CHAPTER 9 SHEAR STRENGTH THEORY

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Must know u_a & u_w
Mohr-Coulomb Failure Criterion
Types of Triaxial Tests
Mohr-Coulomb Failure Criterion

Shear strength designation for a saturated soil
HISTORY OF SHEAR STRENGTH

Modified direct shear box with a colloidon membrane

System for applying a constant negative pore-water pressure

Modified direct shear equipment for testing soils under low matric suction (from Donald, 1956)
Direct Shear Test Results (Donald, 1956)

$$\sigma = 17 \text{ kPa}$$

Results of direct shear tests on sands under low matric suctions (modified from Donald, 1956)
Triaxial Tests
Imperial College
(1960)

Shear Strength of Unsaturated Soils

Results of constant water content triaxial tests on a shale (clay fraction 22%) compacted at a water content of 18.6% (from Bishop, Alpan, Blight and Donald, 1960)
Test Results on a Modified Direct Shear Box (Escario, 1980)

Madrid grey clay (statically compacted)
Liquid limit = 81%
Plasticity index = 43%

AASHTO
\[ \begin{align*}
\rho_d \text{ max} &= 1360 \text{ kg/m}^3 \\
w_{\text{optimum}} &= 29%
\end{align*} \]

Specimen
\[ \begin{align*}
\rho_d &= 1335 \text{ kg/m}^3 \\
w &= 29% \\
(u_a - u_w) &= 750 \text{ kPa}
\end{align*} \]

Zero matric suction

Shear stress, \( \tau \) (kPa)

Net normal stress, \((\sigma - u_a)\) (kPa)

850 kPa

\((u_a - u_w) = 400 \text{ kPa}\)

200 kPa

Increase in shear strength for Madrid clay due to an increase in matric suction, obtained from direct shear tests (from Escario, 1980)

Note the parallelism in the lines
**Test Results on a Modified Triaxial Apparatus**  
(Ho & Fredlund, 1982)

![Failure envelope projection onto the $\tau$ versus ($\sigma - u_a$) plane](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>$(u_a - u_w)_t$ (kPa)</th>
<th>$C$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>34.5</td>
<td>32.5</td>
</tr>
<tr>
<td>2</td>
<td>68.9</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>137.9</td>
<td>76</td>
</tr>
</tbody>
</table>

Parallelism assumed in the interpretation

Multi-stage triaxial test on 1 specimen to obtain $\phi^b$
Test Results on a Modified Triaxial Apparatus (Ho & Fredlund, 1982)

Some nonlinearity evident in $\phi^b$

$\phi' = 33.4$ degrees
Test Results on a Modified Direct Shear Box (Gan & Fredlund, 1986)

Departure of $\phi^b$ from $\phi'$ around 75 kPa

Direct shear test results exhibiting a nonlinear relationship between $\tau$ versus $(u_a - u_w)$ (from Gan, 1986)
Test Results on a Modified Direct Shear Box (Escario & Saez, 1986)

Linearity of $\phi'$

Shear stress versus net confining pressure relationship for various matric suctions

Nonlinearity of $\phi^b$

Direct shear test results for Madrid grey clay, under controlled matric suctions (from Escario and Sáez, 1986)
When is a Soil a New Soil?

When the soil fabric is changed?

The particle structure of clay specimens compacted at various dry densities and water contents (from Lambe, 1958)

Unsaturated Soil Technology
Same soil compacted at different water contents can have different soil parameters due to different soil fabrics.
How should “Failure” be defined?

Consolidated Drained Triaxial

- $(\sigma - u_a) = 386$ kPa
- $(\sigma - u_a) = 96$ kPa

Consolidated drained triaxial test results on Dhanauri clay (from Satija, 1978)
Consolidated Undrained triaxial test with pore pressure measurements

Undrained triaxial tests on a compacted shale (from Bishop, Alpan, Blight and Donald, 1960)
In its simplest form, an “Extended Mohr-Coulomb” type failure surface can be drawn.

