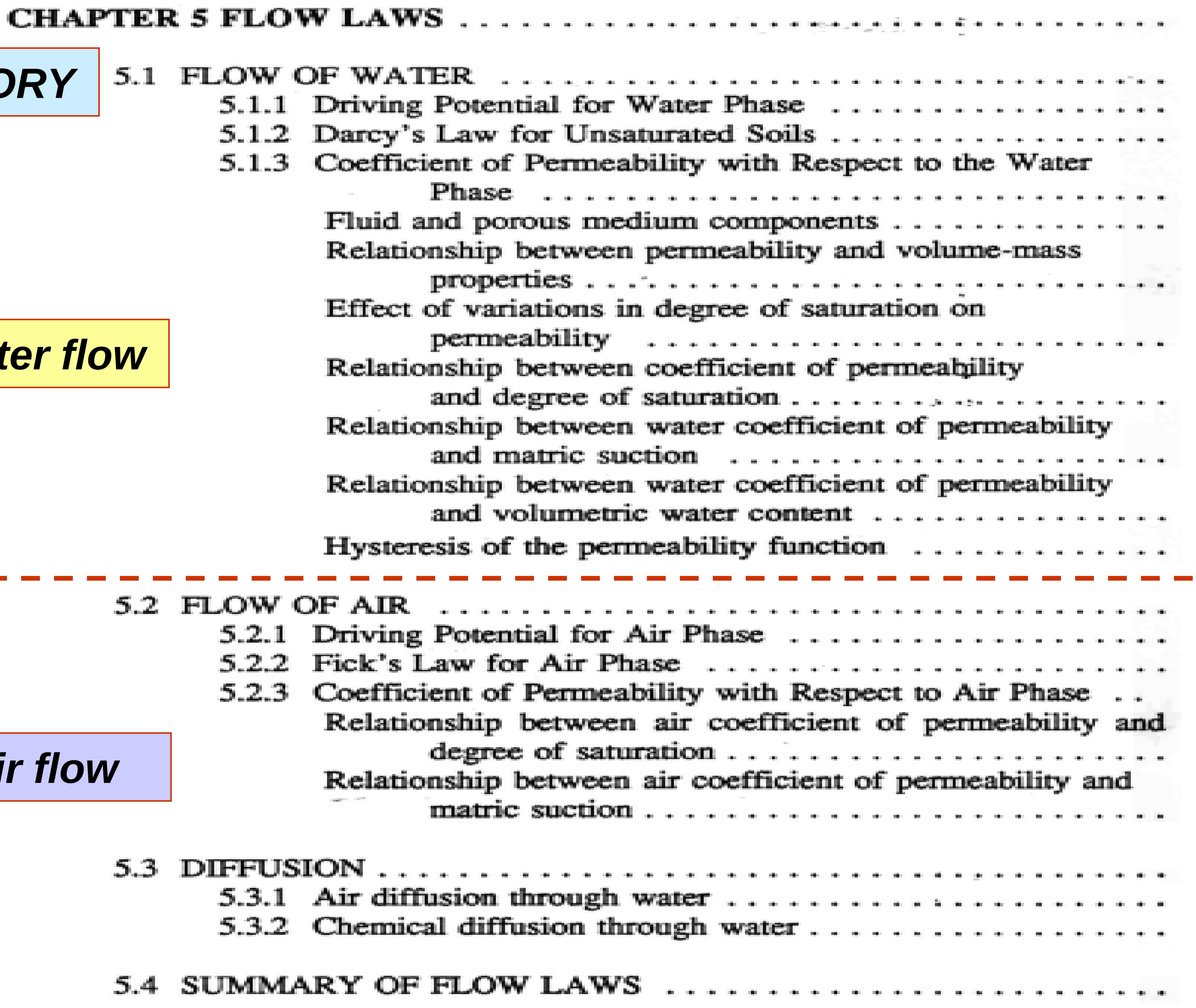


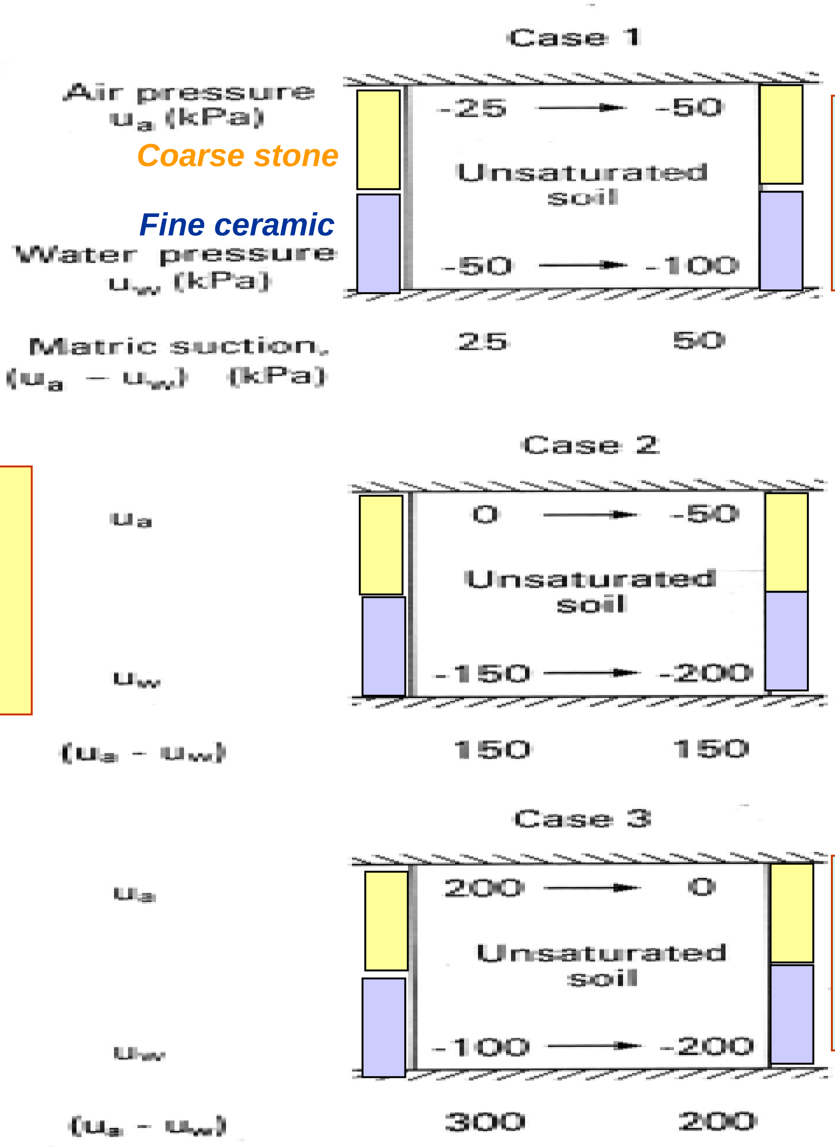
CHAPTER 5 FLOW LAWS

THEORY

Water flow

Air flow

Does Suction Gradient Cause Flow?



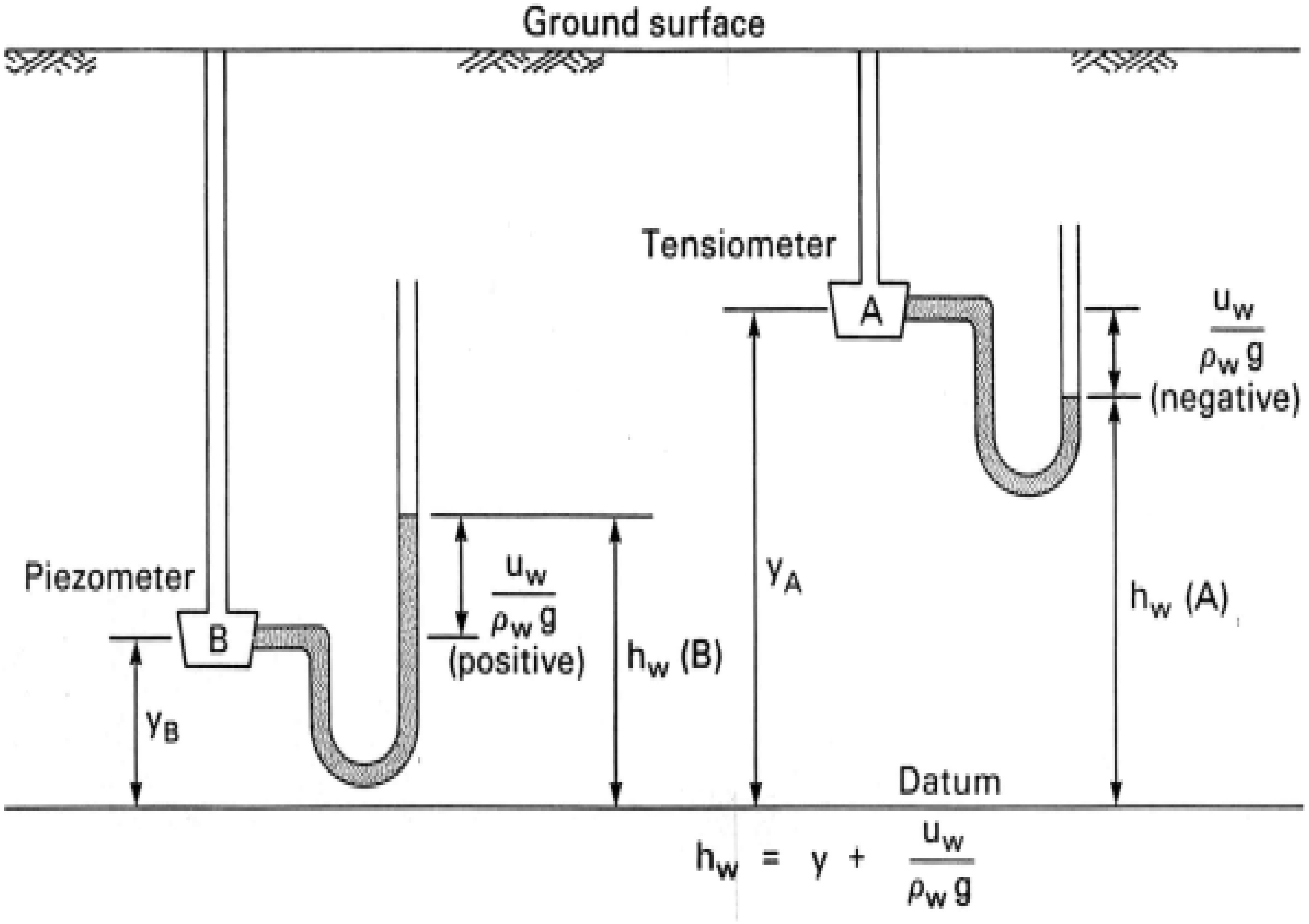
Pressure and matric suction gradients across an unsaturated soil element

Suction gradient to the right

No suction gradient but still flow

Suction gradient to the left

Does water content gradient cause Flow?



