

# ***Soil Mechanics for Unsaturated Soils***

**Delwyn G. Fredlund  
University of Saskatchewan  
Saskatoon, Sask., Canada**

and

**Harianto Rahardjo  
Nanyang Technological University  
Singapore**

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# SOIL MECHANICS FOR UNSATURATED SOILS

D. G. Fredlund  
H. Rahardjo



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# Soil Mechanics for Unsaturated Soils

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## Notes at the bottom of each PowerPoint Slide

- **Notes:**
  - *Different approaches could be taken towards synthesizing the material on the behavior of unsaturated soil mechanics. The approach deemed most desirable was to maintain a relatively close parallel to classical saturated soil mechanics.*
  - *The emphasis was not focused solely on “problematic soils”, but rather on the areas of seepage, shear strength and volume change of all soils with negative pore-water pressures.*
  - *The prepared notes closely follow the material in the textbook. Additional notes have been prepared on new and important subjects that have emerged during the years subsequent to 1993.*
  - *The textbook appears to have filled an important role in centralizing our understanding of the behavior of unsaturated soils from the standpoint of two independent stress state variables.*
  - **Additional notes:**



# ***Format For Each Constitutive Behavior***

- ***Constitutive Relationship*** (e.g., seepage, shear strength, volume change)
  - ***Theory*** associated with the Constitutive Relationship
  - ***Measurement*** of the associated unsaturated soil properties
  - ***Estimation*** of the unsaturated soil property functions (***through use of SWCC***)
  - ***Application*** of the constitutive relationship to practical engineering problems



# ***Important Objective of This Course***

- ***To teach Geotechnical Engineers to think the way the **Unsaturated Soil** behaves***
- ***The **Physics** must be correct***
- ***Many behavioral aspects related to **Unsaturated Soils** are the opposite to saturated soil behavior (e.g., hydraulic conductivity of sands capillary barriers)***

