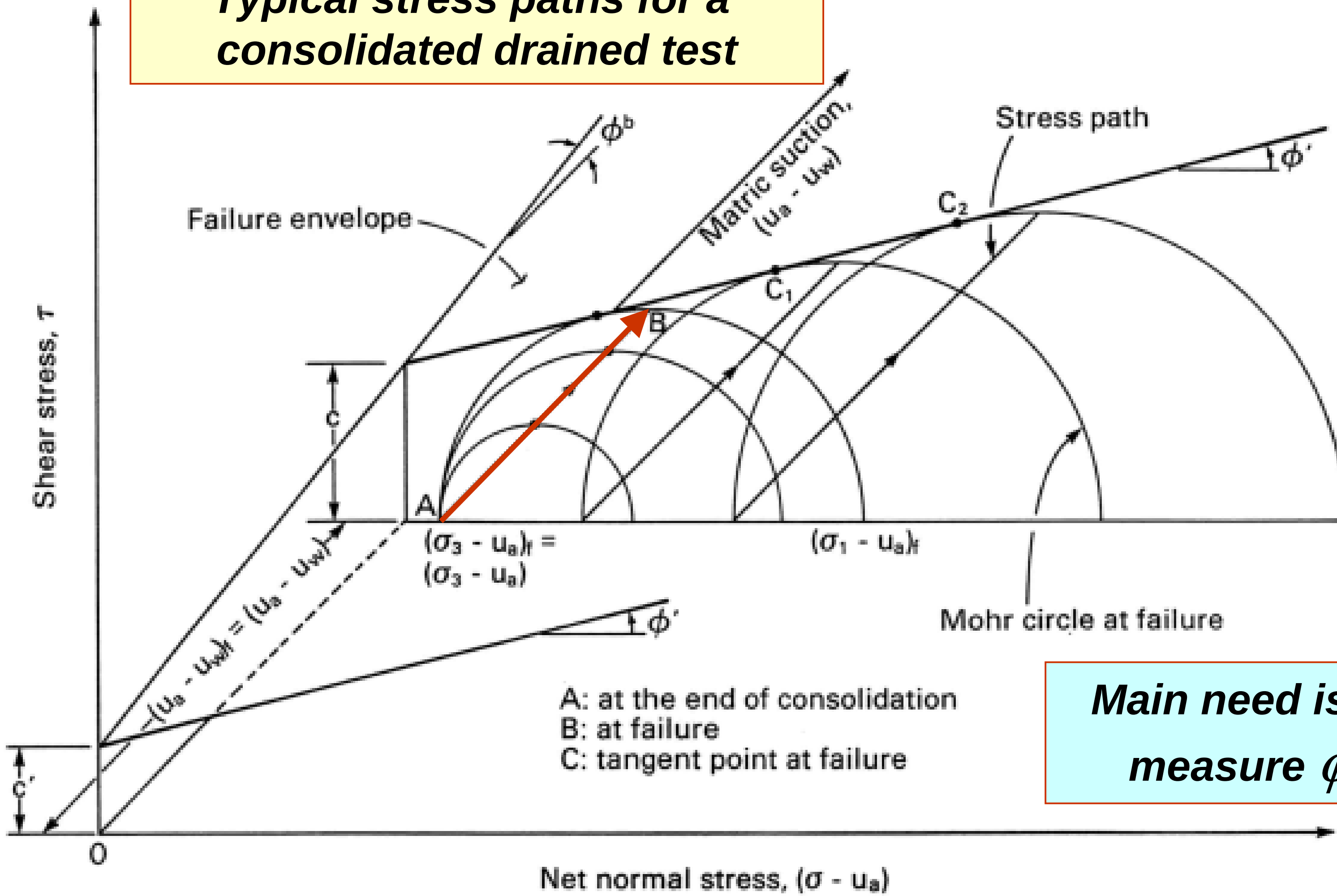
Unsaturated Soil Technology

Consolidated Drained Test



Stress conditions during a consolidated drained triaxial compression test

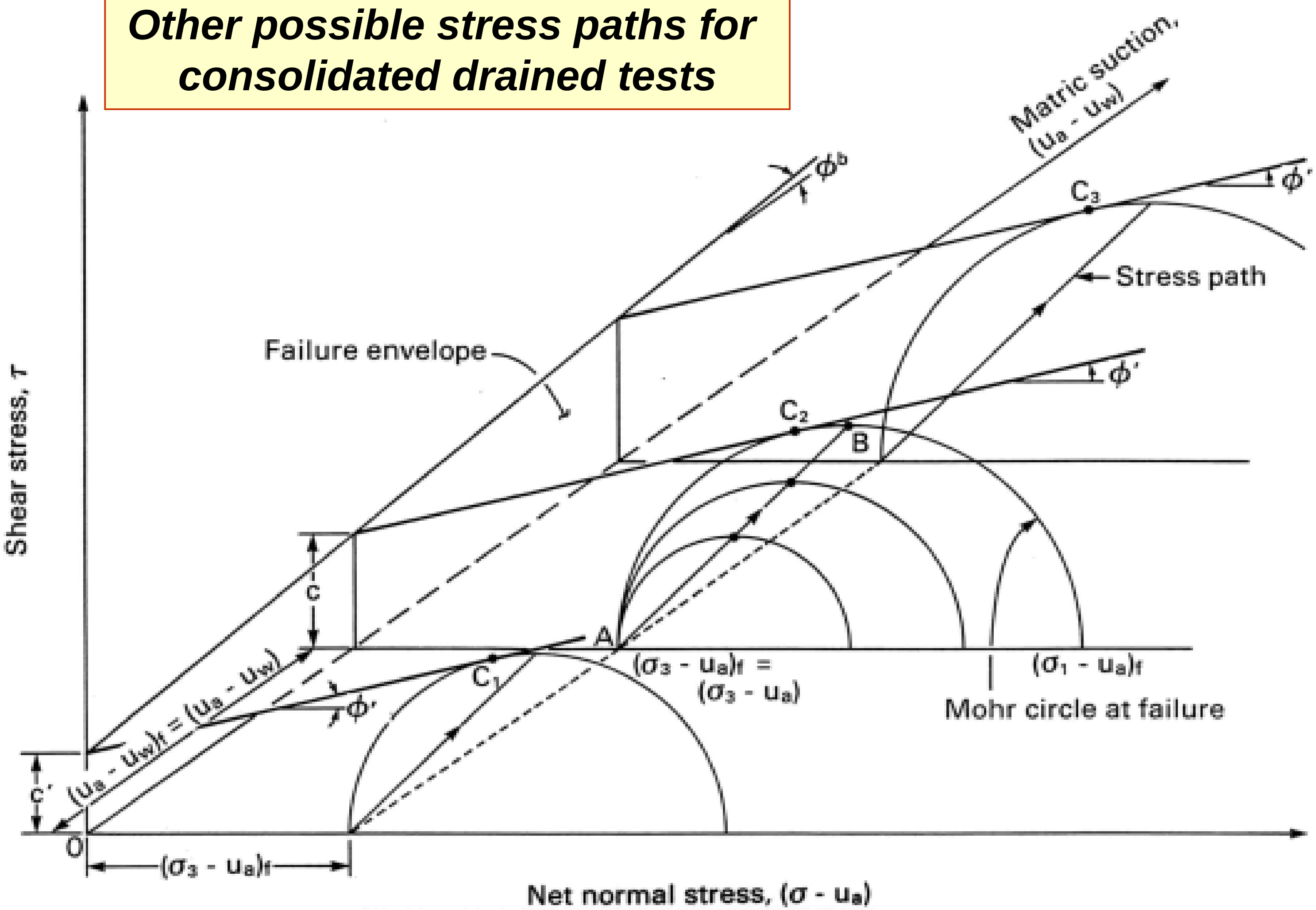
Typical stress paths for a consolidated drained test



Main need is to measure ϕ^b

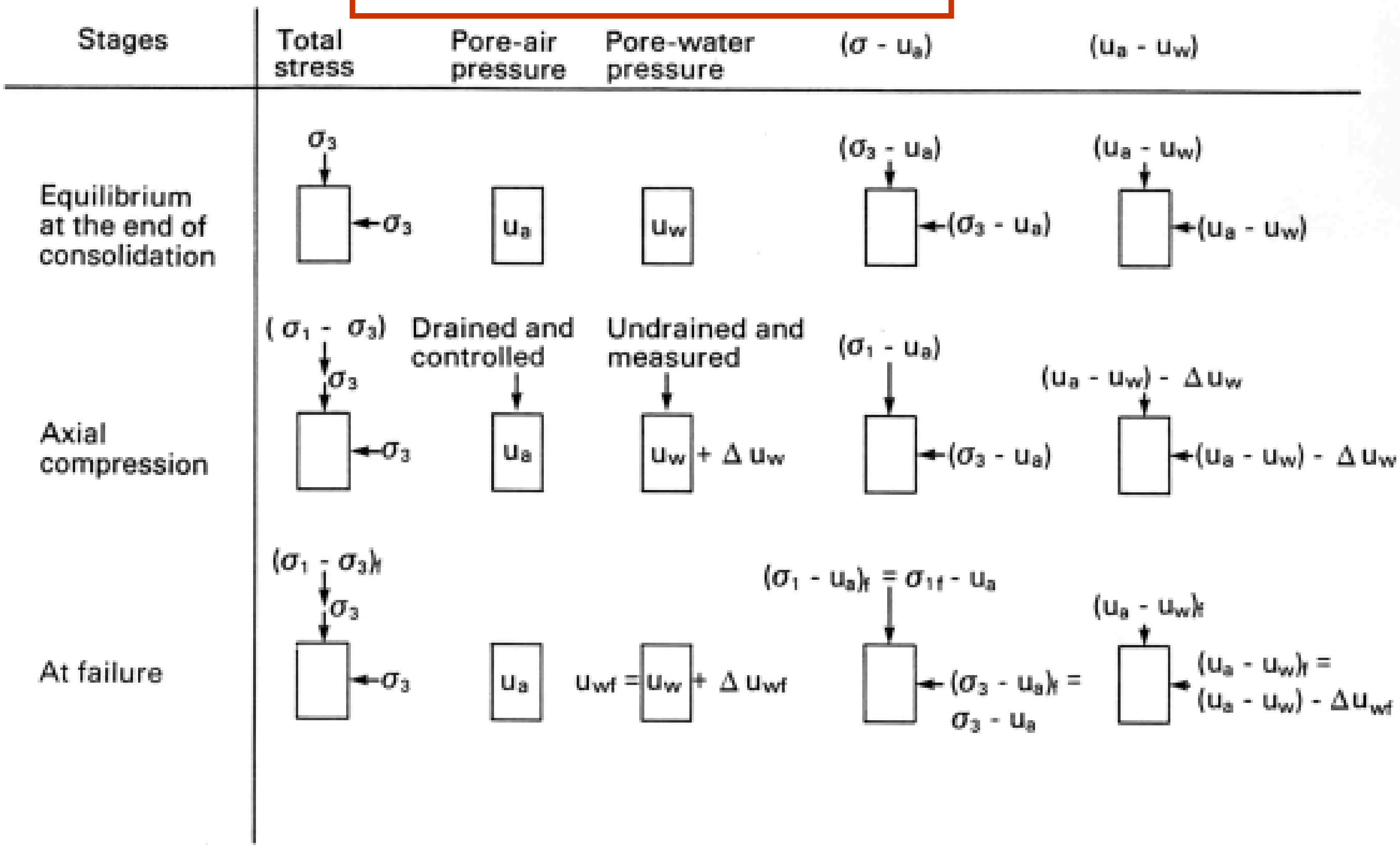
Stress paths followed during a consolidated drained test at various net confining pressures under a constant matric suction