Concept of potential and head for saturated and unsaturated soils

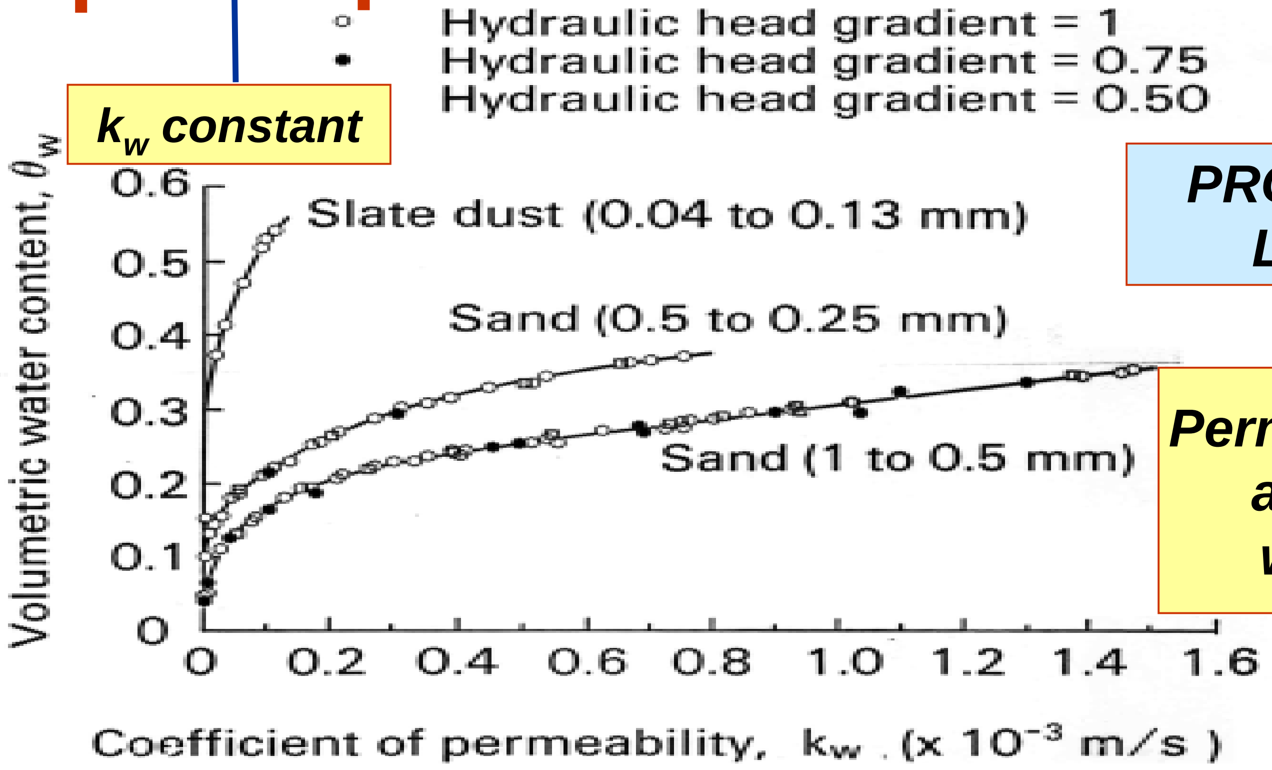
Does water flow from A to B?, or B to A?

Darcy's law (1856)

$$v_w = -k_w \frac{\partial h_w}{\partial y}$$

v_w = flow rate of water
k_w = coefficient of permeability

k_w constant



PROOF: Darcy's Law is valid

Permeability is fixed at a particular water content

Experimental verification of Darcy's law for water flow through an unsaturated soil (from Childs and Collis-George, 1950)

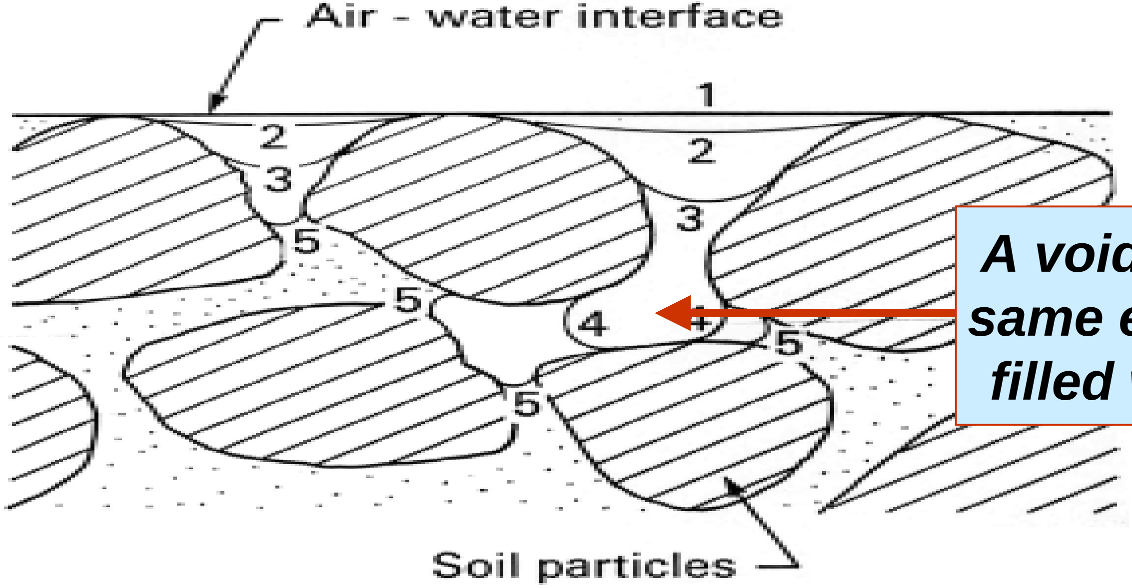
Childs and Collis-George, 1950

$$k_w = \frac{\rho_w g}{\mu_w} K$$

where:

μ_w = absolute (dynamic) viscosity of water

K = intrinsic permeability of the soil



A void of air has the same effect as a void filled with carbowax

Development of an unsaturated soil by the withdrawal of the air-water interface at different stages of matric suction or degree of saturation (i.e., stages 1 to 5) (from Childs, 1969)

$$k_w = k_w(S, e)$$

or

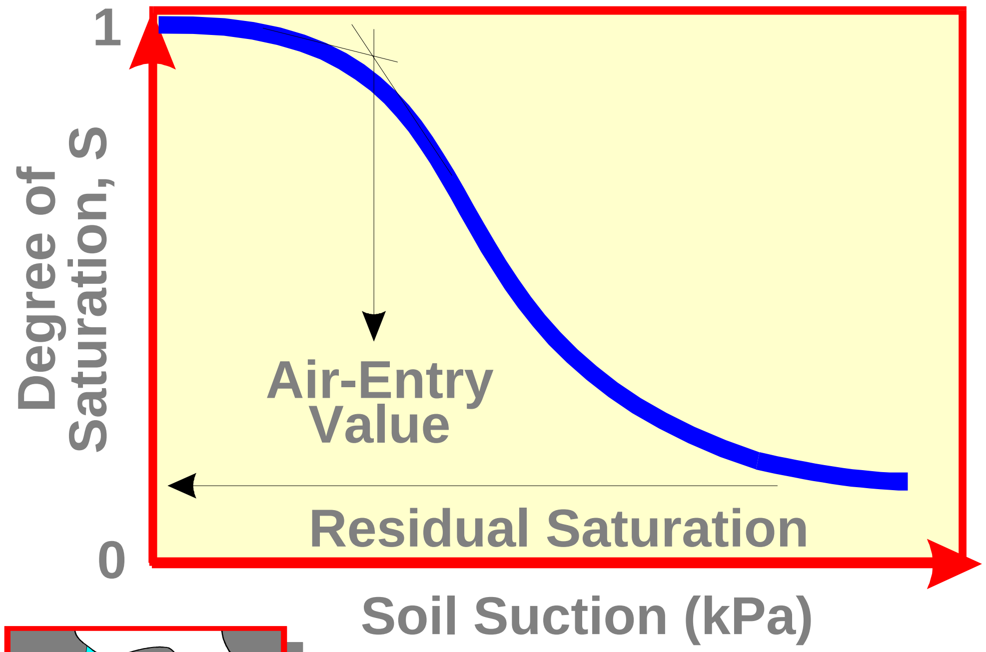
$$k_w = k_w(e, w)$$

or

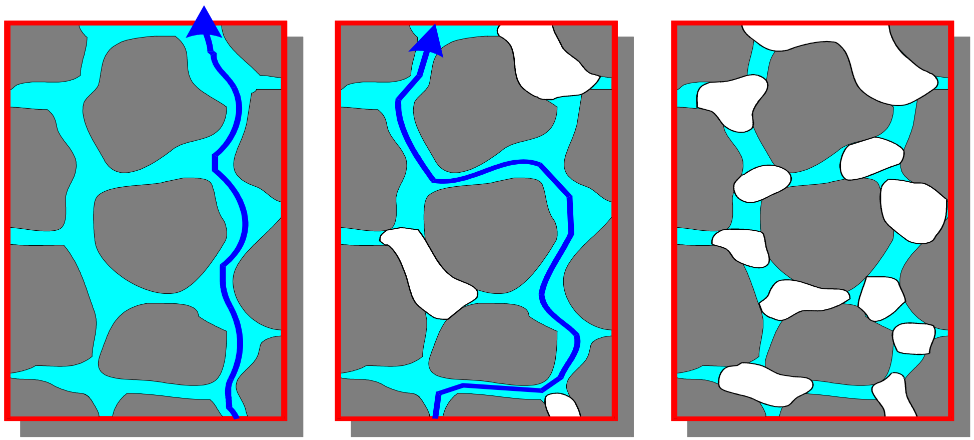
$$k_w = k_w(w, S)$$

For a saturated soil, k = k(e) or k(w)