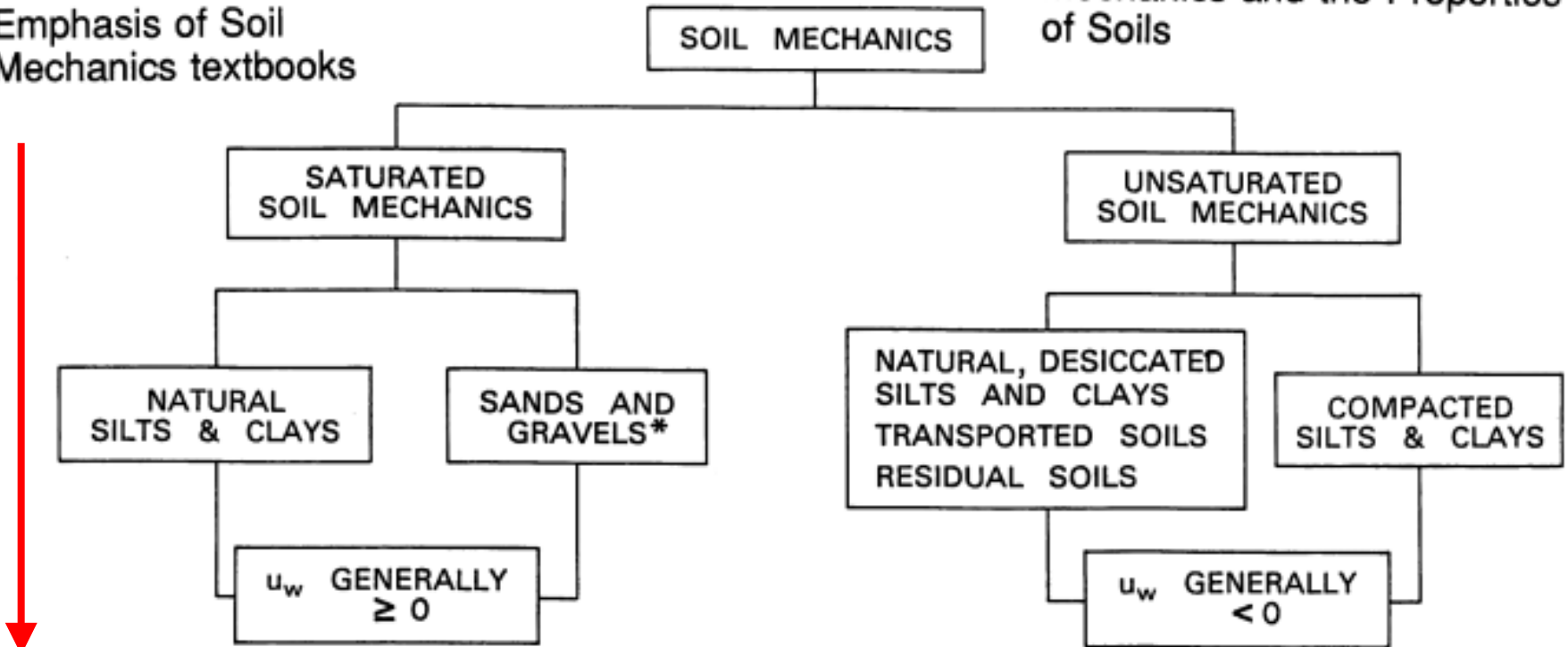


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# Categorization of Soil Mechanics

Emphasis of Soil Mechanics textbooks

- Combination of Engineering Mechanics and the Properties of Soils



\*may be saturated or dry

Two-Phase Behavior

$$(\sigma - u_w) \text{ or } (\sigma - u_a)$$

More than Two-Phases

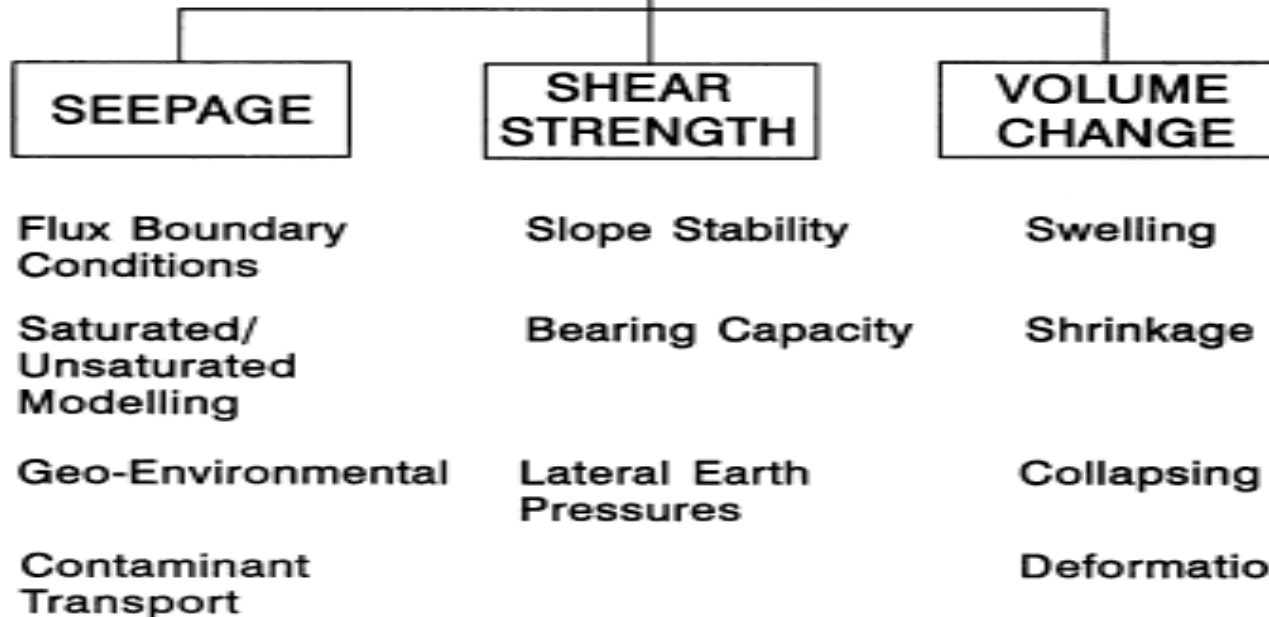
$$(\sigma - u_a) \text{ and } (u_a - u_w)$$