Other possible stress paths for consolidated drained tests



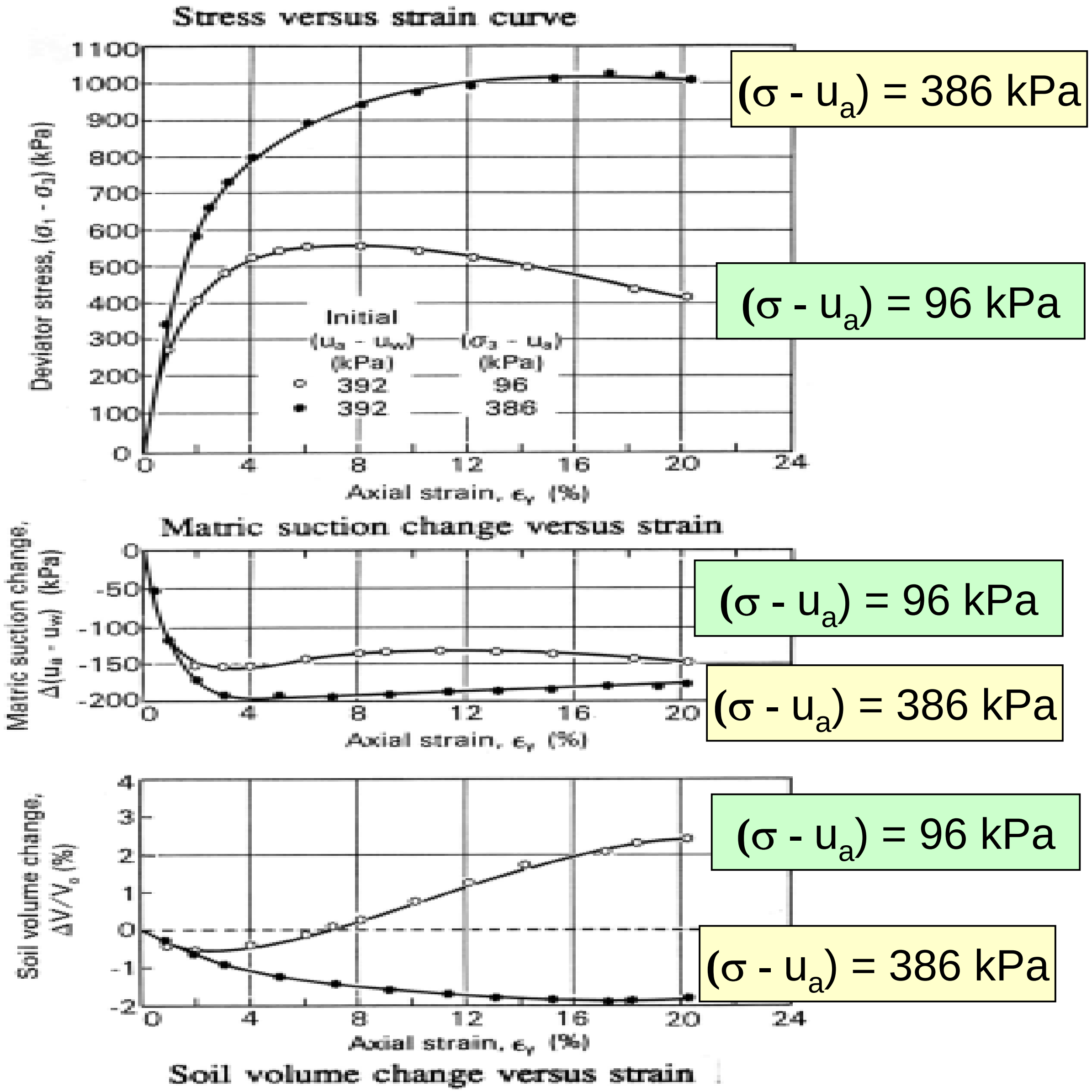
Stress paths followed during consolidated drained tests at various matric suctions under a constant net confining pressure

Constant Water Content Test



Stress conditions during a constant water content triaxial compression test

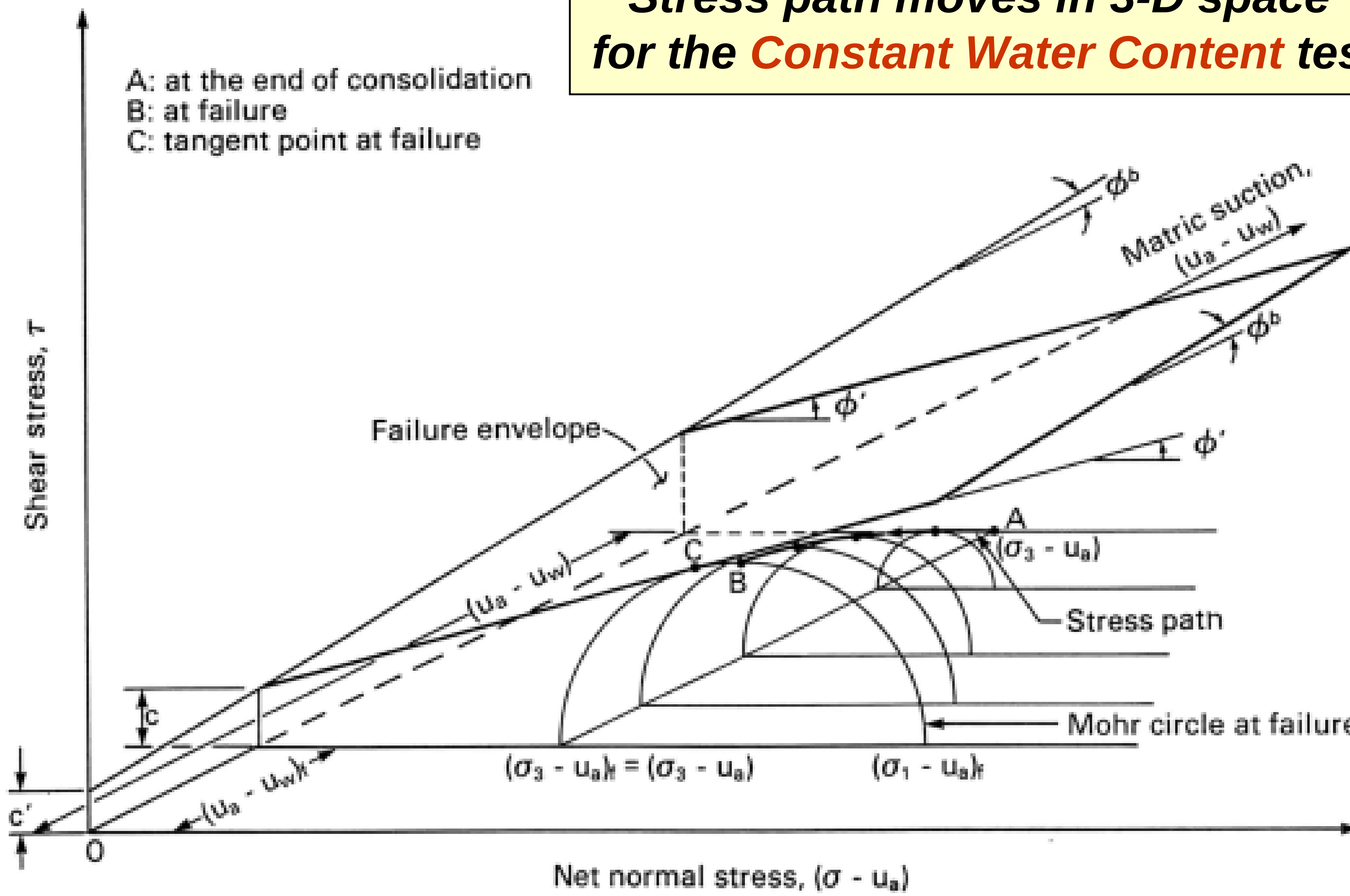
Constant water content test



Constant water content triaxial tests on Dhanauri clay (from Satija, 1978)