SWCC - Soil-Water Characteristic Curve

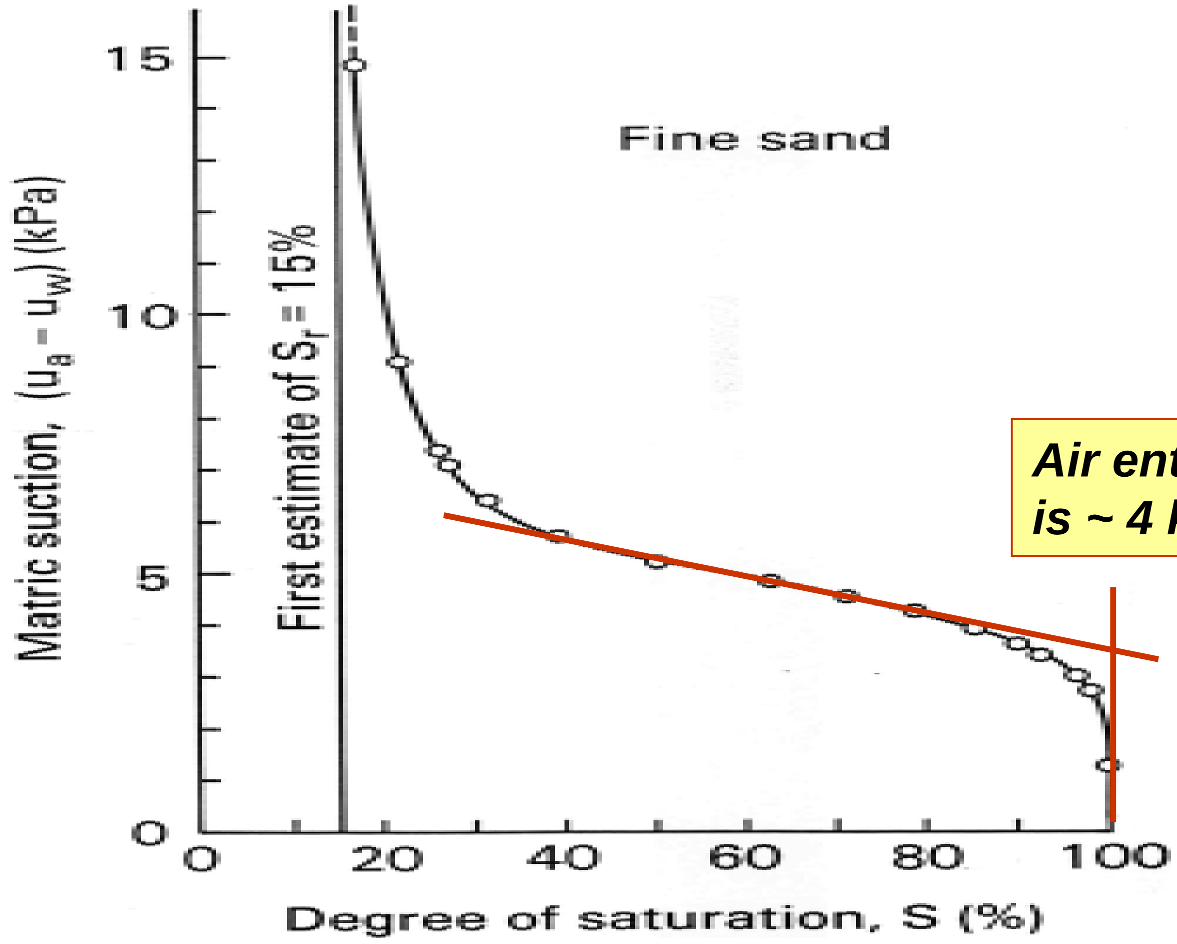


Water can only flow where there is water in the voids



Tortuosity dramatically changes permeability

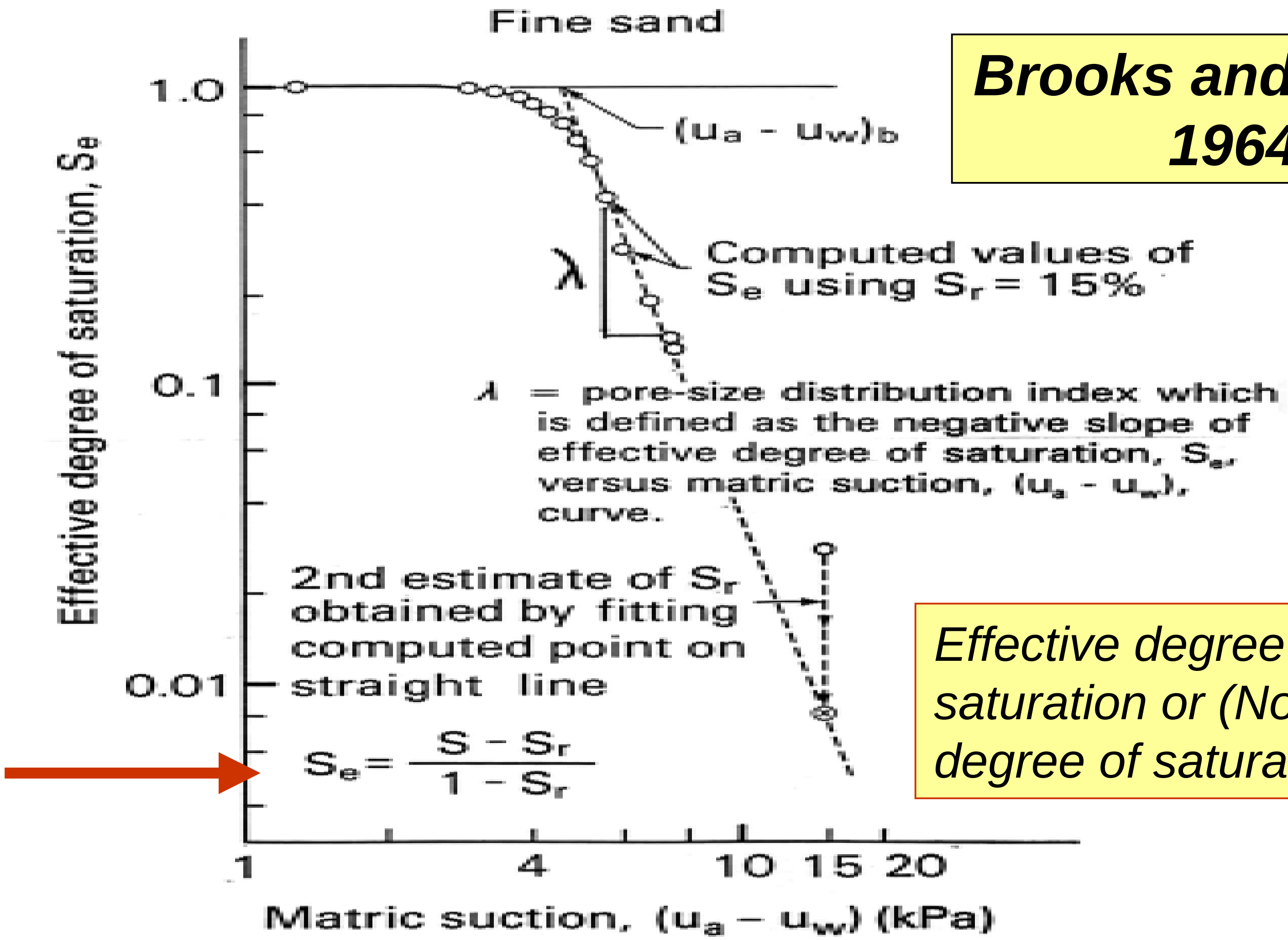
SWCC - Soil-Water Characteristic Curve



Air entry value is ~ 4 kPa

Determination of the air entry value, $(u_a - u_w)_b$, residual degree of saturation, S_r, and pore size distribution index, λ (from Brooks and Corey, 1964)

Matric suction versus effective degree of saturation curve

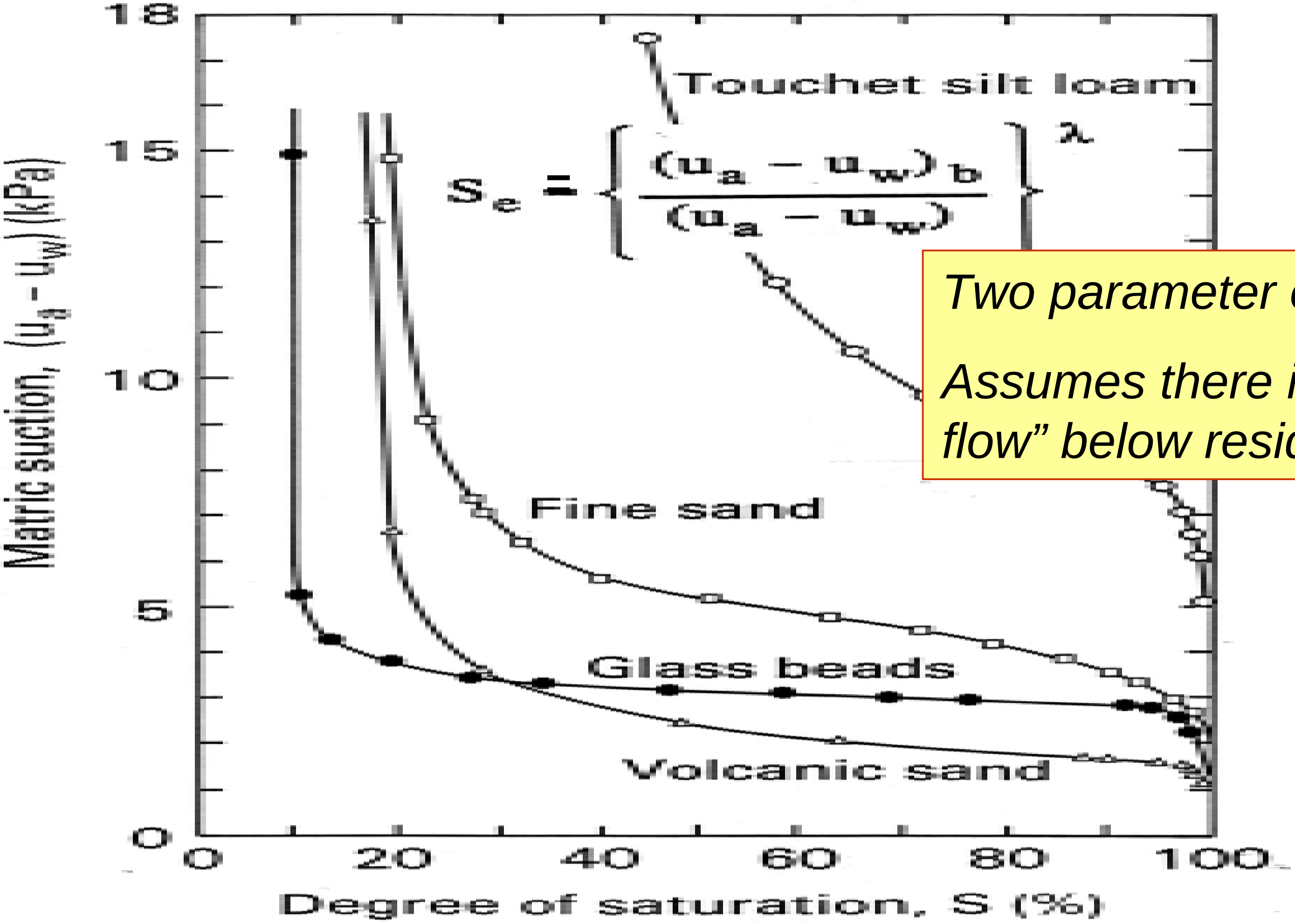


Determination of the air entry value, $(u_a - u_w)_b$, residual degree of saturation, S_r, and pore size distribution index, λ (from Brooks and Corey, 1964)

Brooks and Corey 1964

Effective degree of saturation or (Normalized) degree of saturation

Becomes a straight line on a log-log plot

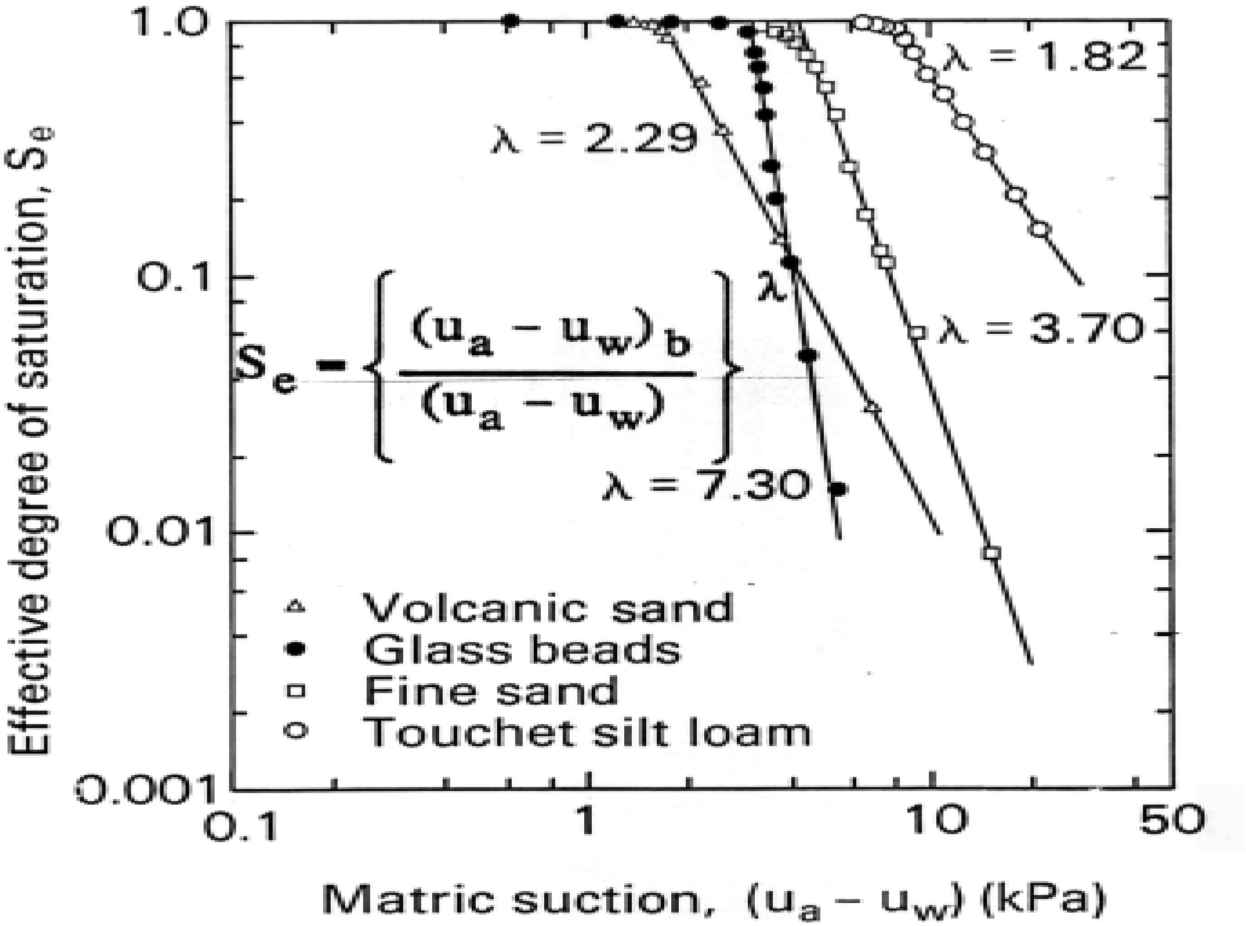


$$S_e = \left\{ \frac{(u_a - u_w)_b}{(u_a - u_w)} \right\}^{\lambda}$$

Two parameter equation

Assumes there is “no-flow” below residual w%

Typical matric suction versus degree of saturation curves for various soils with their corresponding λ values (from Brooks and Corey, 1964)