Fredlund et al (1978)

Failure is defined by a 3-D plane
**Linear form**

**Proposed by Fredlund et al (1978)***

**Extended Mohr-Coulomb Failure Envelope**

\[ \tau_{ff} = c' + (\sigma_f - u_a)_f \tan\phi' + (u_a - u_w)_f \tan\phi^b \]

where:

- \( c' \) = intercept of the "extended" Mohr-Coulomb failure envelope on the shear stress axis where the net normal stress and the matric suction at failure are equal to zero. It is also referred to as "effective cohesion".
- \( (\sigma_f - u_a)_f \) = net normal stress state variable on the failure plane at failure
- \( u_{af} \) = pore-air pressure on the failure plane at failure
- \( \phi' \) = angle of internal friction associated with the net normal stress state variable, \( (\sigma_f - u_a)_f \)
- \( (u_a - u_w)_f \) = matric suction on the failure plane at failure
- \( \phi^b \) = angle indicating the rate of increase in shear strength relative to the matric suction, \( (u_a - u_w)_f \)
Let us assume that cohesion has two components; namely effective cohesion and cohesion due to matric suction.
Linear, Extended Mohr-Coulomb Failure Envelope (Fredlund et al, 1978)
Modifications to a conventional triaxial cell

Primary modification is the high air entry disk sealed into the base pedestal

Modified triaxial cell for testing unsaturated soils
## Experimental Values of $\phi^b$

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$c'$</th>
<th>$\phi'$</th>
<th>$\phi^b$</th>
<th>Test Procedure</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compacted shale; $w = 18.6%$</td>
<td>15.8</td>
<td>24.8</td>
<td>18.1</td>
<td>Constant water content triaxial</td>
<td>Bishop et al. (1960)</td>
</tr>
<tr>
<td>Boulder clay; $w = 11.6%$</td>
<td>9.6</td>
<td>27.3</td>
<td>21.7</td>
<td>Constant water content triaxial</td>
<td>Bishop et al. (1960)</td>
</tr>
<tr>
<td>Dhanauri clay; $w = 22.2%$, $\rho_d$</td>
<td>37.3</td>
<td>28.5</td>
<td>16.2</td>
<td>Consolidated drained triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Constant drained triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td>Dhanauri clay; $w = 22.2%$, $\rho_d$</td>
<td>20.3</td>
<td>29.0</td>
<td>12.6</td>
<td>Consolidated water content triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Constant water content triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td>Dhanauri clay; $w = 22.2%$, $\rho_d$</td>
<td>15.5</td>
<td>28.5</td>
<td>22.6</td>
<td>Consolidated water content triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Constant water content triaxial</td>
<td>Satija, (1978)</td>
</tr>
<tr>
<td>Dhanauri clay; $w = 22.2%$, $\rho_d$</td>
<td>11.3</td>
<td>29.0</td>
<td>16.5</td>
<td>Consolidated drained direct shear</td>
<td>Escario (1980)</td>
</tr>
<tr>
<td>Madrid grey clay; $w = 29%$,</td>
<td>23.7</td>
<td>22.5*</td>
<td>16.1</td>
<td>Consolidated drained direct shear</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Consolidated drain multistage triaxial</td>
<td>Ho and Fredlund (1982a)</td>
</tr>
<tr>
<td>Undisturbed decomposed granite;</td>
<td>28.9</td>
<td>33.4</td>
<td>15.3</td>
<td>Consolidated drain multistage triaxial</td>
<td>Ho and Fredlund (1982a)</td>
</tr>
<tr>
<td>Hong Kong</td>
<td></td>
<td></td>
<td></td>
<td>Consolidated drained multistage triaxial</td>
<td></td>
</tr>
<tr>
<td>Undisturbed decomposed rhyolite;</td>
<td>7.4</td>
<td>35.3</td>
<td>13.8</td>
<td>Consolidated drained multistage triaxial</td>
<td></td>
</tr>
<tr>
<td>Hong Kong</td>
<td></td>
<td></td>
<td></td>
<td>Consolidated drained multistage triaxial</td>
<td></td>
</tr>
<tr>
<td>Tappen–Notch Hill silt; $w = 21.5%$,</td>
<td>0.0</td>
<td>35.0</td>
<td>16.0</td>
<td>Consolidated drained multistage triaxial</td>
<td>Krahn et al. (1989)</td>
</tr>
<tr>
<td>$\rho_d = 1590$ kg/m$^3$</td>
<td></td>
<td></td>
<td></td>
<td>Consolidated drained multistage triaxial</td>
<td></td>
</tr>
<tr>
<td>Compacted glacial till; $w = 12.2%$,</td>
<td>10</td>
<td>25.3</td>
<td>7–25.5</td>
<td>Consolidated drained multistage direct shear</td>
<td>Gan et al. (1988)</td>
</tr>
<tr>
<td>$\rho_d = 1810$ kg/m$^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Average value.