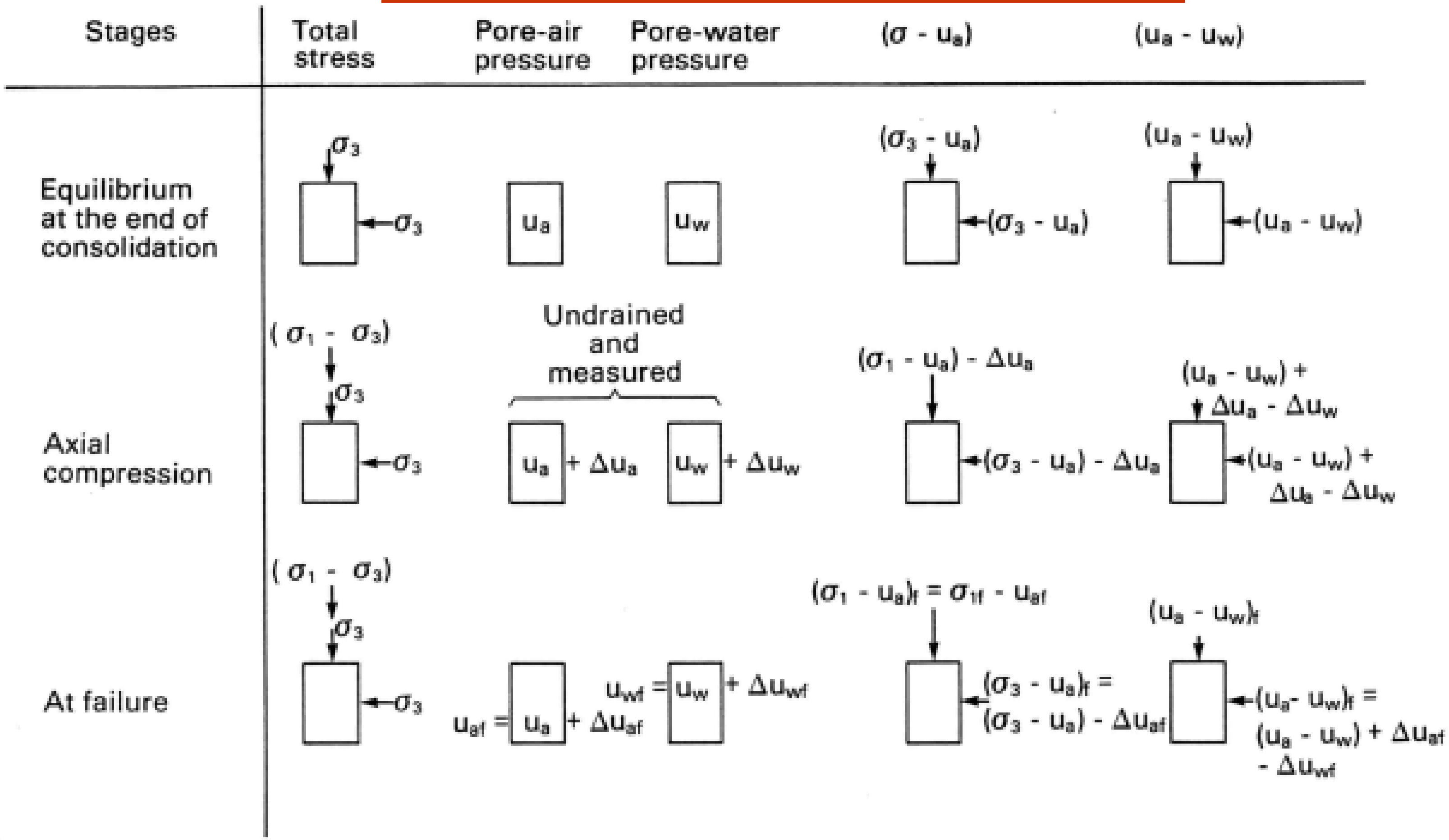
Matric suction decreased during test

Stress path moves in 3-D space for the Constant Water Content test



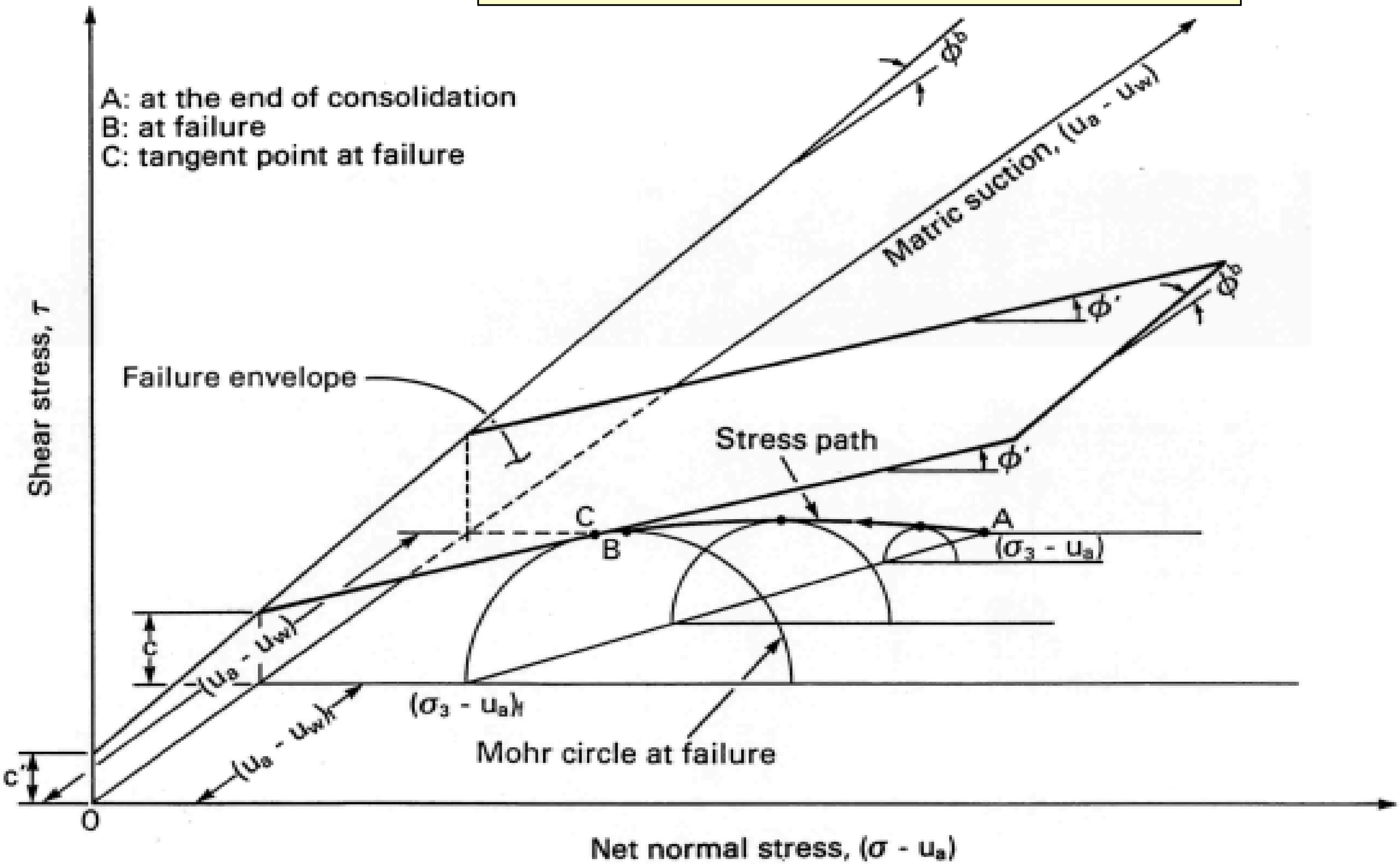
Stress path followed during a constant water content test

Consolidated Undrained Test with Pore Pressure Measurements



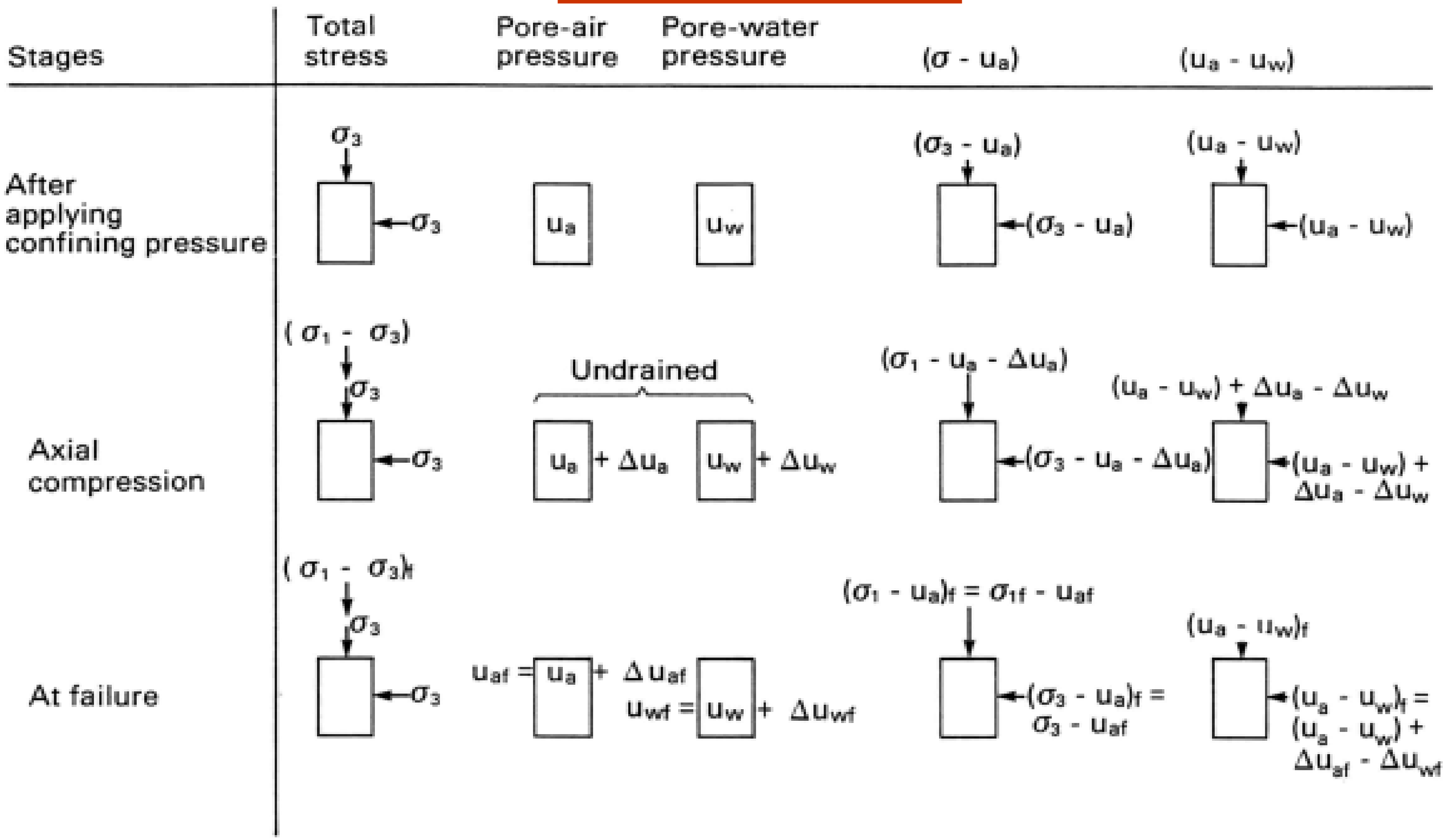
Stress conditions during a consolidated undrained triaxial compression test with pore pressure measurements

Stress path moves in 3-D space for the Consolidated Undrained test



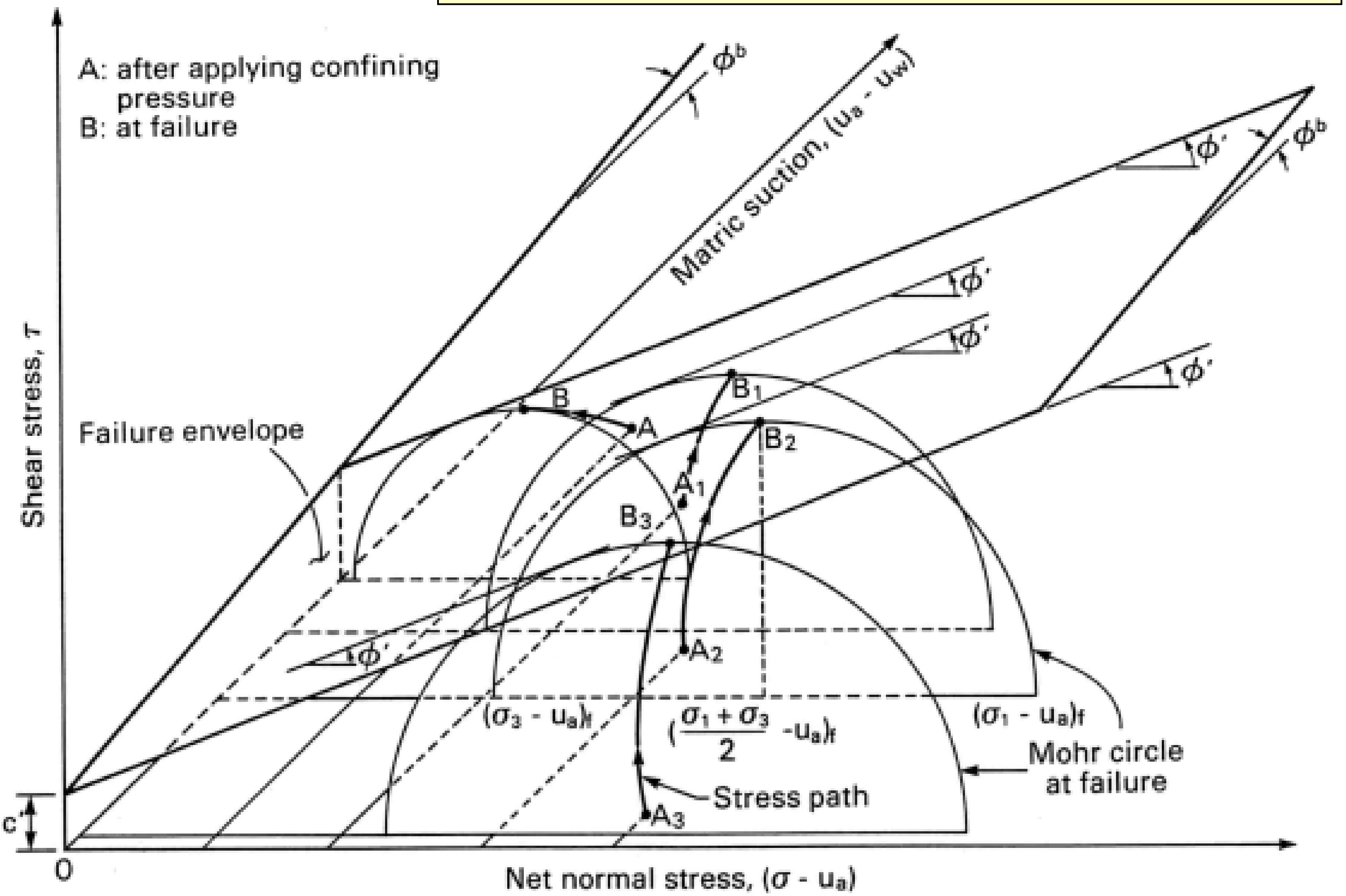
Typical stress path followed during a consolidated undrained test