The Brooks and Corey (1964) equation for SWCC

Typical matric suction versus degree of saturation curves for various soils with their corresponding λ values (from Brooks and Corey, 1964)

$k_w = k_s$ for $(u_a - u_w) \leq (u_a - u_w)_b$

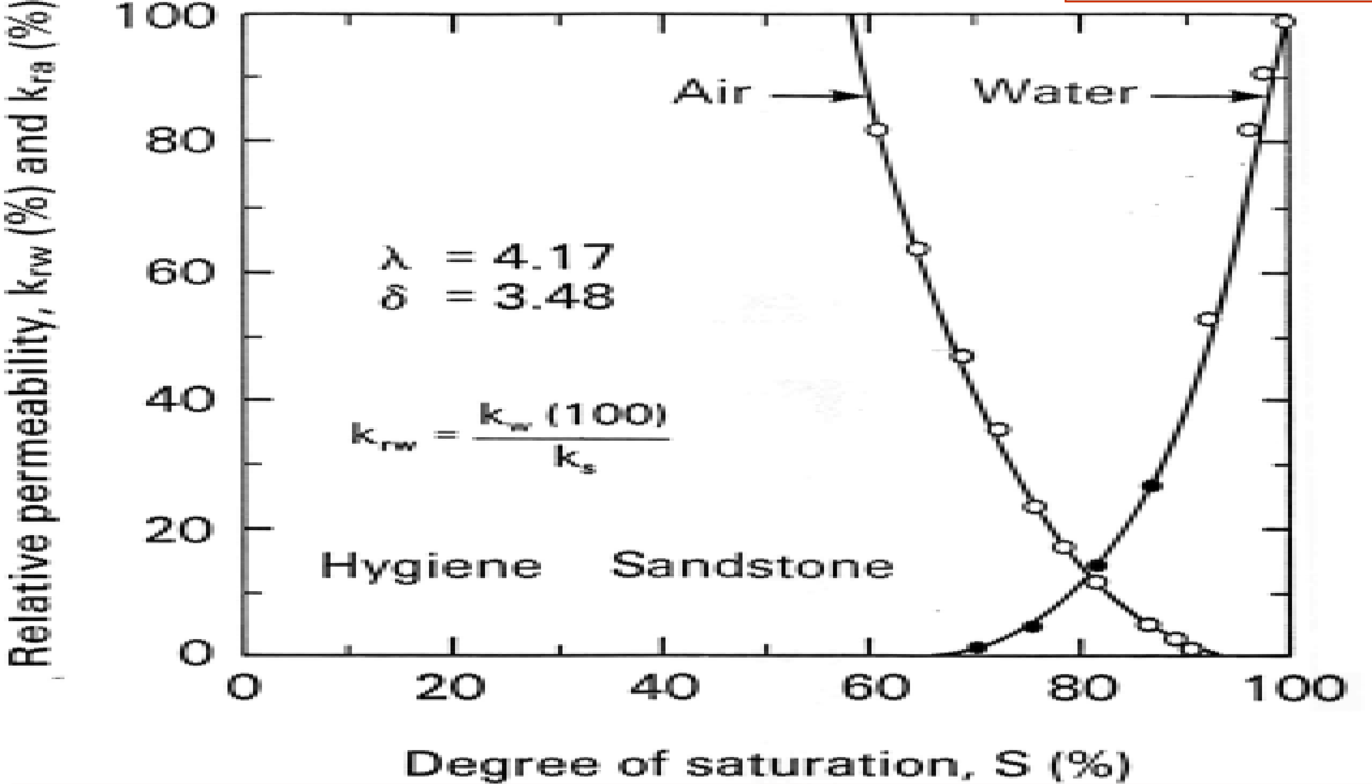
$k_w = k_s S_e^{\delta}$ for $(u_a - u_w) > (u_a - u_w)_b$

Delta is related to the Pore Size Distribution Index

δ = an empirical constant

$$\delta = \frac{2 + 3\lambda}{\lambda}$$

Becomes linear on a log scale)



The Brooks and Corey (1964) equation for Permeability

Relative permeability of water and air as a function of the degree of saturation during drainage (from Brooks and Corey, 1964)

Summary of Brooks and Corey (1964) Equation for the Coefficient of Permeability

$$S_e = \left\{ \frac{(u_a - u_w)_b}{(u_a - u_w)} \right\}^{\lambda}$$

$k_w = k_s$ for $(u_a - u_w) \leq (u_a - u_w)_b$

$k_w = k_s S_e^{\delta}$ for $(u_a - u_w) > (u_a - u_w)_b$

δ = an empirical constant

$$\delta = \frac{2 + 3\lambda}{\lambda}$$

$S_e = (S - S_r) / (1 - S_r)$

S_e = Measure of amount of water

S = Any degree of saturation

S_r = Residual degree of saturation

λ = Pore size distribution index

$$k_w = k_s \left\{ \frac{(u_a - u_w)_b}{(u_a - u_w)} \right\}^{2+3\lambda}$$

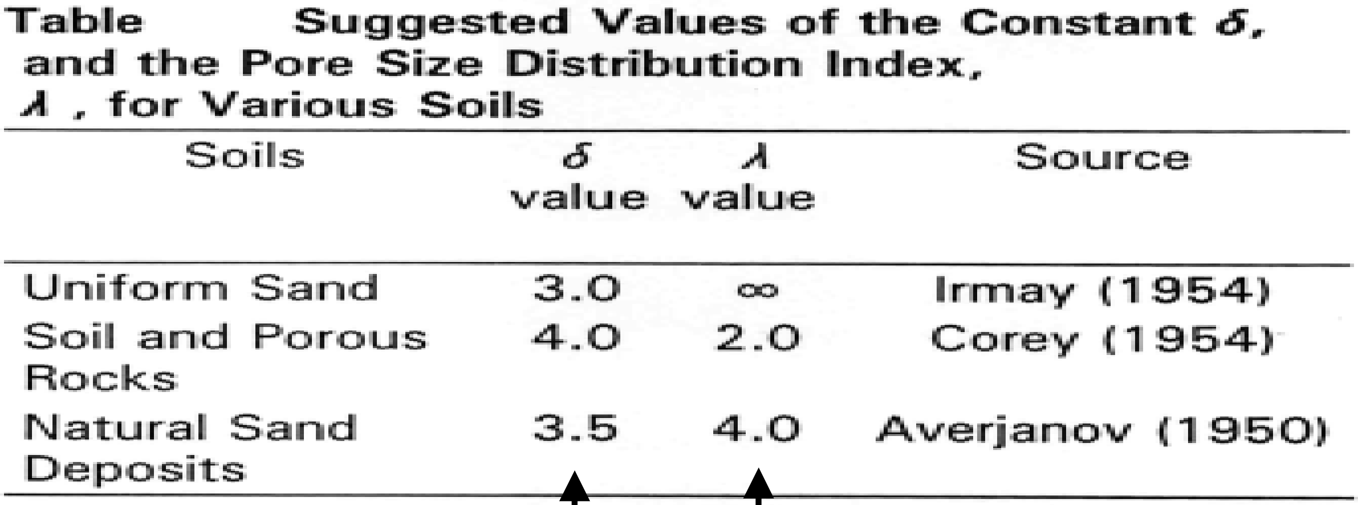


Table Suggested Values of the Constant δ, and the Pore Size Distribution Index, λ, for Various Soils

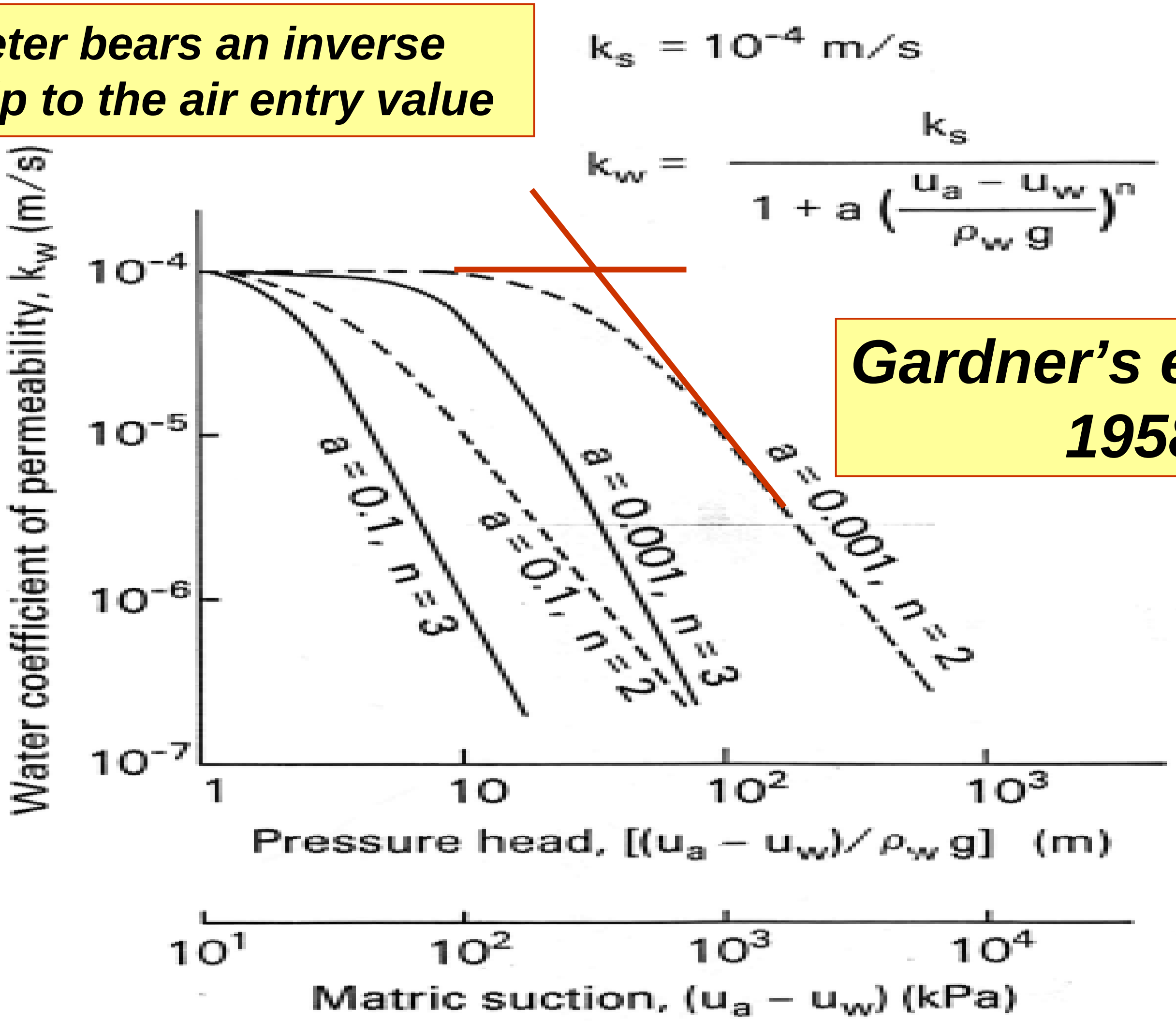
Soils	δ value	λ value	Source
Uniform Sand	3.0	∞	Irmay (1954)
Soil and Porous Rocks	4.0	2.0	Corey (1954)
Natural Sand Deposits	3.5	4.0	Averjanov (1950)