Average = $17^\circ$
Focus on the Matric Suction plane and cohesion has two components

\[ c = c' + (u_a - u_w)\tan \phi^b \]

Matric suction, \((u_a - u_w)\)

Line of intercepts along the failure plane on the \(\tau\) versus \((u_a - u_w)\) plane

**Unsaturated Soil Technology**
Making a 3-Dimensional surface into a 2-Dimensional plot by projecting onto the front surface.
Making a 3-Dimensional surface into a 2-Dimensional plot by projecting onto the front surface.
An increase in matric suction translates into an increase in cohesion

Form of a two parameter equation

Contour lines of the failure envelope onto the $\tau$ versus $(\sigma - u_a)$ plane

$$\tau_f = c + (\sigma - u_a) \tan \phi'$$

or

$$\tau_f = c' + (u_a - u_w) \tan \phi' + (\sigma - u_a) \tan \phi'$$

Horizontal projection of the failure envelope onto the $\tau$ versus $(\sigma - u_a)$ plane, viewing parallel to the $(u_a - u_w)$ axis (continued)
Extended Mohr-Coulomb type failure surface
tangent to the Mohr circles at failure

\[ \tau_{ff} = c' + (\sigma_f - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \]
Use of the Stress Point Method to get the Soil Parameters

Comparisons of the failure envelope and the corresponding stress point envelope
Water and air phases can be “controlled” or “measured”

Primary modification is the high air entry disk sealed into the base pedestal

Modified triaxial cell for testing unsaturated soils
<table>
<thead>
<tr>
<th>Test Methods</th>
<th>Consolidation Prior to Shearing Process</th>
<th>Drainage</th>
<th>Shearing Process</th>
<th>Pore-Air Pressure, $u_a$</th>
<th>Pore-Water Pressure, $u_w$</th>
<th>Soil Volume Change, $\Delta V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consolidated Drained (CD)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>$C$</td>
<td>$C$</td>
<td>$M$</td>
</tr>
<tr>
<td>Constant water content (CW)</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>$C$</td>
<td>$M$</td>
<td>$M$</td>
</tr>
<tr>
<td>Consolidated undrained (CU)</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>$M$</td>
<td>$M$</td>
<td>$-$</td>
</tr>
<tr>
<td>Undrained</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Unconfined compression (UC)</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

$M = \text{Measurement}, \ C = \text{controlled}.$
<table>
<thead>
<tr>
<th>Stages</th>
<th>Total stress</th>
<th>Pore-air pressure</th>
<th>Pore-water pressure</th>
<th>$(\sigma - u_a)$</th>
<th>$(u_a - u_w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium at the end of consolidation</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>Axial compression</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td></td>
<td>$(\sigma_1 - \sigma_3)$</td>
<td></td>
<td></td>
<td>$(\sigma_1 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>At failure</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td></td>
<td>$(\sigma_1 - \sigma_3)$</td>
<td></td>
<td></td>
<td>$(\sigma_1 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
</tbody>
</table>

Stress conditions during a consolidated drained triaxial compression test
Typical stress paths for a consolidated drained test

Main need is to measure $\phi^b$

Stress paths followed during a consolidated drained test at various net confining pressures under a constant matric suction
Other possible stress paths for consolidated drained tests

Stress paths followed during consolidated drained tests at various matric suctions under a constant net confining pressure.
### Constant Water Content Test

<table>
<thead>
<tr>
<th>Stages</th>
<th>Total Stress</th>
<th>Pore-air Pressure</th>
<th>Pore-water Pressure</th>
<th>$(\sigma - u_a)$</th>
<th>$(u_a - u_w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equilibrium at the end of consolidation</strong></td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>Drained and controlled</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w + \Delta u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w - \Delta u_w)$</td>
</tr>
<tr>
<td>Undrained and measured</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w + \Delta u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w - \Delta u_w)$</td>
</tr>
<tr>
<td>At failure</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_{wf} = u_w + \Delta u_{wf}$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_{wf}) = (u_a - u_w) - \Delta u_{wf}$</td>
</tr>
</tbody>
</table>