Undrained Test



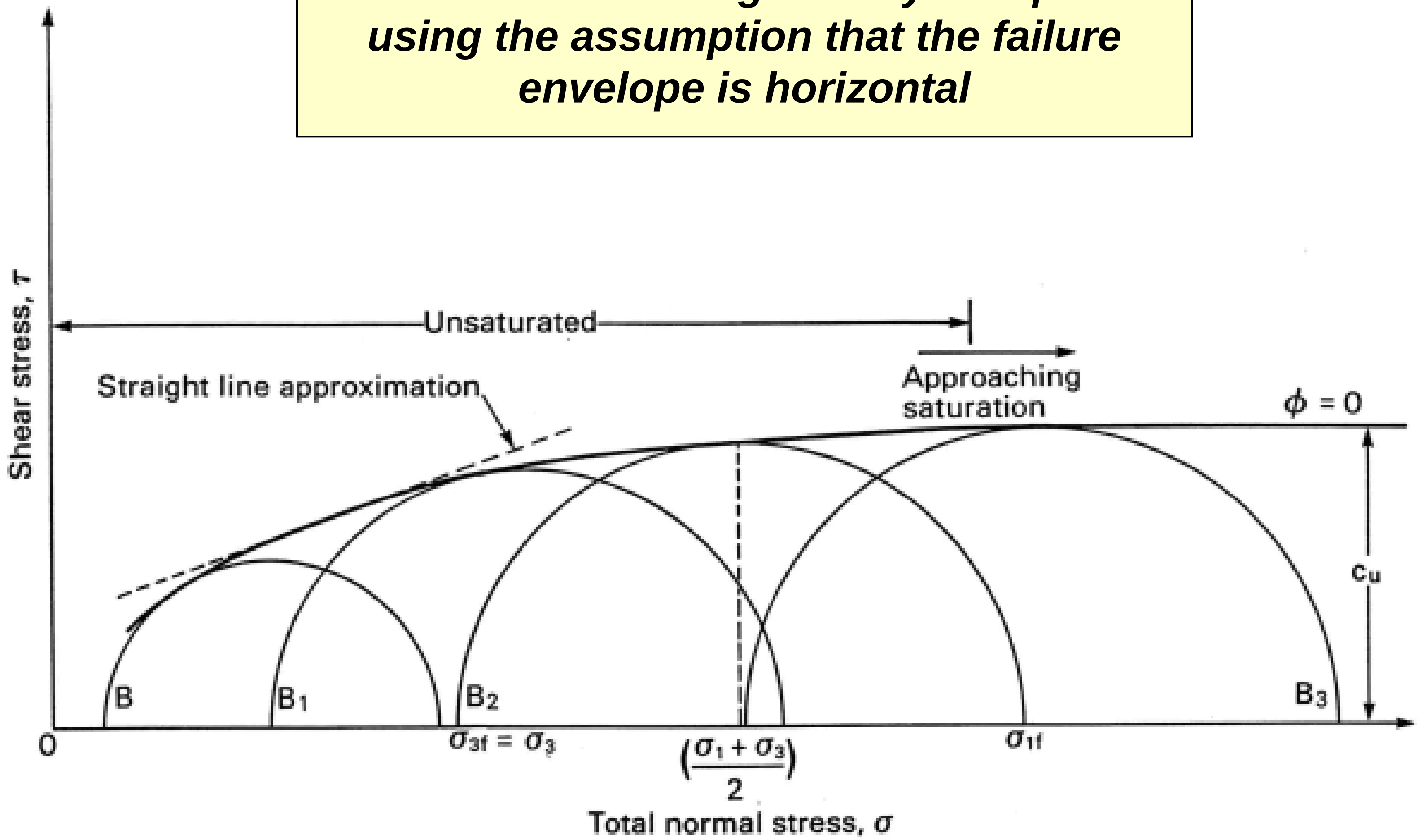
Stress conditions during an undrained triaxial compression test

Undrained test path moves in 3-D space



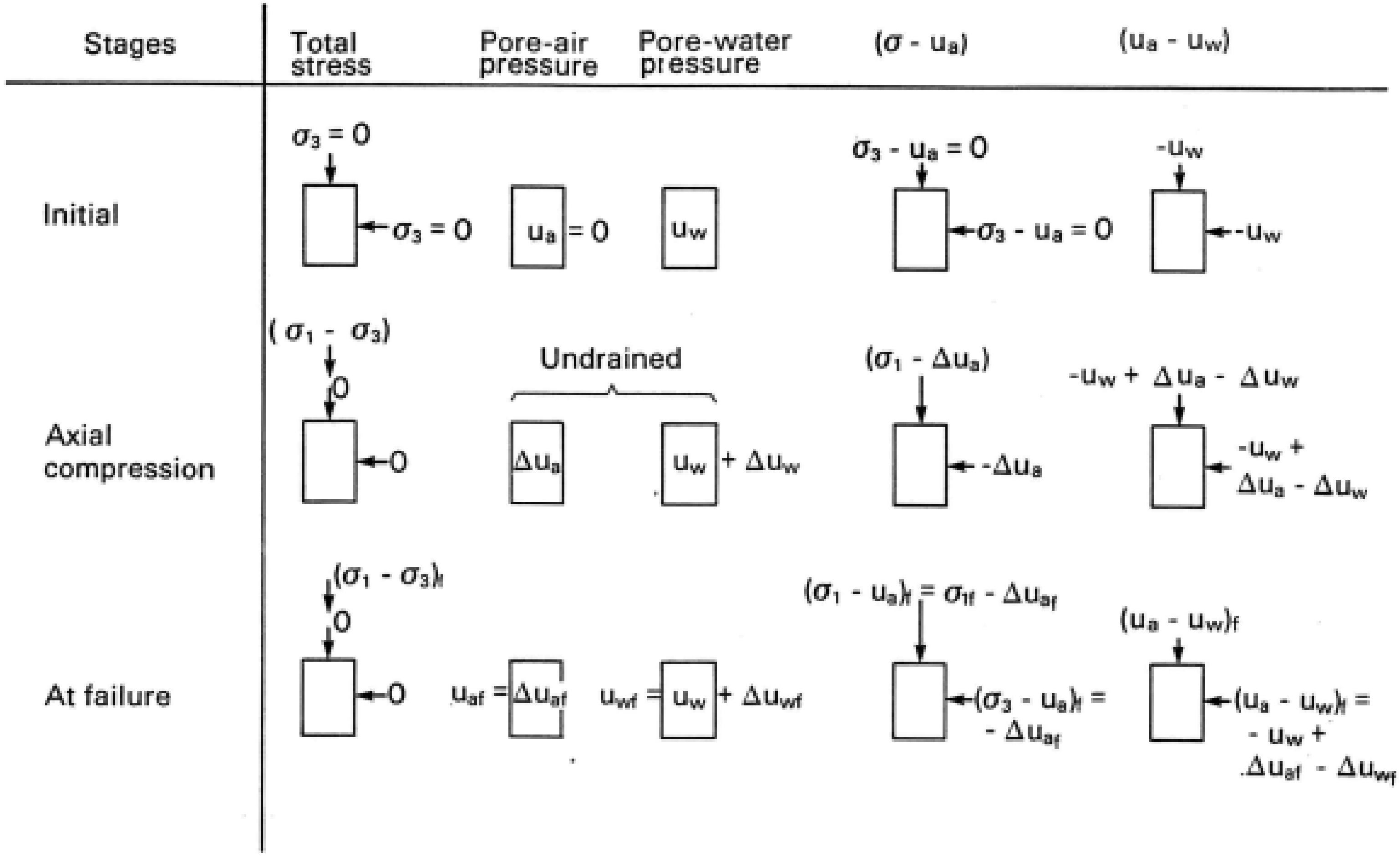
Stress paths followed during an undrained test

Undrained tests ***are generally interpreted using the assumption that the failure envelope is horizontal***



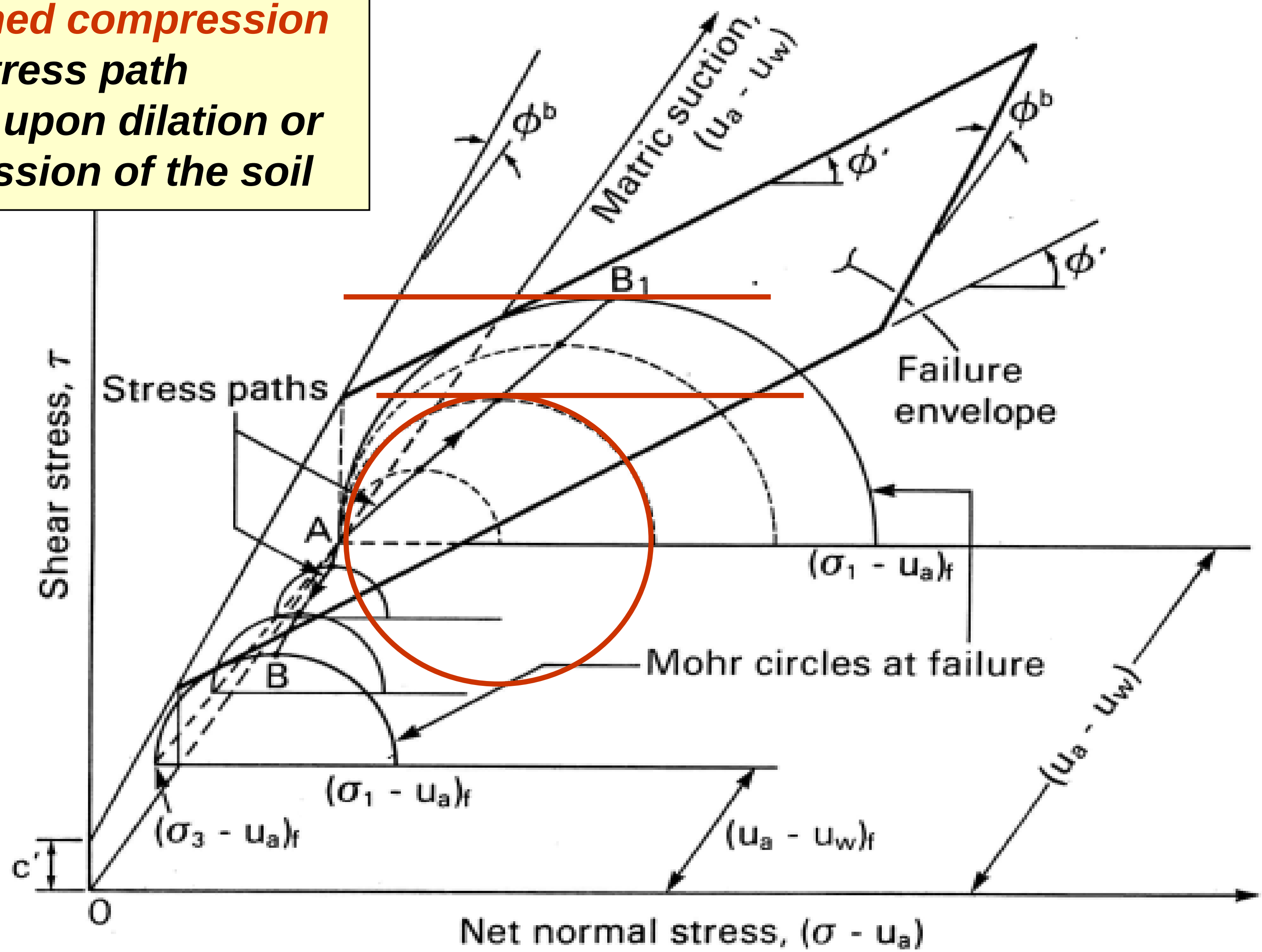
Shear stress versus total normal stress relationship for the undrained test

Unconfined Compression Test



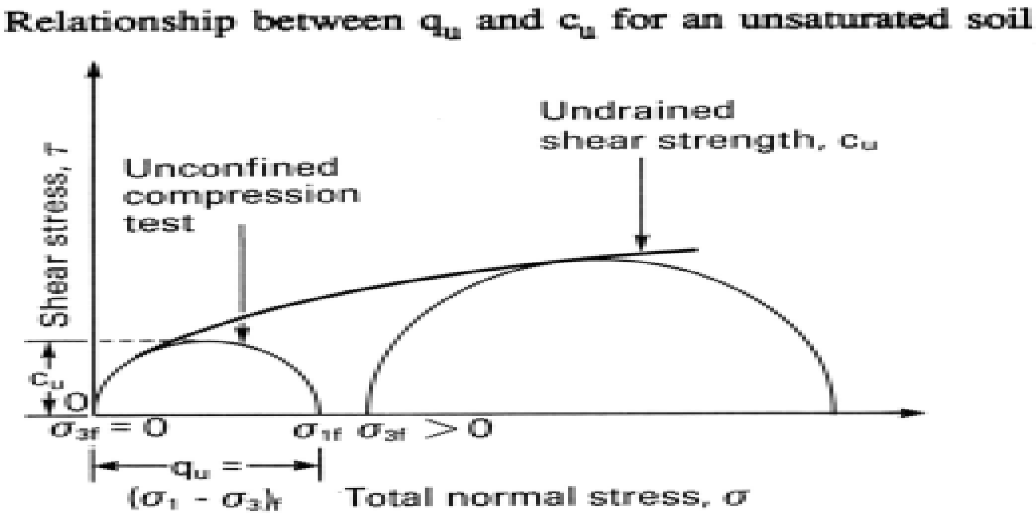
Stress conditions during an unconfined compression test