Pore size distribution for SWCC

Constant δ for Permeability

Brooks and Corey is a discontinuous function since it starts at the Air Entry Value of the soil

'a' parameter bears an inverse relationship to the air entry value



$$k_s = 10^{-4}\ \text{m/s}$$

$$k_w = \frac{k_s}{1 + a\left(\frac{u_a - u_w}{\rho_w g}\right)^n}$$

Gardner's equation 1958

Gardner's equation for the water coefficient of permeability as a function of the matric suction

Integration Forms for the SWCC and Permeability Function

- ***(Childs and Collis-George, 1950); assumed that the soil has a random distribution of pores of various sizes***
- ***Used the summation of a series of terms from the statistical probability of interconnections between the pores***
- ***SWCC was used as an indication of the configuration of the water-filled pores***
- ***Permeability equation was derived based on the Poiseuille equation***

Prediction of the coefficient of permeability from the soil-water characteristic curve

$$k_w(\theta_w)_i = \frac{k_s}{k_{sc}} \frac{T_s^2 \rho_w g}{2\mu_w} \frac{\theta_s^p}{N} \sum_{j=i}^{m} \{(2j + 1 - 2i)(u_a - u_w)_j^{-2}\}$$

$i = 1, 2, \ldots, m$

Childs and Collis-George (1950)

where:

$k_w(\theta_w)_i$ = calculated water coefficient of permeability, (m/s), for a specified volumetric water co[illegible]nt, $(\theta_w)_i$, corresponding to the i th interval

i = interval number which increase with the decreasing volumetric water content. For example, i = 1 identifies the first interval that closely corresponds to the saturated volumetric water content, θ_s ; i = m identifies the last interval corresponding to the lowest volumetric water content, θ_L, on the experimental soil-water characteristic curve

j = a counter from "i" to "m"

k_s = measured saturated coefficient of permeability, (m/s)

k_{sc} = calculated saturated coefficient of permeability, (m/s)

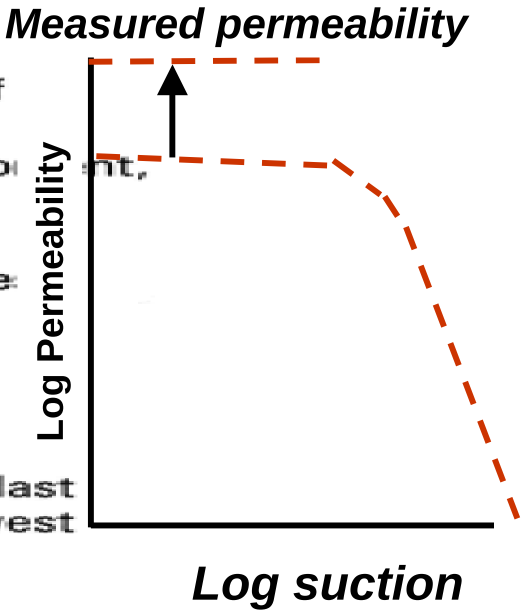
T_s = surface tension of water (kN/m)

ρ_w = water density (kg/m^3)

g = gravitational acceleration (m/s^2)

μ_w = absolute viscosity of water (Ns/m^2)

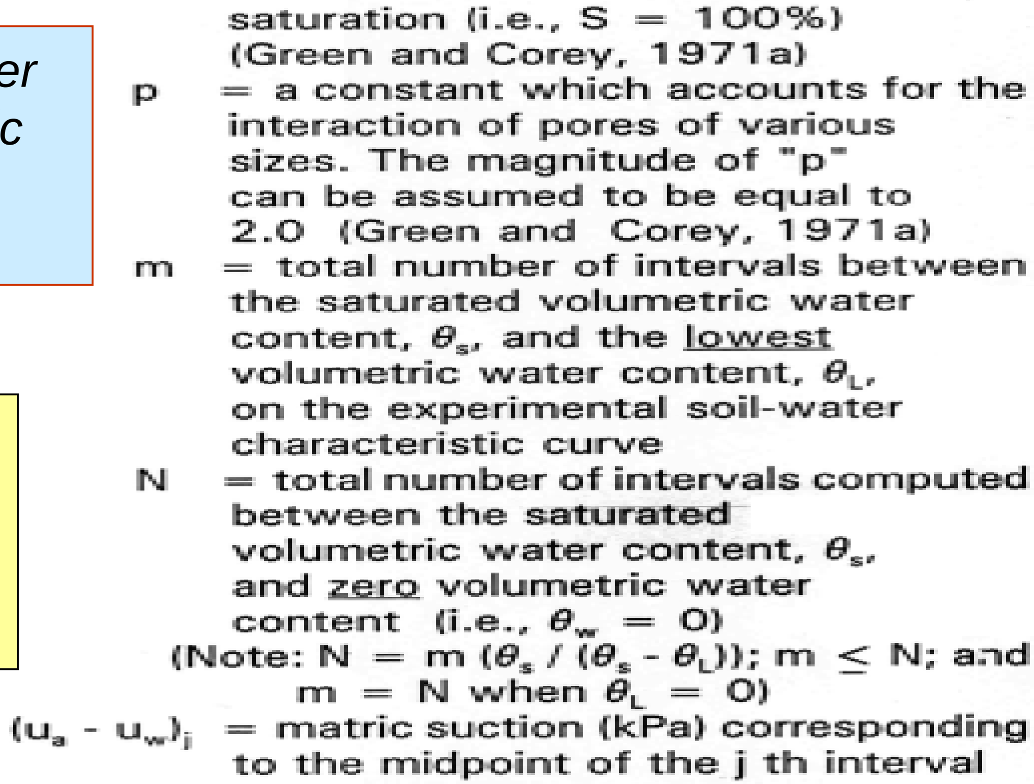
θ_s = volumetric water content at



Variable “p” is a power applied to volumetric water content

Assume p = 2.0

Childs and Collis-George (1950)



saturation (i.e., S = 100%) (Green and Corey, 1971a)

p = a constant which accounts for the interaction of pores of various sizes. The magnitude of "p" can be assumed to be equal to 2.0 (Green and Corey, 1971a)

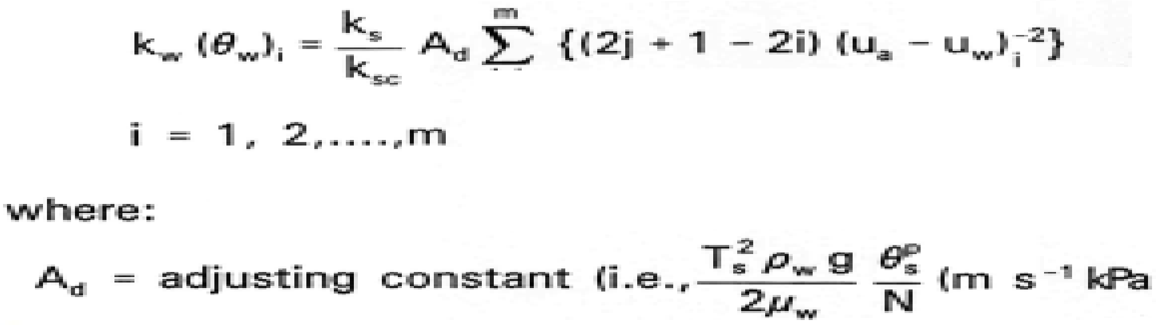
m = total number of intervals between the saturated volumetric water content, θ_s, and the <u>lowest</u> volumetric water content, θ_L, on the experimental soil-water characteristic curve

N = total number of intervals computed between the saturated volumetric water content, θ_s, and <u>zero</u> volumetric water content (i.e., $\theta_w = 0$)

(Note: $N = m(\theta_s / (\theta_s - \theta_L))$; $m \leq N$; and $m = N$ when $\theta_L = 0$)

$(u_a - u_w)_j$ = matric suction (kPa) corresponding to the midpoint of the j th interval

Based on summation (or integration) along the SWCC

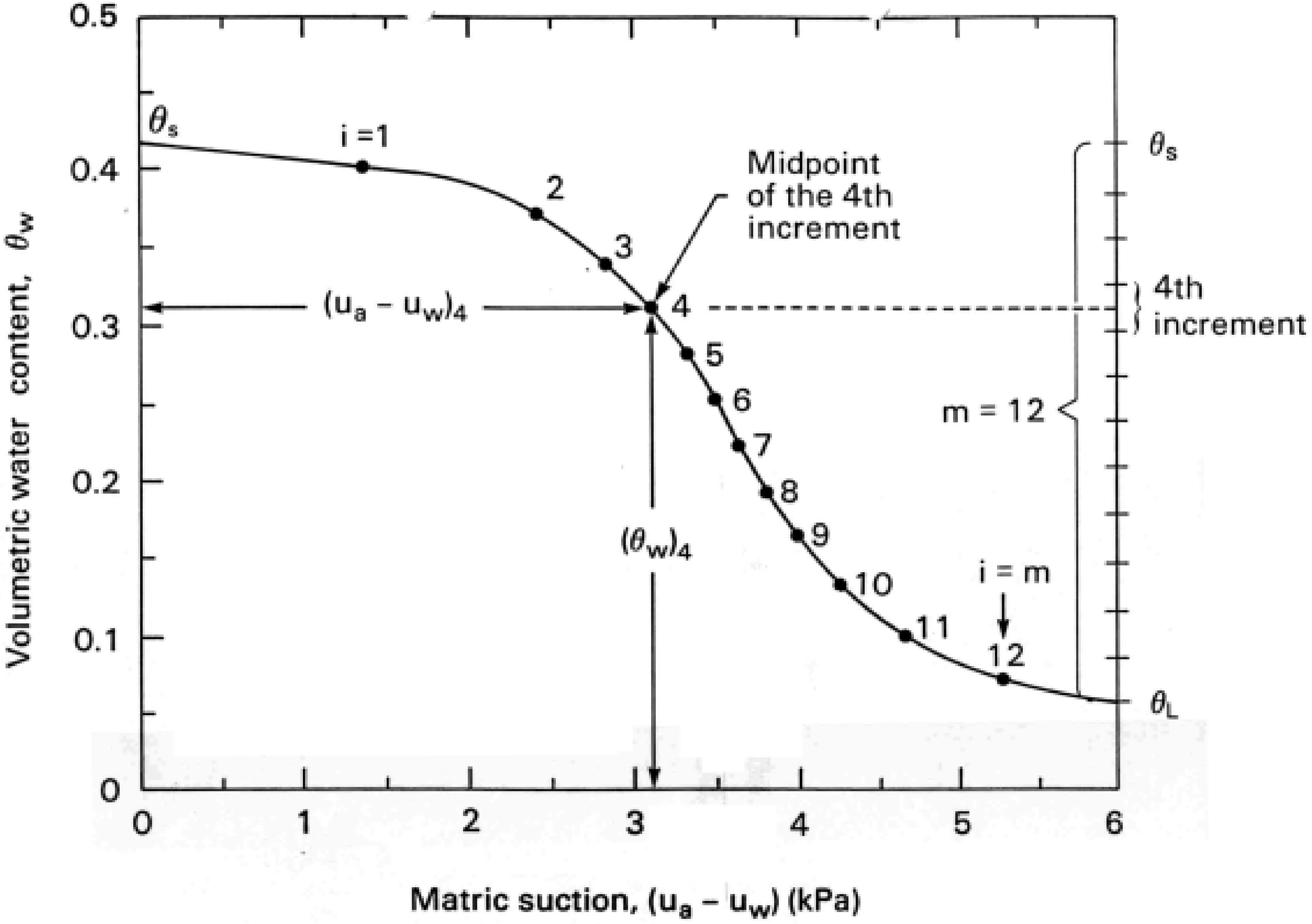


$$k_w(\theta_w)_i = \frac{k_s}{k_{sc}} A_d \sum^{m} \{(2j + 1 - 2i)(u_a - u_w)_i^{-2}\}$$