**Stress conditions during a constant water content triaxial compression test**
Matric suction decreased during test

Constant water content test

\[ (\sigma - u_a) = 96 \text{ kPa} \]

\[ (\sigma - u_a) = 386 \text{ kPa} \]
Stress path moves in 3-D space for the Constant Water Content test.
### Consolidated Undrained Test with Pore Pressure Measurements

<table>
<thead>
<tr>
<th>Stages</th>
<th>Total stress</th>
<th>Pore-air pressure</th>
<th>Pore-water pressure</th>
<th>((\sigma - u_a))</th>
<th>((u_a - u_w))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equilibrium</strong></td>
<td>(\sigma_3)</td>
<td>(u_a)</td>
<td>(u_w)</td>
<td>((\sigma_3 - u_a))</td>
<td>((u_a - u_w))</td>
</tr>
<tr>
<td>at the end of consolidation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Axial compression</strong></td>
<td>((\sigma_1 - \sigma_3))</td>
<td>(u_a + \Delta u_a)</td>
<td>(u_w + \Delta u_w)</td>
<td>((\sigma_1 - u_a) - \Delta u_a)</td>
<td>((u_a - u_w) + \Delta u_a - \Delta u_w)</td>
</tr>
<tr>
<td><strong>At failure</strong></td>
<td>((\sigma_1 - \sigma_3))</td>
<td>(u_{af} = u_a + \Delta u_{af})</td>
<td>(u_{wf} = u_w + \Delta u_{wf})</td>
<td>((\sigma_3 - u_a) = \sigma_{1f} - u_{af})</td>
<td>((u_a - u_w) = (u_a - u_w) + \Delta u_{af} - \Delta u_{wf})</td>
</tr>
</tbody>
</table>

Stress conditions during a consolidated undrained triaxial compression test with pore pressure measurements.
Stress path moves in 3-D space for the Consolidated Undrained test
## Undrained Test

<table>
<thead>
<tr>
<th>Stages</th>
<th>Total stress</th>
<th>Pore-air pressure</th>
<th>Pore-water pressure</th>
<th>$(\sigma - u_a)$</th>
<th>$(u_a - u_w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>After applying confining pressure</td>
<td>$\sigma_3$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_3 - u_a)$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>(Axial compression)</td>
<td>$(\sigma_1 - \sigma_3)$</td>
<td>$u_a + \Delta u_a$</td>
<td>$u_w + \Delta u_w$</td>
<td>$(\sigma_1 - u_a - \Delta u_a)$</td>
<td>$(u_a - u_w) + \Delta u_a - \Delta u_w$</td>
</tr>
<tr>
<td>At failure</td>
<td>$(\sigma_1 - \sigma_3)_f$</td>
<td>$u_{af} = u_a + \Delta u_{af}$</td>
<td>$u_{wf} = u_w + \Delta u_{wf}$</td>
<td>$(\sigma_1 - u_a)<em>f = \sigma</em>{1f} - u_{af}$</td>
<td>$(u_a - u_{wf})<em>f = (u_a - u_w) + \Delta u</em>{af} - \Delta u_{wf}$</td>
</tr>
</tbody>
</table>

Stress conditions during an undrained triaxial compression test.
Undrained test path moves in 3-D space

Stress paths followed during an undrained test
Undrained tests are generally interpreted using the assumption that the failure envelope is horizontal.
### Unconfined Compression Test

<table>
<thead>
<tr>
<th>Stages</th>
<th>Total stress</th>
<th>Pore-air pressure</th>
<th>Pore-water pressure</th>
<th>$(\sigma - u_a)$</th>
<th>$(u_a - u_w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initial</strong></td>
<td>$\sigma_3 = 0$</td>
<td>$\sigma_3 = 0$</td>
<td>$u_a = 0$</td>
<td>$\sigma_3 - u_a = 0$</td>
<td>$-u_w$</td>
</tr>
<tr>
<td><strong>Axial compression</strong></td>
<td>$(\sigma_1 - \sigma_3)$</td>
<td>$\Delta u_a$</td>
<td>$u_w + \Delta u_w$</td>
<td>$-u_w + \Delta u_a - \Delta u_w$</td>
<td>$-u_w + \Delta u_a - \Delta u_w$</td>
</tr>
<tr>
<td><strong>At failure</strong></td>
<td>$(\sigma_1 - \sigma_3)_f$</td>
<td>$0$</td>
<td>$u_{af} = \Delta u_{af}$</td>
<td>$(\sigma_1 - u_a)<em>f = \sigma_f - \Delta u</em>{af}$</td>
<td>$(u_a - u_w)<em>f = -u_w + \Delta u</em>{af} - \Delta u_{wf}$</td>
</tr>
</tbody>
</table>