***Unconfined compression* stress path depends upon dilation or compression of the soil**

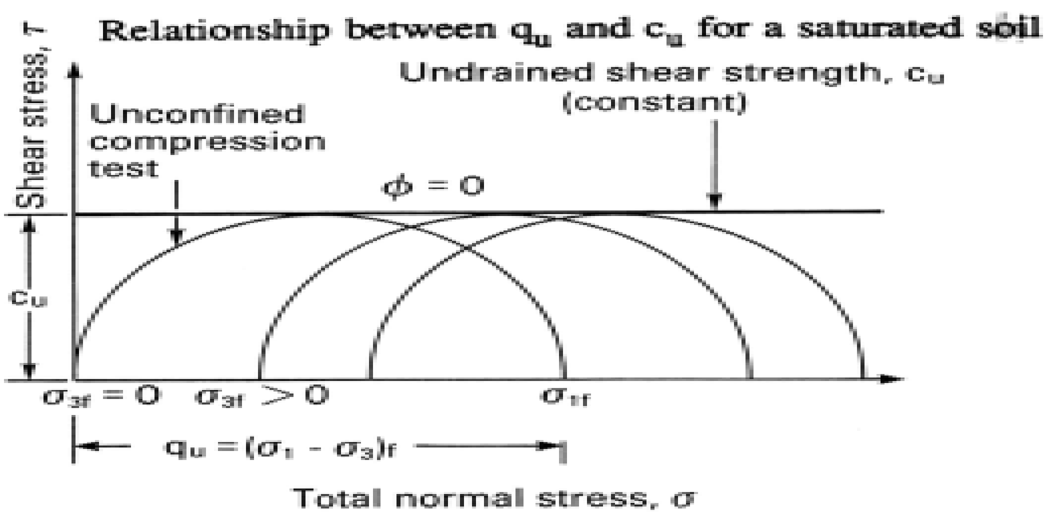


Possible stress paths followed during an unconfined compression test

Unsaturated Soil



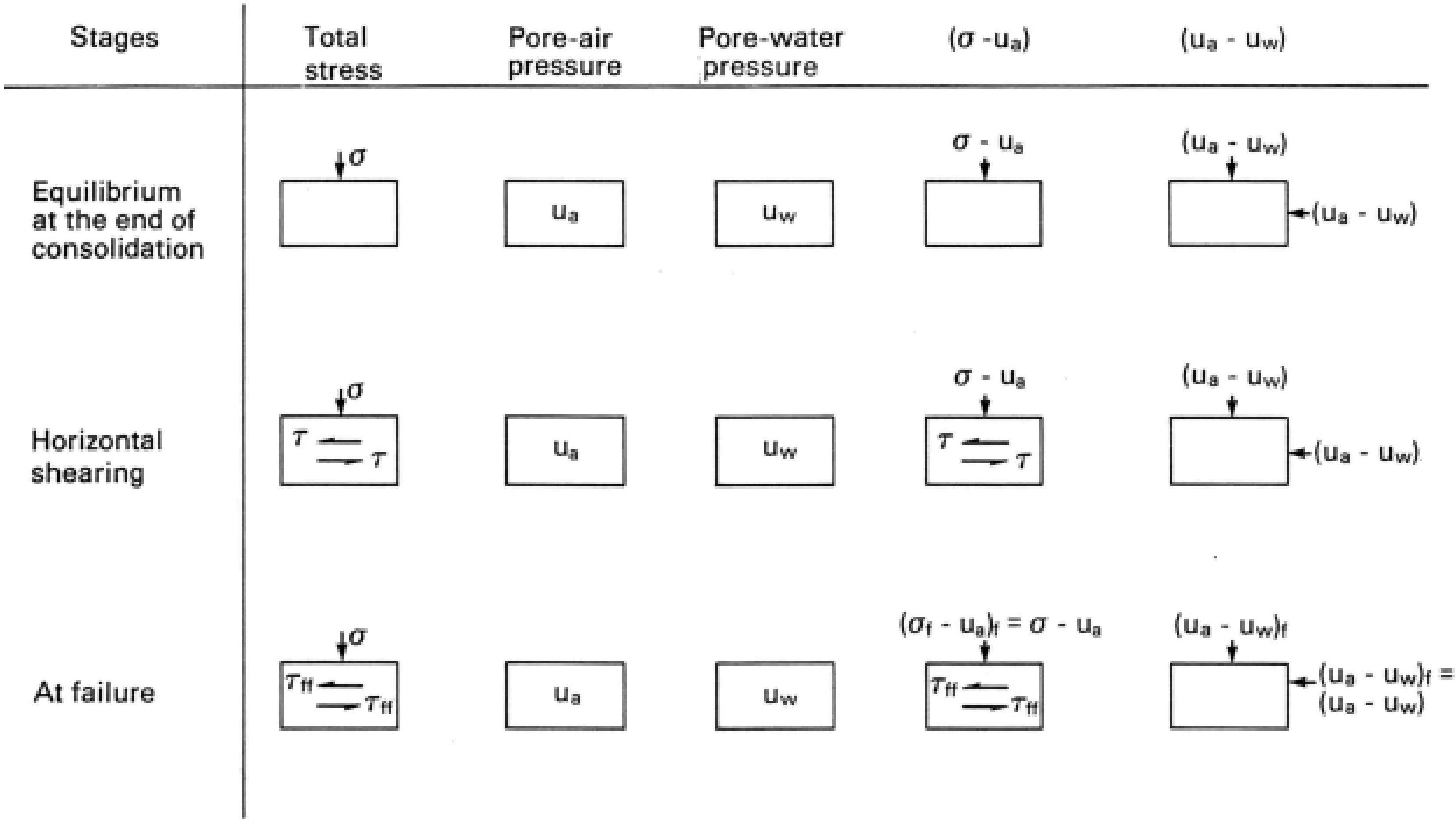
Saturated Soil



Use of the unconfined compressive strength, q_u, to approximate the undrained shear strength, c_u, for an unsaturated and a saturated soil

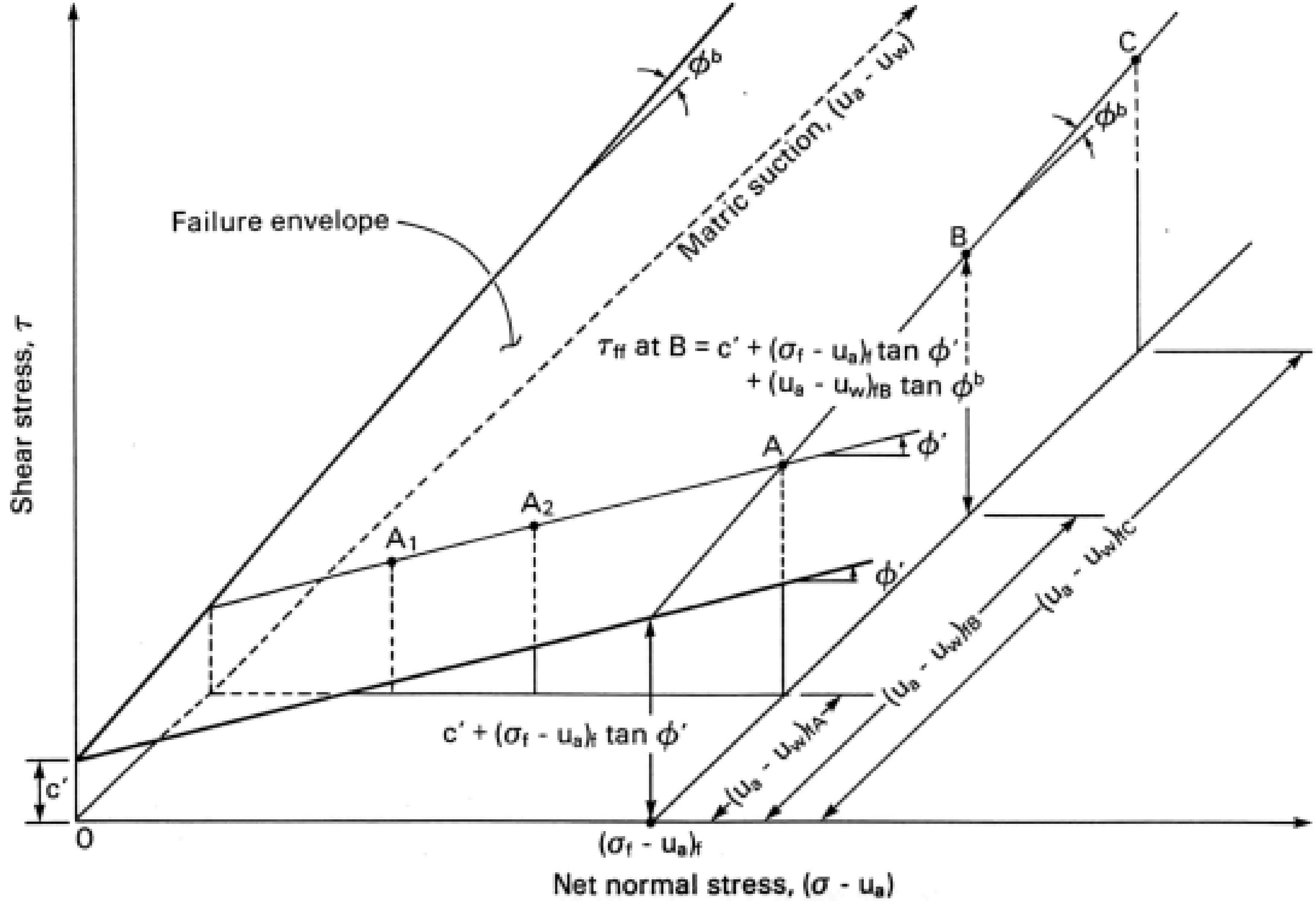
UST Unsaturated Soil Technology

DIRECT SHEAR TESTS



Stress conditions during a consolidated drained direct shear test

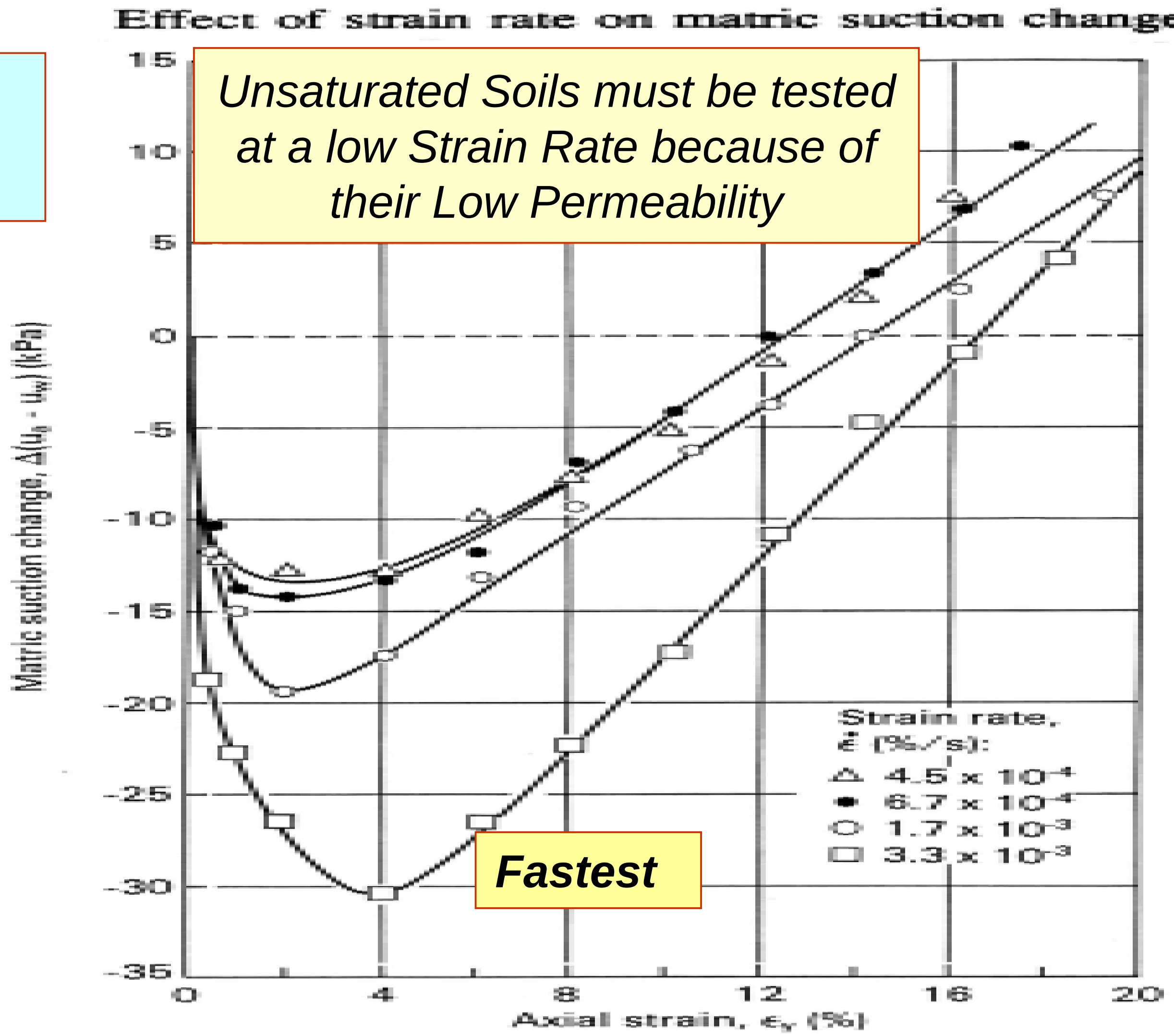
Interpretation generally used for Direct Shear tests