$i = 1, 2, \ldots, m$

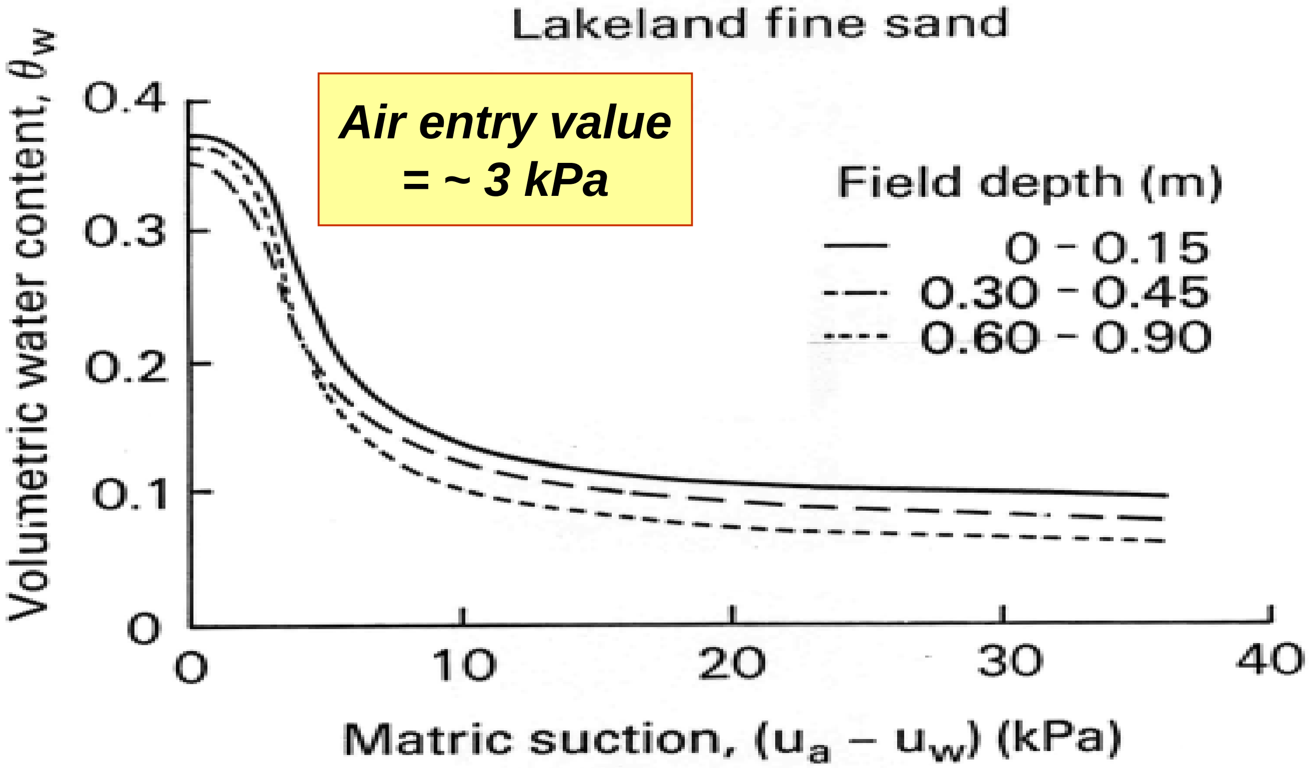
where:

A_d = adjusting constant (i.e., $\frac{T_s^2 \rho_w g}{2\mu_w} \frac{\theta_s^p}{N}$ (m s^{-1} kPa



Prediction of the coefficient of permeability from the soil-water characteristic curve

Soil-water characteristic curves

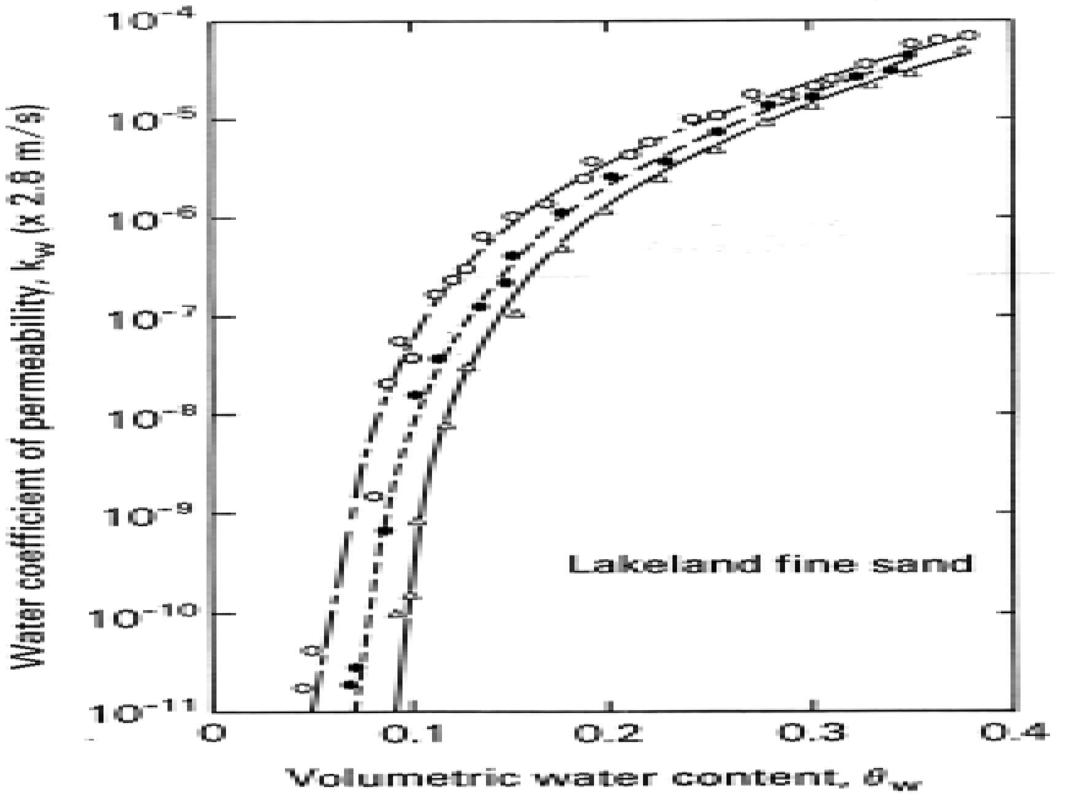


Air entry value
= ~ 3 kPa

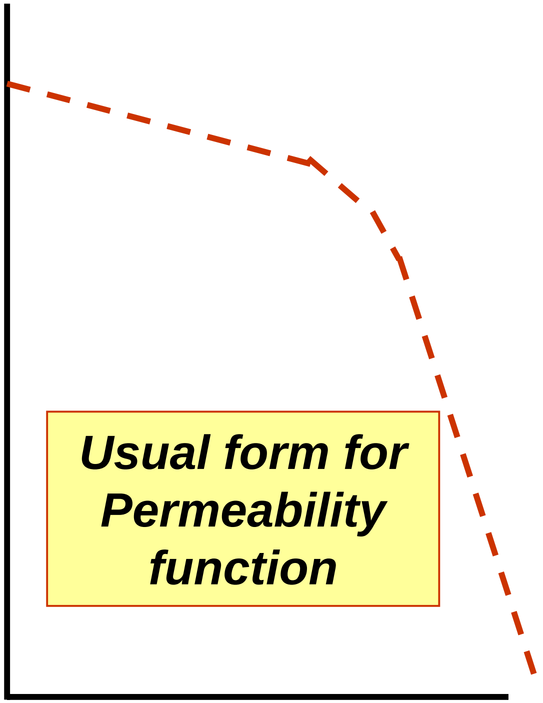
Comparisons between calculated and measured unsaturated permeabilities for Lakeland fine sand (from Elzeftawy and Cartwright, 1981)

Coefficient of permeability as a function of volumetric water content

Field Depth (m)	k_w measured	k_w calculated	k_s (m/s)
0 – 0.15	△	——	0.41×10^{-4}
0.30 – 0.45	●	- - - - - -	0.36×10^{-4}
0.60 – 0.90	○	—·—·—	0.48×10^{-4}



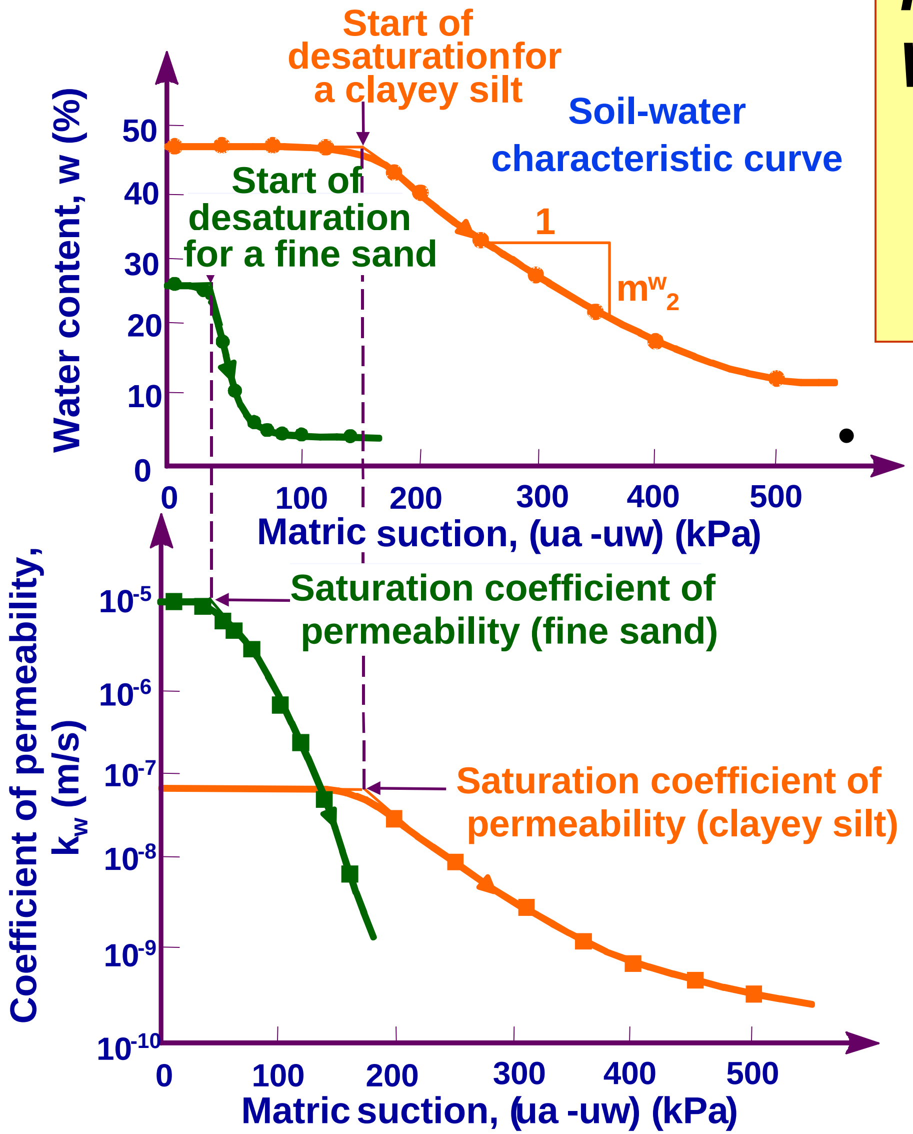
Comparisons between calculated and measured unsaturated permeabilities for Lakeland fine sand (from Elzeftawy and Cartwright, 1981)