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# CATEGORIES OF UNSATURATED SOIL MECHANICS PROBLEMS

Description of STRESS STATE VARIABLES  
( $\sigma - u_a$ ) and ( $u_a - u_w$ )



*Based on Constitutive Behaviour*



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## CLIMATIC CHARACTERISTICS

*Moisture flux  
is an aspect  
originally  
omitted from  
Soil Mechanics*

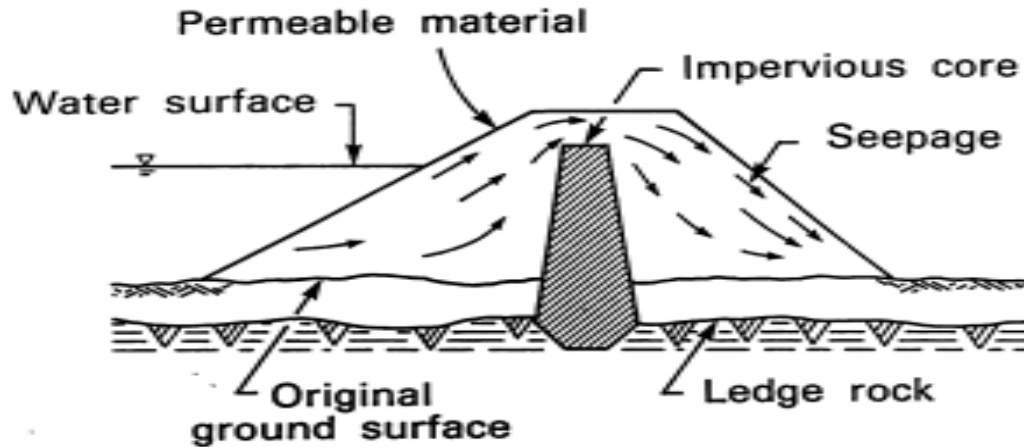
*Coupled mass-  
transport & thermal  
analysis*

- Approximately 33% of the earth's surface is Arid or Semi-arid
- Climate gives rise to a continuously changing FLUX Boundary Condition at the ground surface
- An UPWARD Flux is produced as water is removed through;
  - i.) Evaporation of water from the soil surface,
  - ii.) Evapo-transpiration from vegetative cover
- The result is a Drying, Cracking and Desaturation of the soil
- ACTUAL EVAPORATIVE FLUX depends on the pore-water stress state and is difficult to predict
- POTENTIAL EVAPORATIVE FLUX is from a water surface and depends primarily on temperature (e.g., Thornthwaite Moisture Index)
- A DOWNWARD Flux is produced by rain and other forms of precipitation. The result is an attempt to saturate the soil
- The DIFFERENCE between the UPWARD and the DOWNWARD Flux largely dictates the location of the water table, and therefore the location of the negative pore-water pressure zone



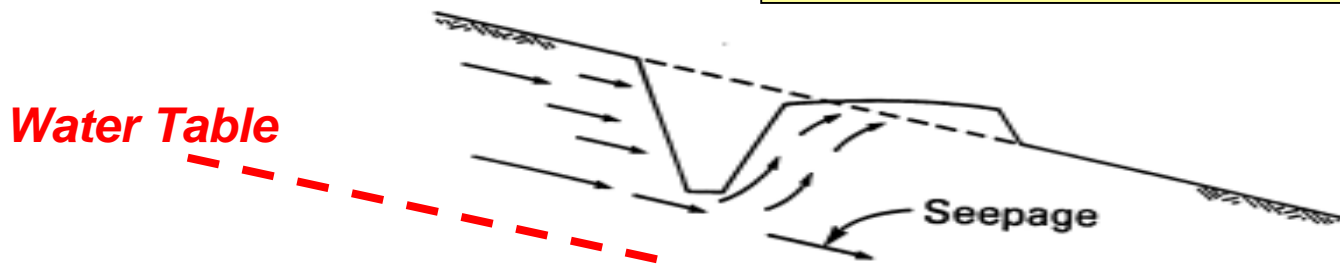
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ILLUSTRATIONS OF UNSATURATED FLOW PHENOMENA IN THE FIELD (from Hogentogler and Barber, 1941)