Extended Mohr-Coulomb failure envelope established from direct shear test results

Strain Rates

Unsaturated Soils must be tested at a low Strain Rate because of their Low Permeability



Strain rate effects for constant water content tests on Dhanauri clay (from Satija, 1978) (continued)

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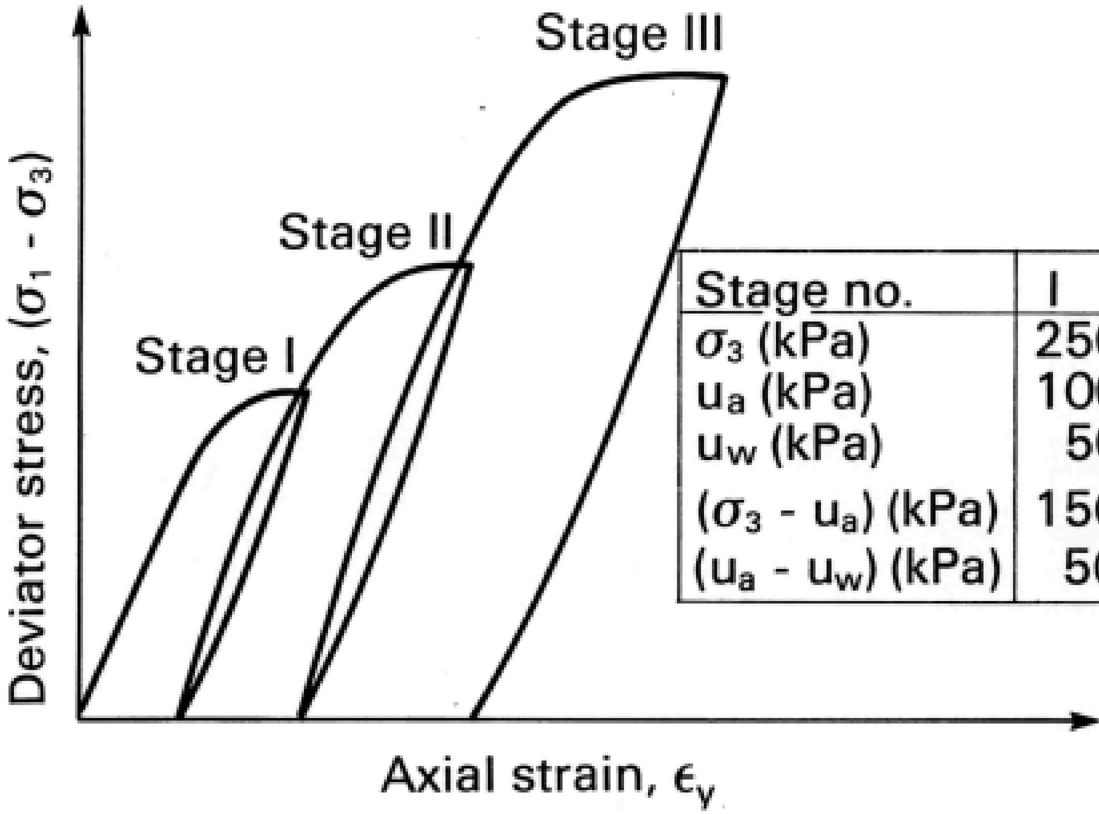
Unsaturated Soil Technology

Strain Rate and Strain at Failure for Triaxial Tests on Unsaturated Soils

Soil Type	Triaxial Test	Strain Rate, $\dot{\epsilon}$ (%/s)	Approximate Strain at Failure, ϵ_f (%)	References
Boulder clay; w = 11.6% and % clay = 18%	CW	3.5×10^{-5}	15	Bishop *et al.* 1960
Braehead silt	CW	4.7×10^{-5}	11	Bishop and Donald (1961)
	CD	8.3×10^{-6}	12	
Talybont boulder clay; w = 9.75% and % clay = 6%	Undrained with pore pressure measurements	4.7×10^{-7}	σ_3 = 83 kPa : 8.5 σ_3 = 207 kPa : 11	Donald (1963)
Dhanauri clay; w = 22.2% and % clay = 25%	CW	6.7×10^{-4}	20	Satija and Gulhati (1979)
	CD	1.3×10^{-4}	20	
Undisturbed decomposed granite and rhyolite	CD	1.7×10^{-5}	Stage I: 3–5	Ho and Fredlund (1982a)
	Multistage	6.7×10^{-5}	Stage II: 1–3 Stage III: 1–3	
Clayey sand; w = 14–17% and % clay = 30%	Undrained and unconfined	1.7×10^{-3}	15–20	Chantawarangul (1983)

MULTISTAGE TESTING

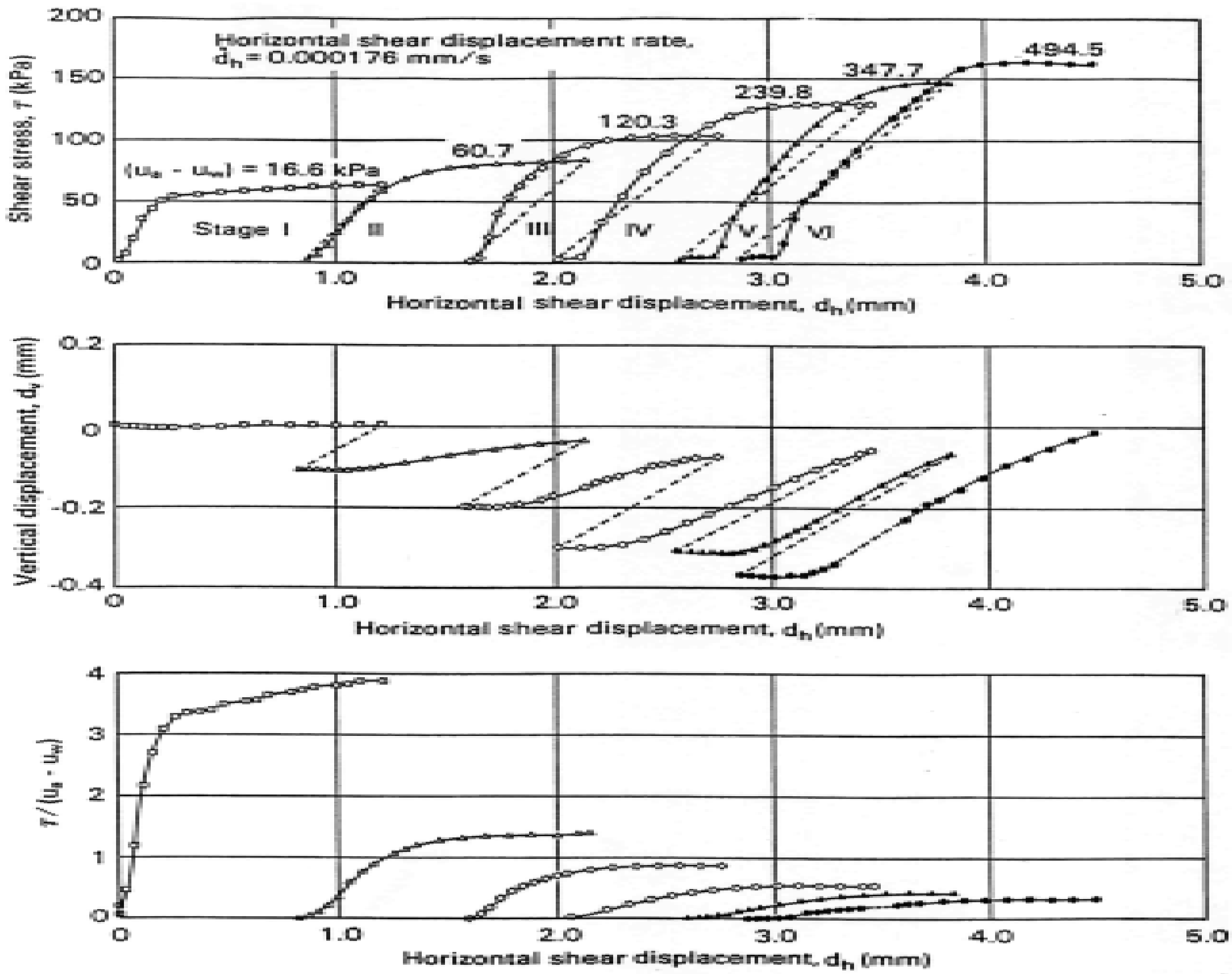
Cyclic: *Loading and Unloading procedure for multistage tests*



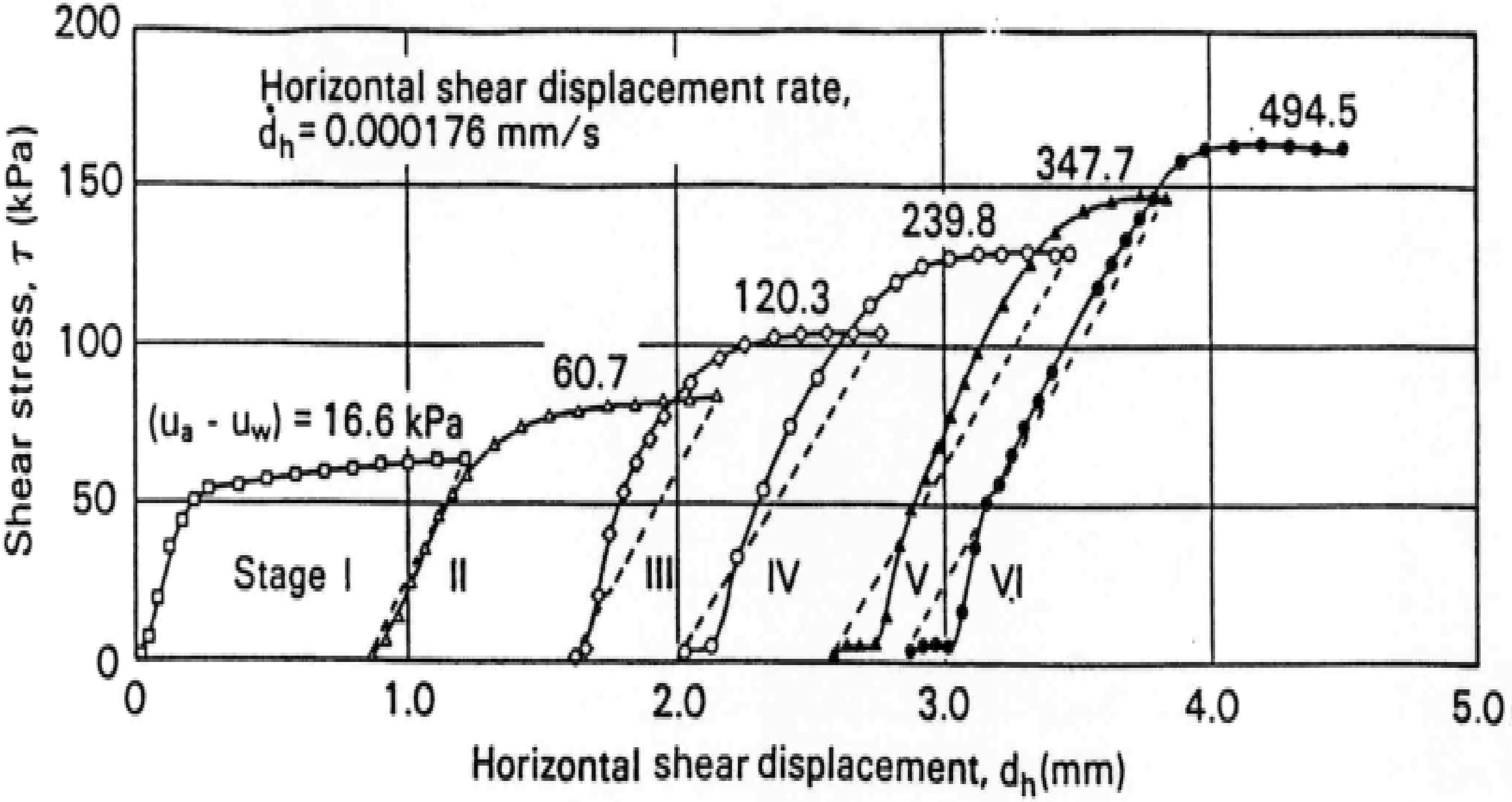
Stage no.	I	II	III
σ_3 (kPa)	250	350	500
u_a (kPa)	100	200	350
u_w (kPa)	50	50	50
$(\sigma_3 - u_a)$ (kPa)	150	150	150
$(u_a - u_w)$ (kPa)	50	150	300