Usual form for Permeability function

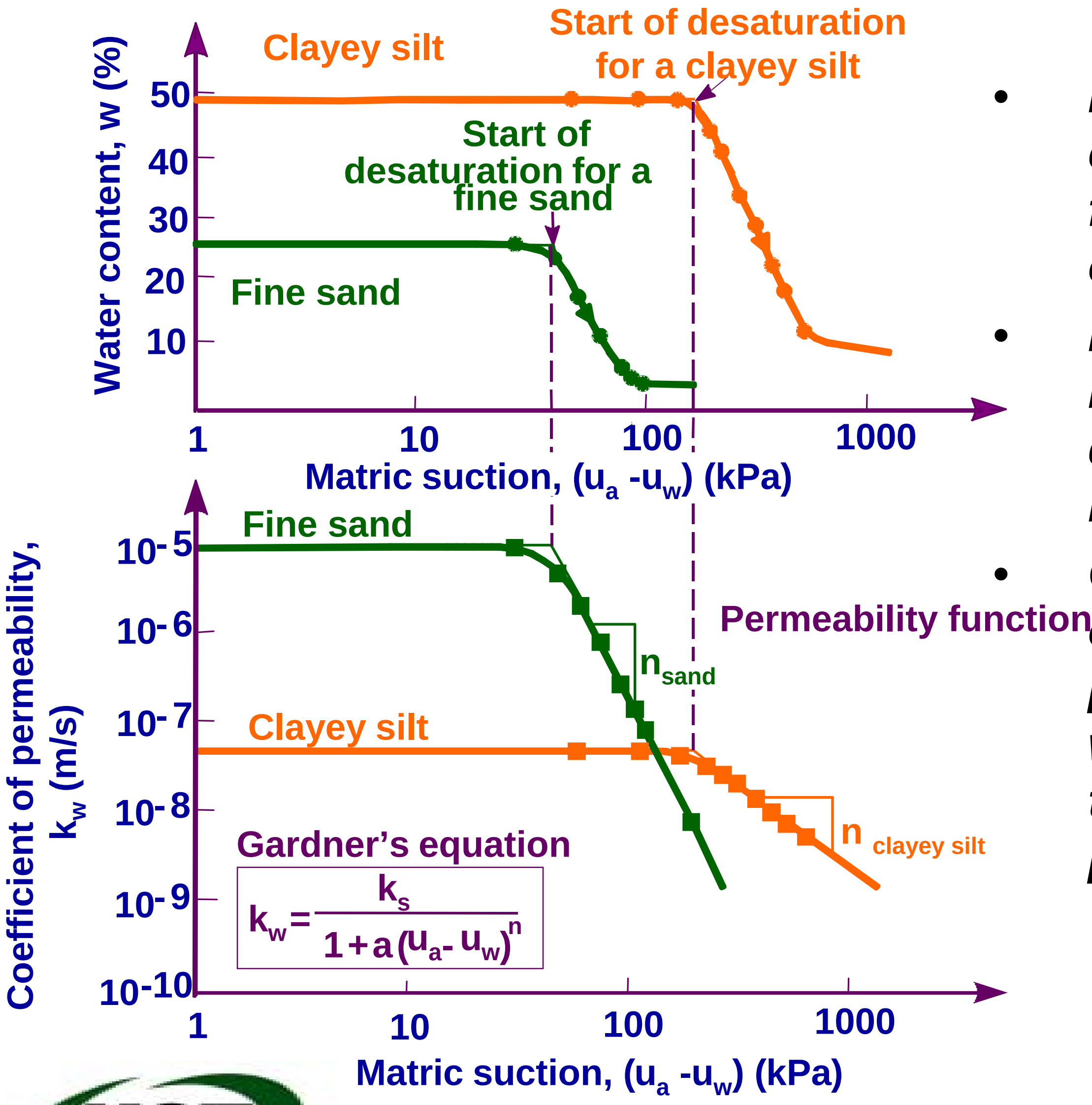
Log suction

Relationship Between Soil-Water Characteristic Curve and the Coefficient of Permeability for sand and a Clayey Silt



- ***The soil-water characteristic curve defines the amount of water in the soil***
  - ***Air entry value initiates a reduction in the coefficient of permeability***
  - ***Is possible for a sandy soil to have a lower permeability than a clayey soil***

Typical Gardner's Empirical Permeability Functions Shown for a Sand and a Clayey Silt



- *Permeability function is commonly plotted as a function of the logarithm of suction*
- *Most soils show a straight line as the soil desaturates towards residual conditions*
- *Gardner's function is one of the simplest permeability functions with physical meaning to the "a" and "n" parameters*

Commonly used Permeability Functions

- *van Genucthen (1980)*
  - **SWCC**

$$S_e = \left[\frac{1}{1 + (\alpha\psi)^n} \right]^m$$

  - ***k-function***

$$k_r = \left\{ \frac{1 - (\alpha\psi)^{n-1} [1 + (\alpha\psi)^n]^{-m}}{[1 + (\alpha\psi)^n]^{m/2}} \right\}^2$$

where m = 1-1/n

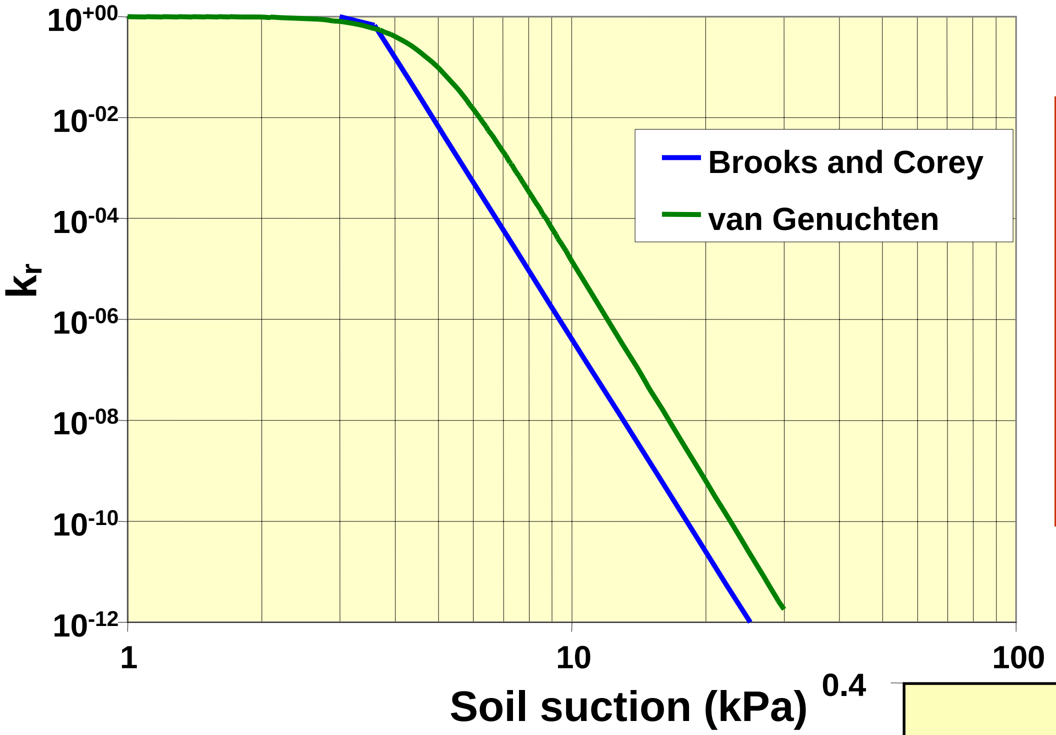
- *Fredlund and Xing (1994)*
  - **SWCC**

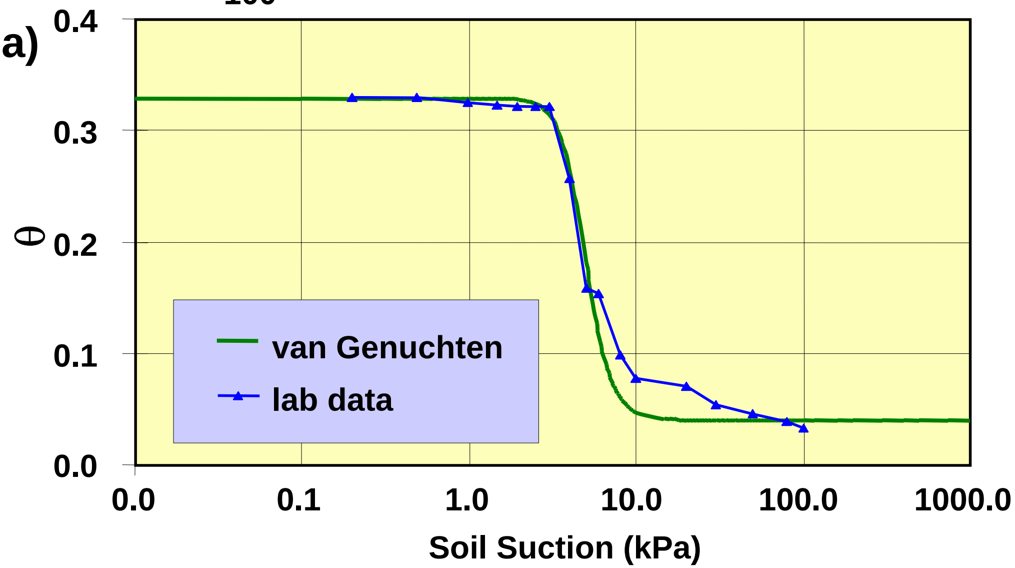
$$S = \left[\frac{1}{ln\left[e + (\frac{\psi}{A})^B \right]} \right]^C C(\psi)$$

  - ***k-function***

$$k_r = \frac{\int_{ln(u_a - u_w)}^{ln 10^6} \frac{\theta_w(e^y) - \theta_w \; f(u_a - u_w)}{e^y} \theta'_w(e^y) dy}{\int_{ln(u_a - u_w)b}^{ln 10^6} \frac{\theta_w(e^y) - \theta_s}{e^y} \theta'_w(e^y) dy}$$

Comparison of van Genuchten (1980) and Brooks and Corey (1964)