Stress conditions during an unconfined compression test
Unconfined compression stress path depends upon dilation or compression of the soil.
Unsaturated Soil

Relationship between $q_u$ and $c_u$ for an unsaturated soil

Unconfined compression test

Undrained shear strength, $c_u$

Unconfined compression test

Use of the unconfined compressive strength, $q_u$, to approximate the undrained shear strength, $c_u$, for an unsaturated and a saturated soil

Saturated Soil

Relationship between $q_u$ and $c_u$ for a saturated soil

Undrained shear strength, $c_u$ (constant)

$\phi = 0$

Total normal stress, $\sigma$
## DIRECT SHEAR TESTS

<table>
<thead>
<tr>
<th>Stages</th>
<th>Total stress</th>
<th>Pore-air pressure</th>
<th>Pore-water pressure</th>
<th>$(\sigma - u_a)$</th>
<th>$(u_a - u_w)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equilibrium at the end of consolidation</td>
<td>$\sigma$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$\sigma - u_a$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>Horizontal shearing</td>
<td>$\sigma$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$\sigma - u_a$</td>
<td>$(u_a - u_w)$</td>
</tr>
<tr>
<td>At failure</td>
<td>$\sigma$</td>
<td>$u_a$</td>
<td>$u_w$</td>
<td>$(\sigma_f - u_a)_f = \sigma - u_a$</td>
<td>$(u_a - u_w)_f = (u_a - u_w)$</td>
</tr>
</tbody>
</table>

Stress conditions during a consolidated drained direct shear test
Interpretation generally used for Direct Shear tests

Extended Mohr-Coulomb failure envelope established from direct shear test results
Unsaturated Soils must be tested at a low Strain Rate because of their Low Permeability.

Strain Rates

Fastest

Strain rate effects for constant water content tests on Dhanauri clay (from Satija, 1978) (continued)
<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Triaxial Test</th>
<th>Strain Rate, $\dot{\varepsilon} (% / s)$</th>
<th>Approximate Strain at Failure, $\varepsilon_f (%)$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulder clay; $w = 11.6%$ and $% clay = 18%$</td>
<td>CW</td>
<td>$3.5 \times 10^{-5}$</td>
<td>15</td>
<td>Bishop et al. 1960</td>
</tr>
<tr>
<td>Brachest silt</td>
<td>CW</td>
<td>$4.7 \times 10^{-5}$</td>
<td>11</td>
<td>Bishop and Donald (1961)</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>$8.3 \times 10^{-6}$</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Talybont boulder clay; $w =$ 9.75% and $% clay = 6%$</td>
<td>Undrained with pore pressure measurements</td>
<td>$4.7 \times 10^{-7}$</td>
<td>$\sigma_3 = 83 \text{ kPa : 8.5}$</td>
<td>Donald (1963)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_3 = 207 \text{ kPa : 11}$</td>
<td></td>
</tr>
<tr>
<td>Dhanauri clay; $w = 22.2%$ and $% clay = 25%$</td>
<td>CW</td>
<td>$6.7 \times 10^{-4}$</td>
<td>20</td>
<td>Satija and Gulhati (1979)</td>
</tr>
<tr>
<td></td>
<td>CD</td>
<td>$1.3 \times 10^{-4}$</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Undisturbed decomposed</td>
<td>CD</td>
<td>$1.7 \times 10^{-5}$</td>
<td>Stage I: 3–5</td>
<td>Ho and Fredlund (1982a)</td>
</tr>
<tr>
<td>granite and rhyolite</td>
<td>Multistage</td>
<td>$6.7 \times 10^{-5}$</td>
<td>Stage II: 1–3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Stage III: 1–3</td>
<td></td>
</tr>
</tbody>
</table>
| Clayey sand; $w =$ 14–17\% and $% clay = 30\%$ | Undrained and unconfined | $1.7 \times 10^{-3}$                     | 15–20                             | Chantawarangul (1983)
**Cyclic:** Loading and Unloading procedure for multistage tests