Idealized stress-strain curves for multistage triaxial testing

Multi-Stage Direct Shear Results on Glacial Till



Multistage direct shear test on a compacted glac[illegible]
- sample GT-16-N4 (from Gan, 1986)



Multi-Stage Direct Shear Results on Glacial Till

FAILURE ENVELOPE FOR UNSATURATED SOILS

Extended Mohr-Coulomb Failure Envelope

$$\tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b$$

Proposed by Fredlund et al (1978)

Recall: Bishop's effective stress equation

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w)$$

Let: $\tau = c' + (\sigma') \tan \phi'$

Then substitute the effective stress into the shear strength equation.

$$\tau = c' + ((\sigma - u_a) + \chi (u_a - u_w)) \tan \phi'$$

Comparison of the ϕ^b and χ methods of designating shear strength

Bishop proposed a shear strength equation for unsaturated soils which had the following form:

$$\tau_{ff} = c' + \{(\sigma_f - u_a)_f + \chi (u_a - u_w)_f\} \tan\phi'$$

Bishop's shear strength equation

where:

χ = a parameter related to the degree of saturation of the soil

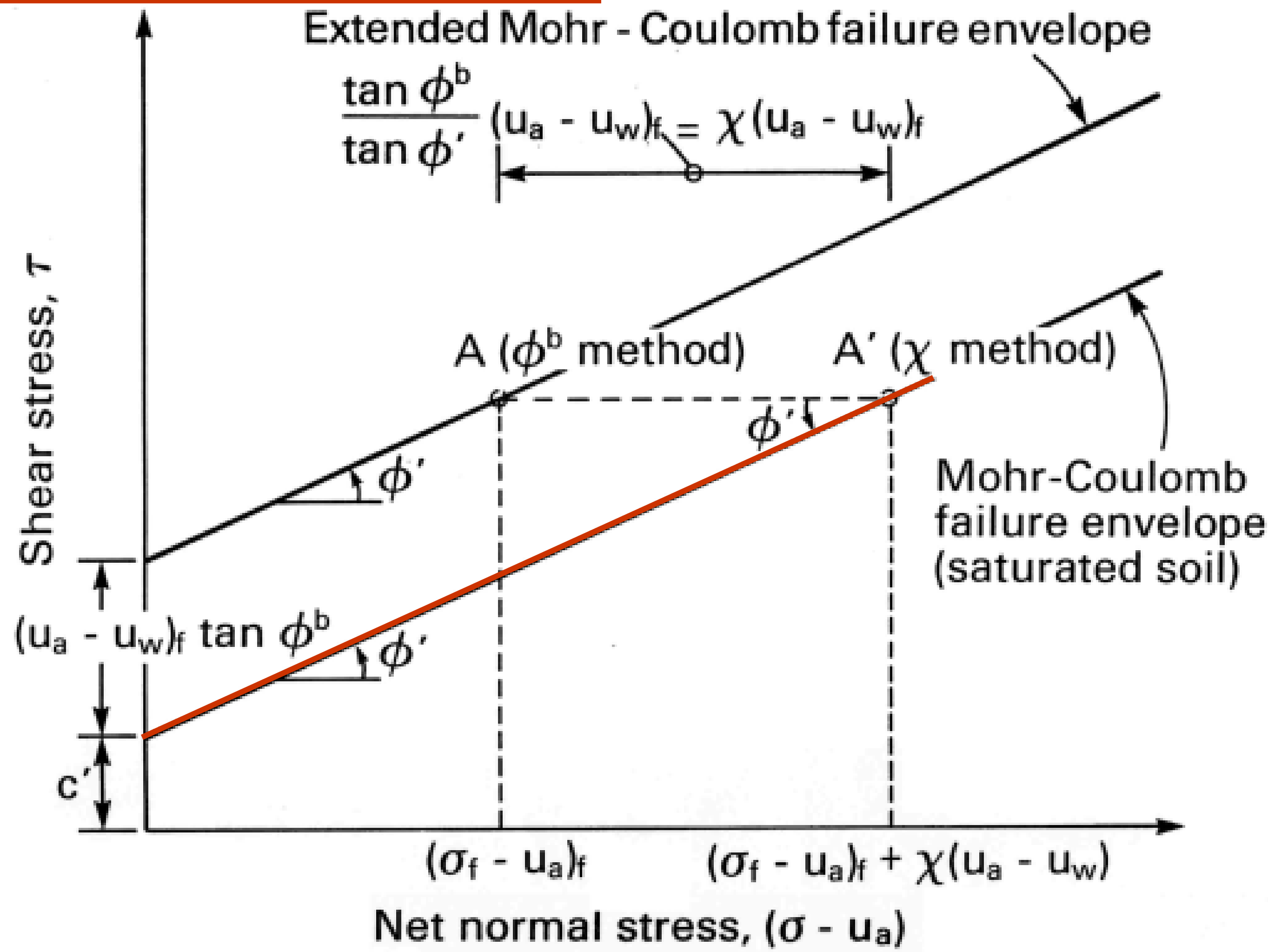
$$(u_a - u_w)_f \tan\phi^b = \chi (u_a - u_w)_f \tan\phi'$$

$$\chi = \frac{\tan\phi^b}{\tan\phi'}$$

Arises from a comparison of unsaturated shear strength behavior and saturated soil behavior

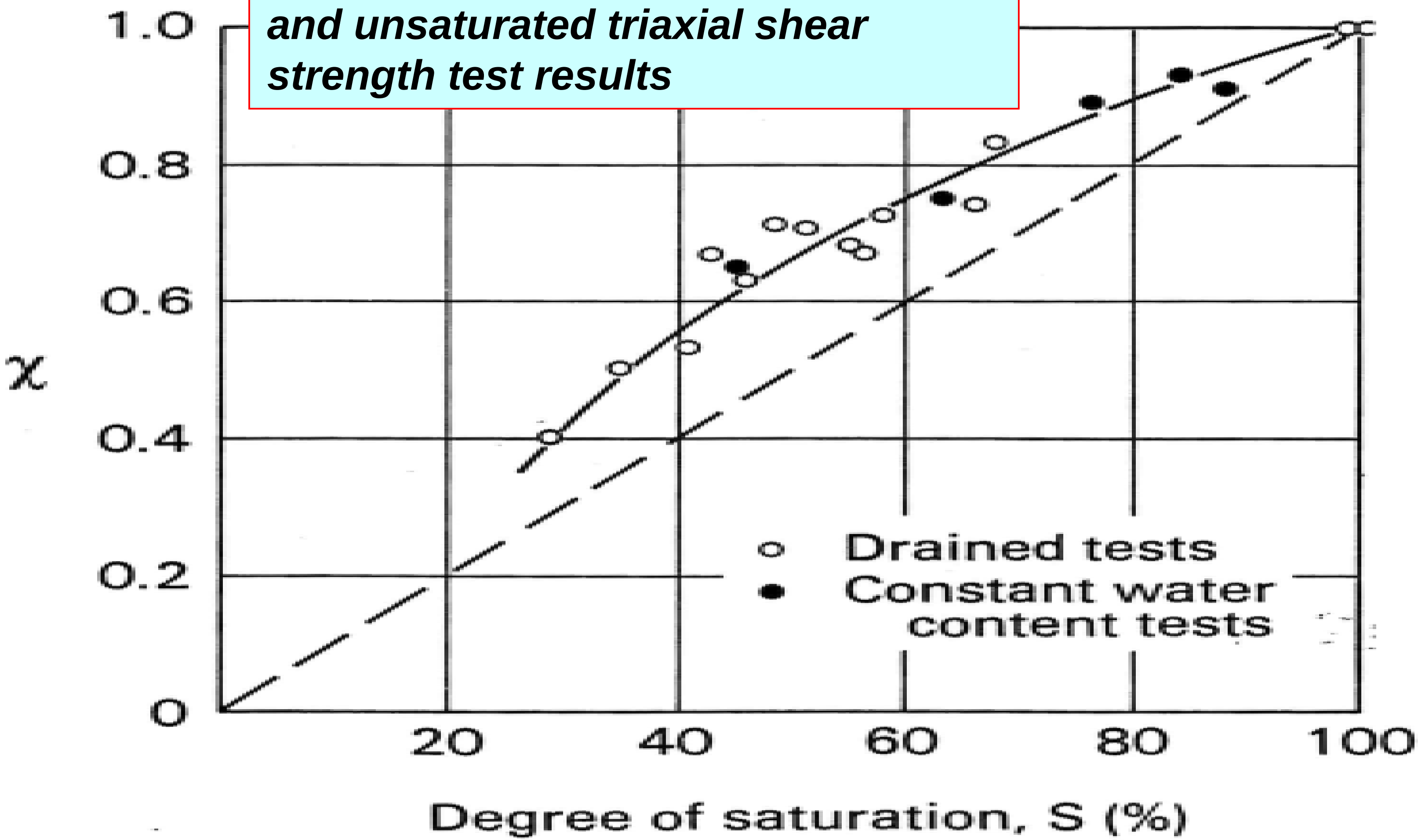
Must remember that χ is determined from the degree of saturation of the soil at the point of failure; therefore, requires another prediction for S%

RELATIONSHIPS BETWEEN ϕ^b and χ



Comparison of the ϕ^b and χ methods of designating shear strength

Results from comparing saturated and unsaturated triaxial shear strength test results

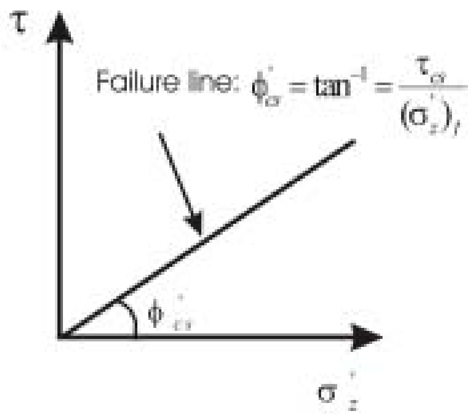


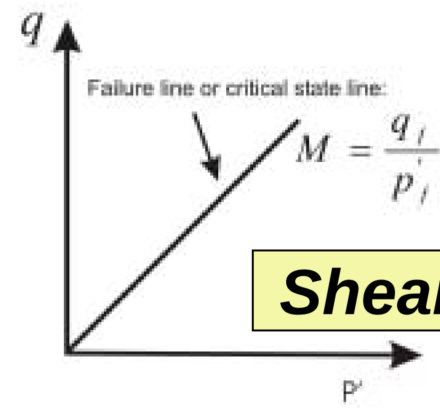
χ values for a cohesionless silt
(after Donald, 1961)

Description of Stress State for Saturated Soils

- ***Mohr-Coulomb Terminology***
  - ***Shear stress =*** $$\tau = \frac{(\sigma_1 - \sigma_3)}{2}$$
  - ***Principal Stresses;*** σ_1 *and* σ_3
  - ***Principal Effective Stresses,*** $$\sigma_1' = (\sigma_1 - u_w);$$ $$\sigma_3' = (\sigma_3 - u_w)$$
- ***Critical State (or Elasto-Plastic) Terminology***
  - ***Shear Stresses*** $q = \sigma_1 - \sigma_3$
  - ***Mean Stress =*** $$p' = [\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} - u_w]$$