Syphon Effect on an Earth Dam with a Core-wall

**CAPILLARY FLOW PHENOMENA**



Intercept Ditch for a Highway on a Side-hill Location



# EXAMPLES OF PROBLEMS REQUIRING AN UNDERSTANDING OF UNSATURATED SOIL MECHANICS

**Compacted  
Soils**

- 1.) THE DESIGN, CONSTRUCTION AND OPERATION OF MAN-MADE STRUCTURES SUCH AS A DAM
  - a.) During construction of the fill
  - b.) During the filling of the reservoir **and Rapid Drawdown**
  - c.) After Steady State seepage conditions are established, but environmental changes occur

**Infiltration**

- 2.) NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES
  - Natural slopes often fail after being subjected to heavy precipitation for a long time
  - The slip surface may be relatively shallow, passing through the unsaturated zone
- 3.) MOUNDING BELOW WASTE RETENTION PONDS
  - Wastes are often stored on the ground surface where the water table is deep
  - Contaminant may move to the groundwater even if the soil remains unsaturated
- 4.) STABILITY OF VERTICAL OR NEAR VERTICAL EXCAVATIONS
  - Excavation back-slopes often fail some time after excavation
  - Negative pore-water pressures are dissipated due to infiltration



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## 5.) LATERAL EARTH PRESSURES

- Dry, clayey soils are often used as backfill
- Can exert high lateral pressures as they become wet

## 6.) BEARING CAPACITY FOR SHALLOW FOUNDATIONS

- Foundation design is generally based on unconfined compression tests on soils from above the ground-water table
- Analysis assumes that negative pore-water pressures are maintained with time

## 7.) GROUND MOVEMENTS INVOLVING EXPANSIVE SOILS

- Light structures suffer distress as a result of environmental changes or man-made effects

## 8.) COLLAPSING SOILS

- There may be volume change or a loss of shear strength resulting from pore-water pressure changes