Idealized stress-strain curves for multistage triaxial testing

<table>
<thead>
<tr>
<th>Stage no.</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_3$ (kPa)</td>
<td>250</td>
<td>350</td>
<td>500</td>
</tr>
<tr>
<td>$u_a$ (kPa)</td>
<td>100</td>
<td>200</td>
<td>350</td>
</tr>
<tr>
<td>$u_w$ (kPa)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$(\sigma_3 - u_a)$ (kPa)</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>$(u_a - u_w)$ (kPa)</td>
<td>50</td>
<td>150</td>
<td>300</td>
</tr>
</tbody>
</table>
Multi-Stage Direct Shear Results on Glacial Till

Multistage direct shear test on a compacted glacial till sample GT-16-N4 (from Gan, 1986)
Multi-Stage Direct Shear Results on Glacial Till

Horizontal shear displacement rate,
\( d_h = 0.000176 \text{ mm/s} \)

\((u_a - u_w) = 16.6 \text{ kPa} \)

Stage I, II, III, IV, V, VI

Shear stress, \( \tau \) (kPa)

Horizontal shear displacement, \( d_h \) (mm)
Recall: Bishop’s effective stress equation

\[ \sigma' = (\sigma - u_a) + \chi (u_a - u_w) \]

Let:

\[ \tau = c' + (\sigma') \tan \phi' \]

Then substitute the effective stress into the shear strength equation.

\[ \tau = c' + ((\sigma - u_a) + \chi (u_a - u_w)) \tan \phi' \]
Bishop’s shear strength equation

\[ \tau_{ff} = c' + \left\{ (\sigma_f - u_a)_f + \chi \,(u_a - u_w)_f \right\} \tan\phi' \]

where:
\[ \chi = \frac{\tan\phi^b}{\tan\phi'} \]

Arises from a comparison of unsaturated shear strength behavior and saturated soil behavior

Must remember that \( \chi \) is determined from the degree of saturation of the soil at the point of failure; therefore, requires another prediction for S%
RELATIONSHIPS BETWEEN $\phi^b$ and $\chi$

Extended Mohr - Coulomb failure envelope

\[
\frac{\tan \phi^b}{\tan \phi'} (u_a - u_w)_f = \chi (u_a - u_w)_f
\]

Shear stress, $\tau$

$\tan \phi^b$ tangent at point $A$ (\(\phi^b\) method)

$\phi'$ tangent at point $A'$ (\(\chi\) method)

Mohr-Coulomb failure envelope (saturated soil)

Net normal stress, $(\sigma - u_a)$

Comparison of the $\phi^b$ and $\chi$ methods of designating shear strength
Results from comparing saturated and unsaturated triaxial shear strength test results

\[ \chi \] values for a cohesionless silt (after Donald, 1961)
Description of Stress State for Saturated Soils

• Mohr-Coulomb Terminology
  – Shear stress = \( \tau = \frac{(\sigma_1 - \sigma_3)}{2} \)
  – Principal Stresses; \( \sigma_1 \) and \( \sigma_3 \)
  – Principal Effective Stresses,
    \( \sigma_1' = (\sigma_1 - u_w) \);
    \( \sigma_3' = (\sigma_3 - u_w) \)

• Critical State (or Elasto-Plastic) Terminology
  – Shear Stresses \( q = \sigma_1 - \sigma_3 \)
  – Mean Stress = \( p' = \left[ \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} - u_w \right] \)
Mapping of Strength and Consolidation Parameters

Shear Strength

Conventional Soil Mechanics

Elasto-Plastic Soil Mechanics

Compression

Unsaturated Soil Technology
Critical State Lines, Normal Compression, and Unloading/Reloading Lines

Shear Strength

Compression
Conditions Contributing to Static Liquifaction or Collapse

Collapse can occur below critical state conditions
Stress Versus Strain for Dilative and Contractive Soils (Saturated Soil)

Must have undrained conditions for collapse to occur
Critical States of Soils

Shear Strength

Compression

Unsaturated Soil Technology