Mapping of Strength and Consolidation Parameters

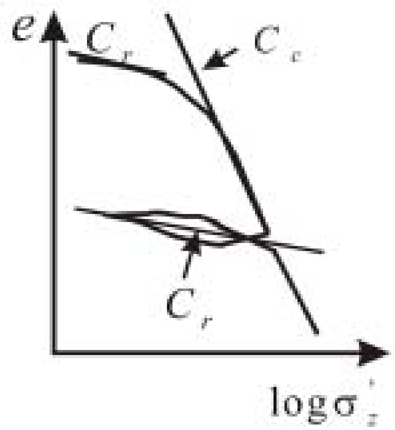


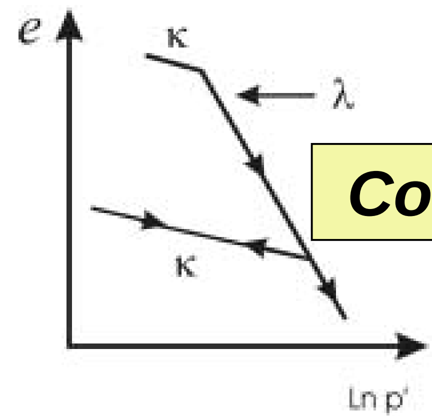


Shear Strength

Conventional Soil Mechanics

Elasto-Plastic Soil Mechanics

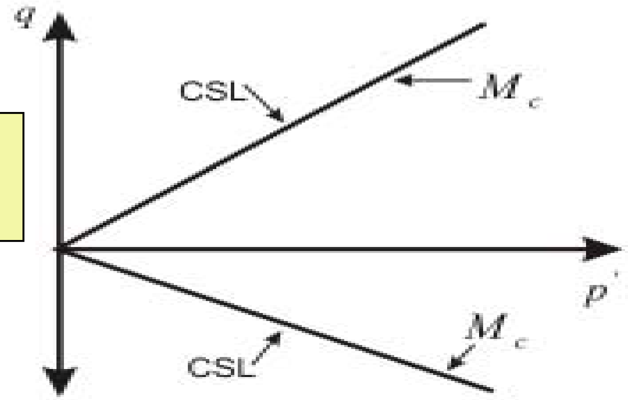


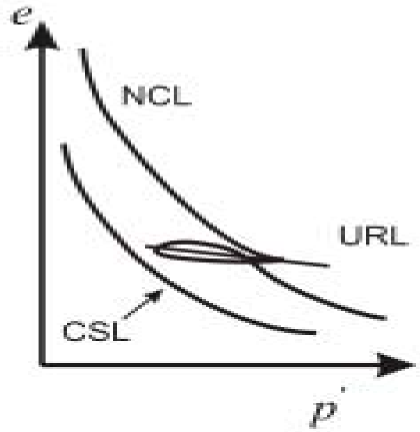


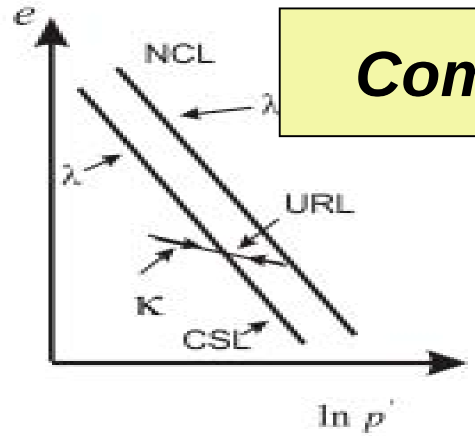
Compression

Critical State Lines, Normal Compression, and Unloading/Reloading Lines

Shear Strength

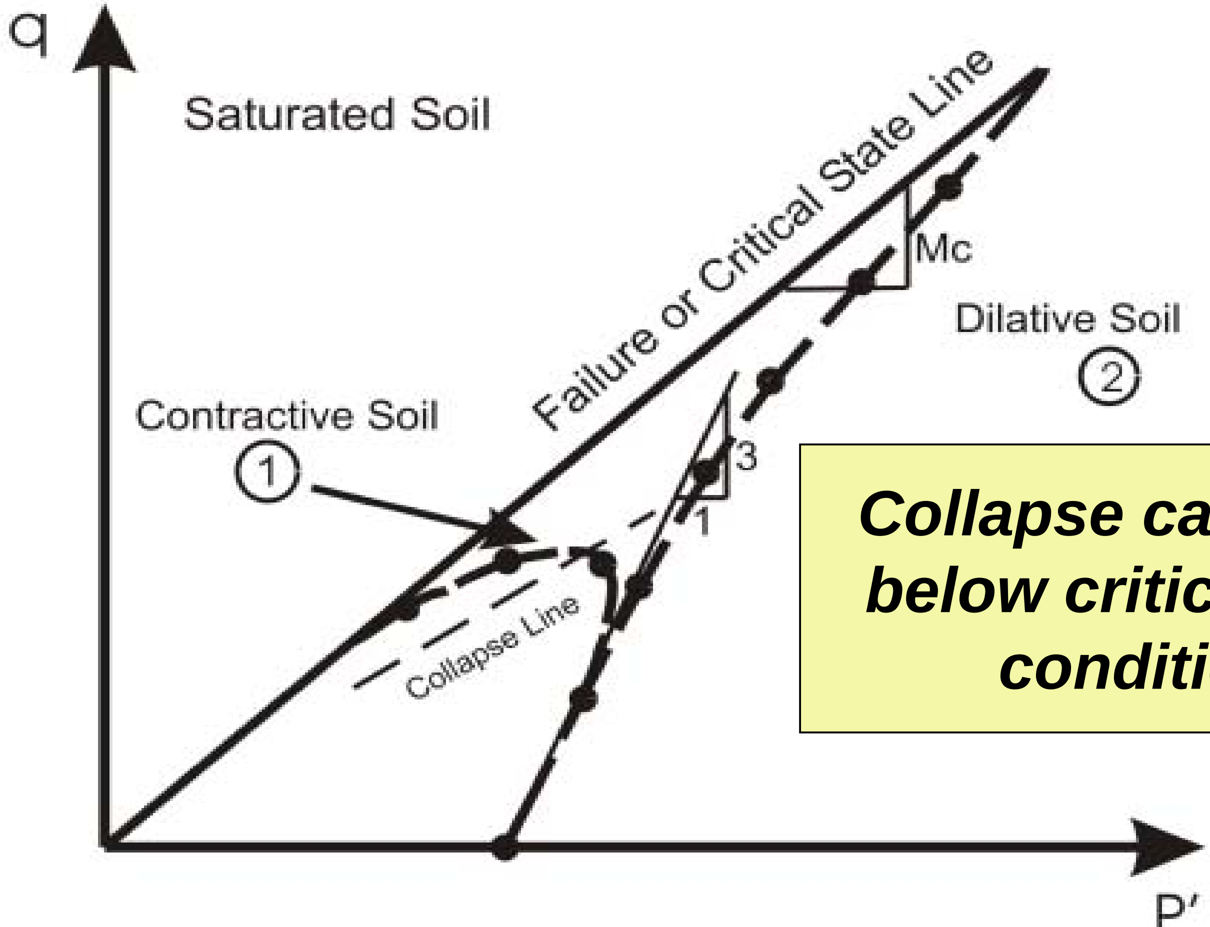






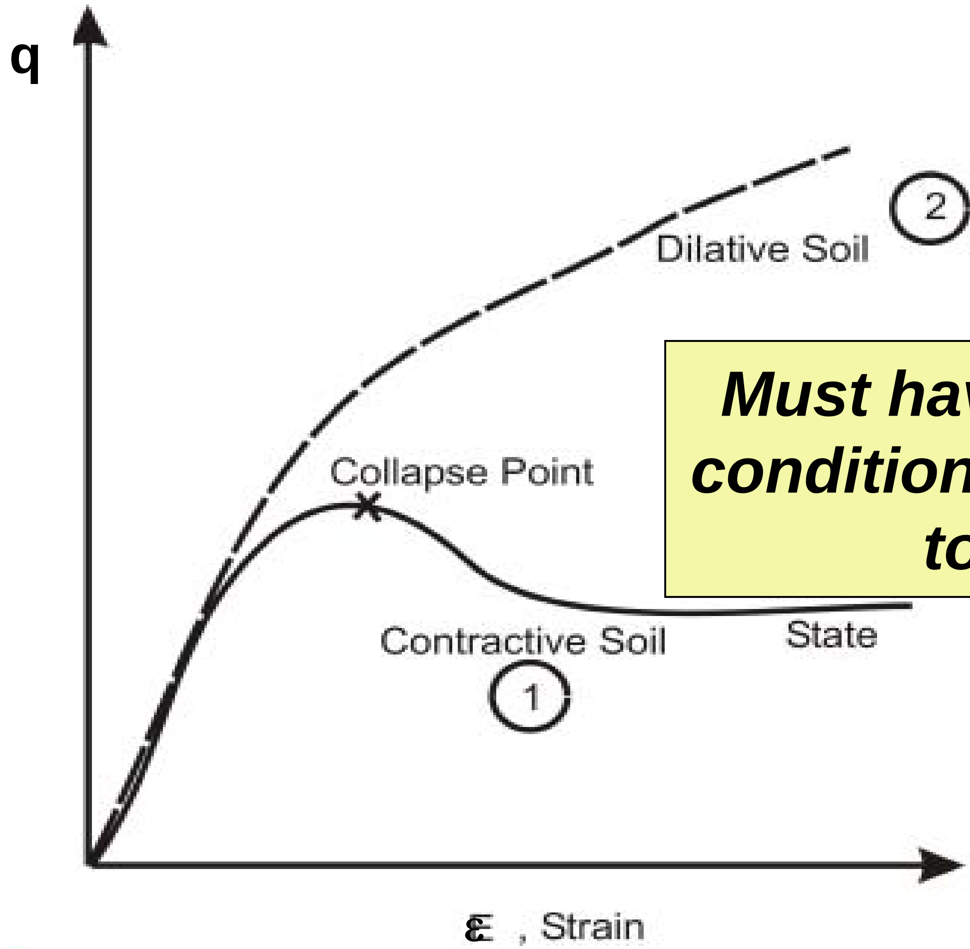
Compression

Conditions Contributing to Static Liquifaction or Collapse



Collapse can occur below critical state conditions

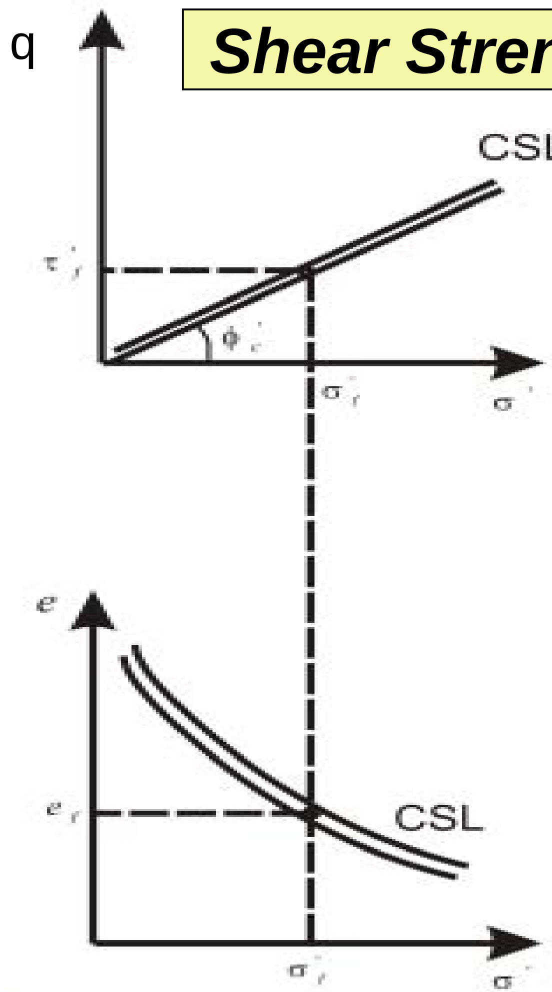
Stress Versus Strain for Dilative and Contractive Soils (Saturated Soil)



Must have undrained conditions for collapse to occur

Critical States of Soils

Shear Strength



Compression