**Problematic  
Soils**

## 9.) FLOW THROUGH RESIDUAL SOILS

- Water infiltration into Residual soils often results in instability of slopes

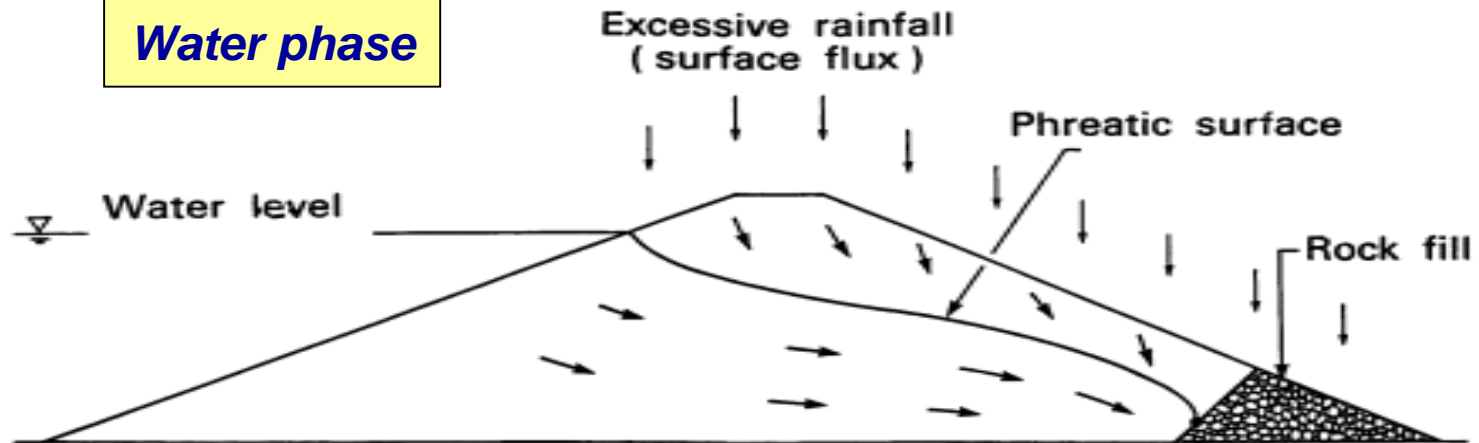


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# ENVIRONMENTAL EFFECTS DURING THE OPERATION OF THE RESERVOIR

The effect of rainfall on steady state flow through a dam

**Water phase**



**SOME RELEVANT QUESTIONS MIGHT BE ASKED AS STEADY STATE CONDITIONS ARE ESTABLISHED**

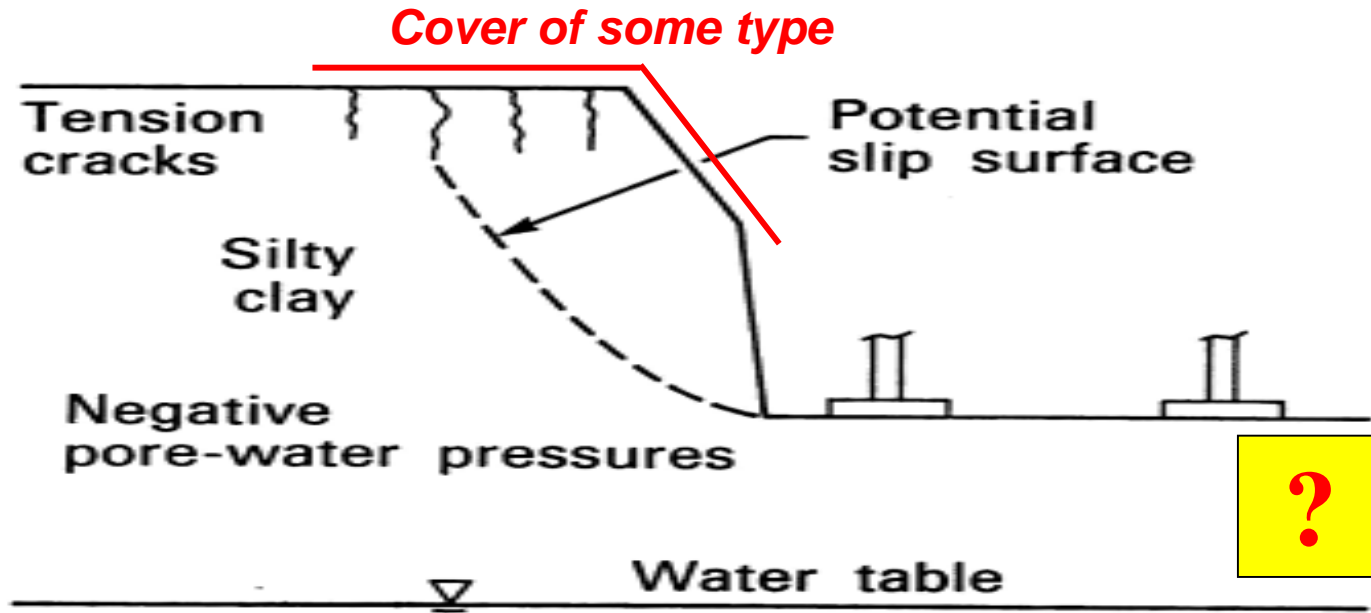
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# STABILITY OF VERTICAL OR NEAR VERTICAL EXCAVATIONS

An example showing potential instability of a near vertical excavation used during the construction of a foundation