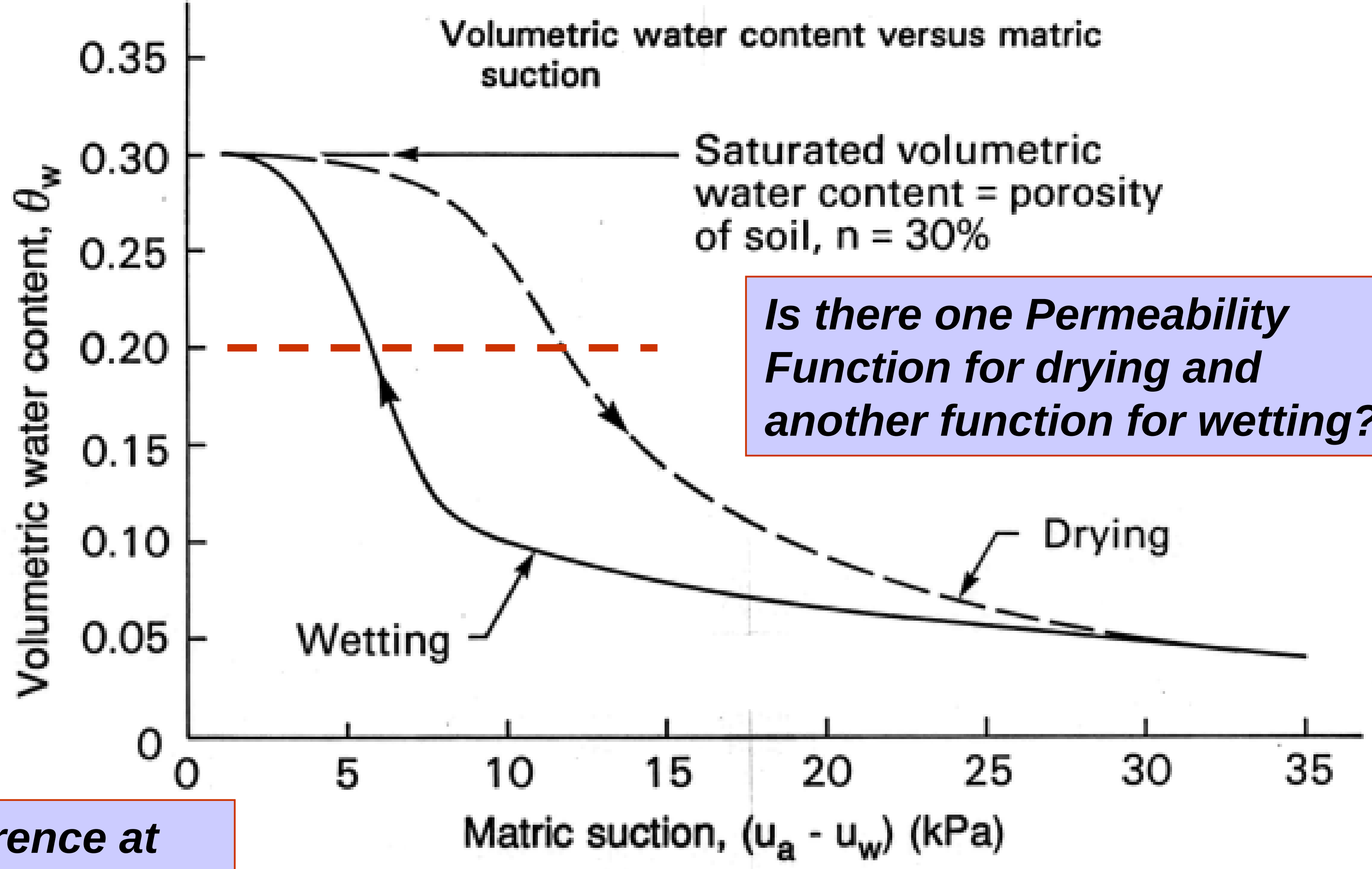


Unsaturated Soil Technology

Forms for the Permeability Function Based on the SWCC

- ***Childs and Collis-George (1950)***
  - ***Summation***
- ***van Genuchten (1980)***
- ***van Genuchten-Burdine (1953, 1980)***
- ***van Genuchten-Maulem (1976, 1980)***
- ***Fredlund, Xing and Huang (1994)***
  - ***Integration form***
- ***Rahardjo and Leong (2000)***
  - ***Closed form; raised SWCC to a power***

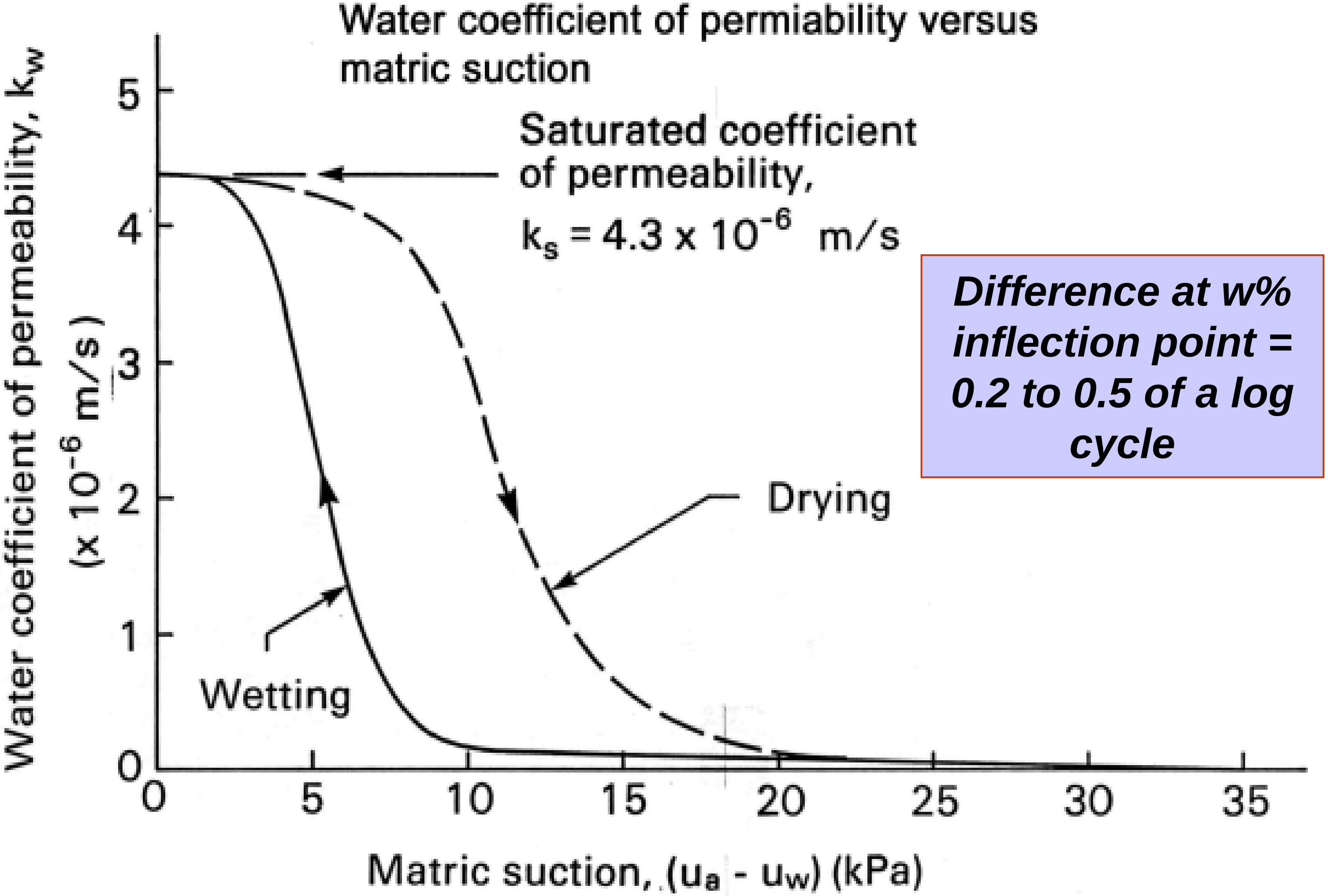
Difficulties with Hysteresis of SWCC and Permeability



Is there one Permeability Function for drying and another function for wetting?

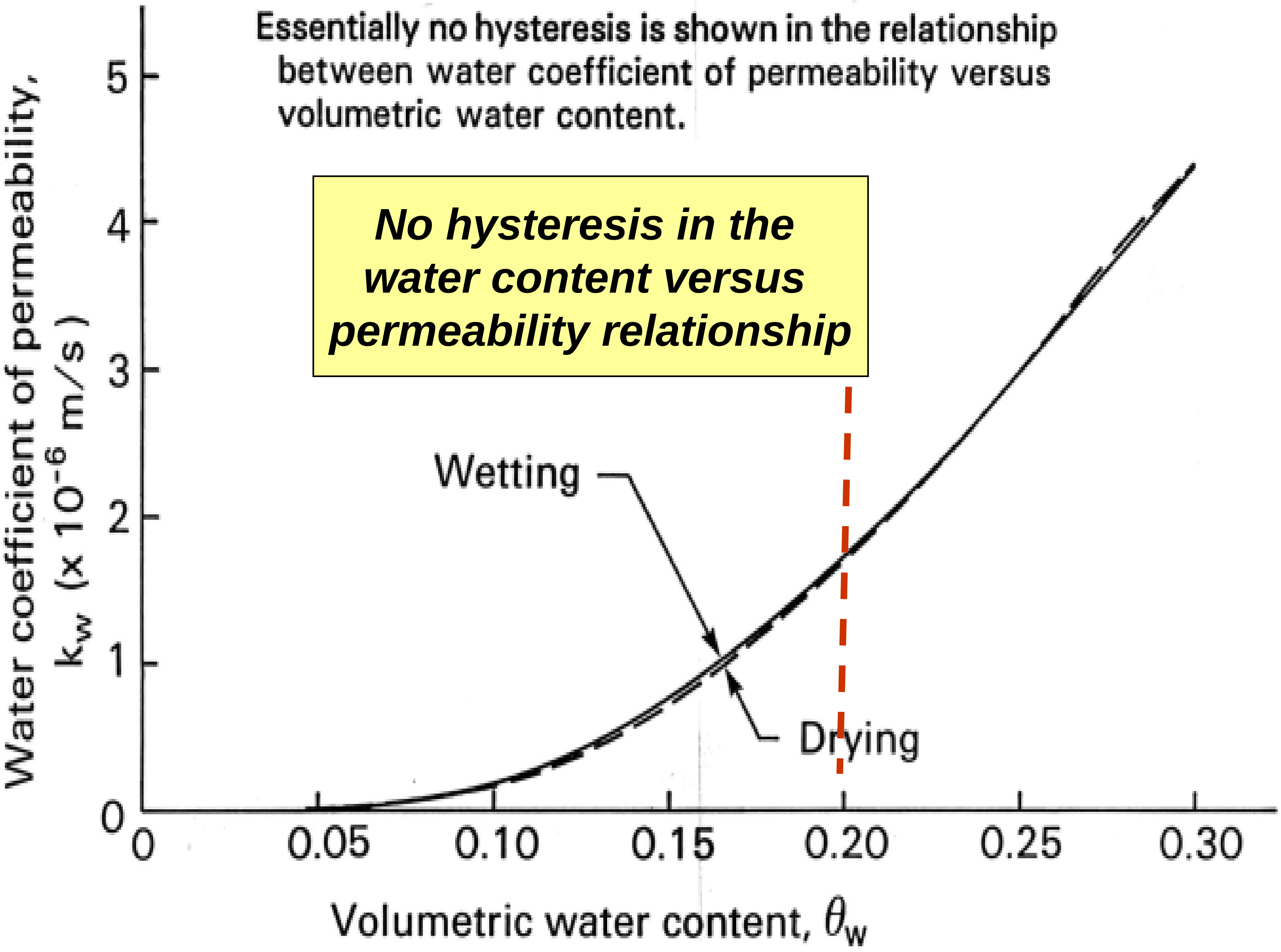
Difference at inflection point = 0.2 to 0.5 of a log cycle

Similar hysteresis forms in the volumetric water content and water coefficient of permeability when plotted as a function of $(u_a - u_w)$ for a naturally deposited sand (from Liakopoulos, 1965a)



Difference at w% inflection point = 0.2 to 0.5 of a log cycle

Similar hysteresis forms in the volumetric water content and water coefficient of permeability when plotted as a function of $(u_a - u_w)$ for a naturally deposited sand (from Liakopoulos, 1965a)



Essentially no hysteresis is shown in the relationship between water coefficient of permeability versus volumetric water content.

No hysteresis in the water content versus permeability relationship

Must Live with Hysteresis in SWCC and Permeability

- Generally it is the Drying (or desorption) curve that is measured or estimated
- Sometimes the Wetting (or Adsorption) curve might be measured or estimated
- The Wetting Curve might be estimated as being shifted to the left by approximately (one half) log cycle at the inflection point
- Independent permeability functions can be determined for both the Drying and the Wetting processes
- Some rigorous permeability models have been proposed with scanning curves