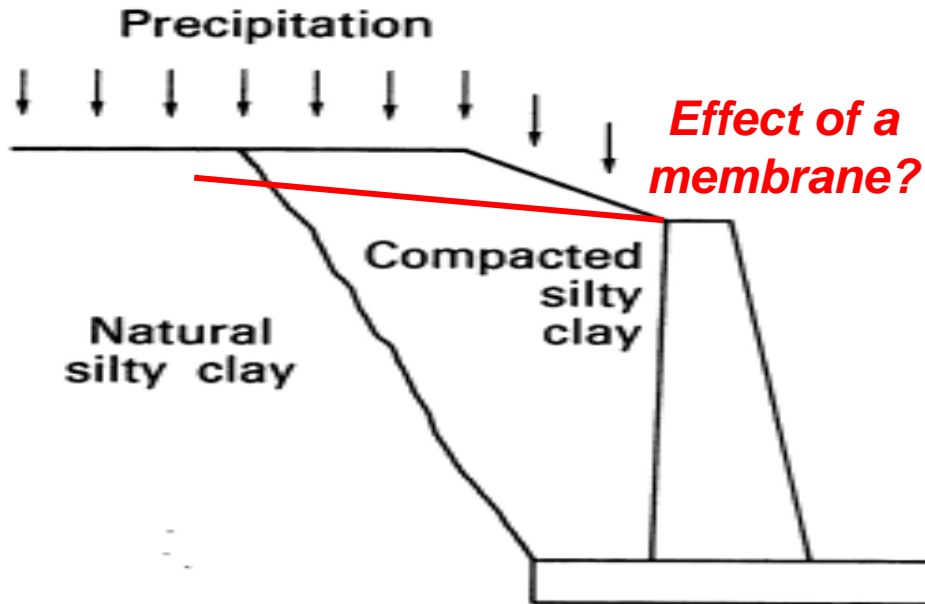
**SOME RELEVANT QUESTIONS THAT MIGHT BE ASKED ARE REFERRED TO THE STABILITY OF THE EXCAVATION SLOPES**



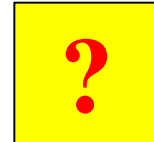
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# LATERAL EARTH PRESSURES

Lateral earth pressures against a retaining wall as water infiltrates the compacted backfill



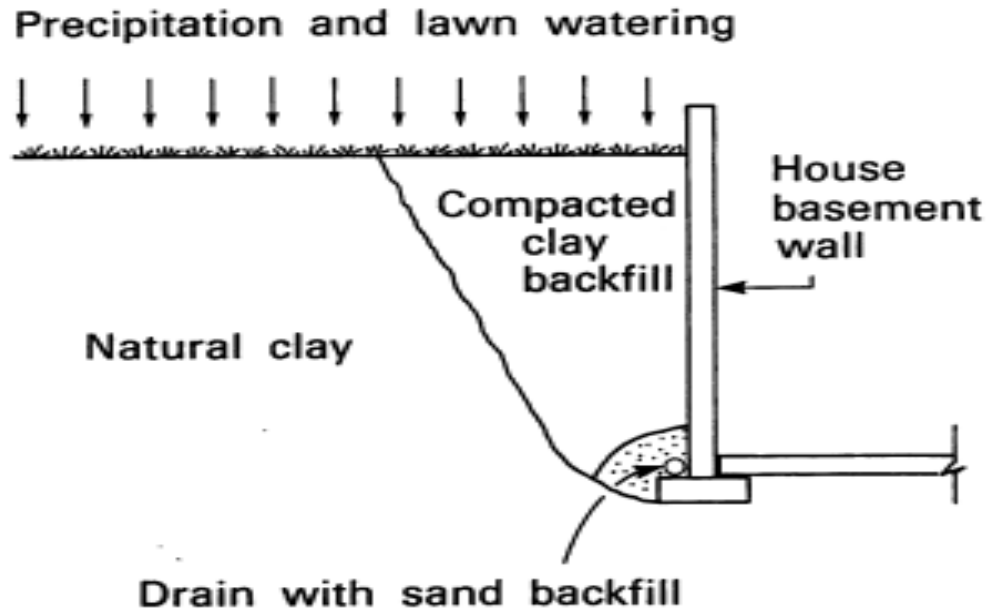
SOME RELEVANT QUESTIONS MIGHT BE ASKED  
PERTAINING TO LATERAL EARTH PRESSURES



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# LATERAL EARTH PRESSURES AGAINST WALLS

Example of lateral earth pressures generated subsequent to backfilling with dry soils



Lateral earth pressure against a house basement wall



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# ***IS THERE A NEED FOR UNSATURATED SOIL MECHANICS?***

**YES!**

***The Geotechnical Engineer has the greatest potential to assist the public in circumventing problems associated with Unsaturated Soils***

There is need for an APPROPRIATE TECHNOLOGY which is:

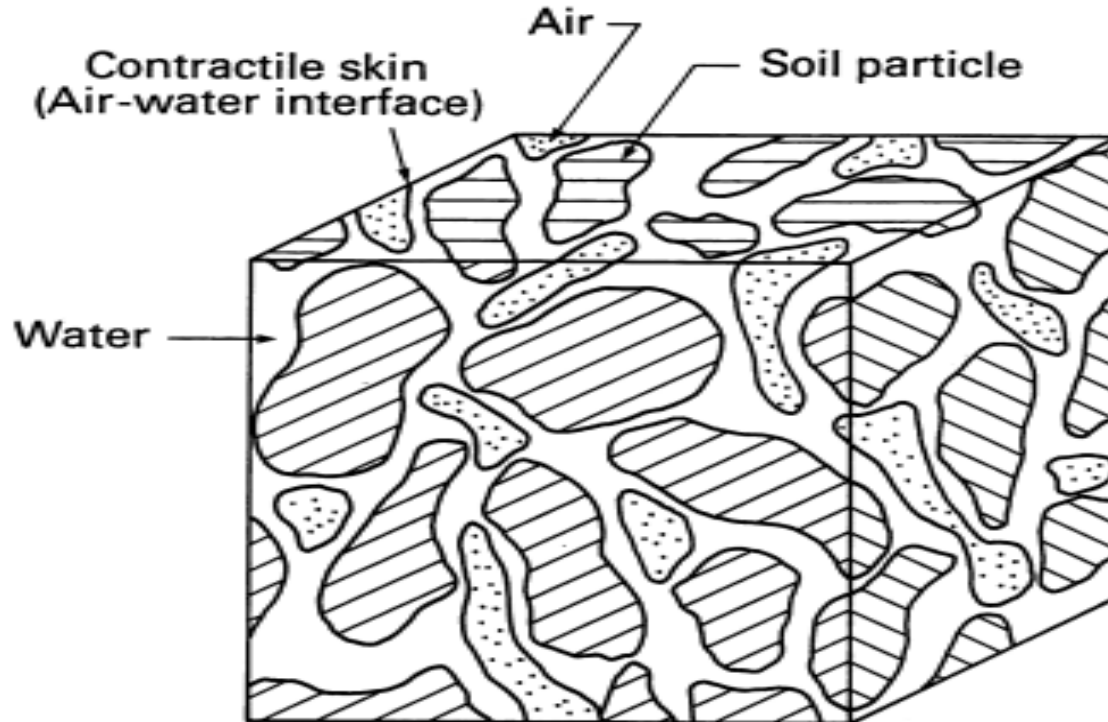
- a.) Practical
- b.) Not too costly to employ
- c.) Has a sound theoretical basis, and
- d.) Runs parallel in concept to Saturated Soil Mechanics

***Concern for the environment and advances in computing power greatly assisted in the promotion of **Unsaturated Soil Mechanics*****



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# AN UNSATURATED SOIL ELEMENT WITH A CONTINUOUS AIR PHASE



## PHASES OF AN UNSATURATED SOIL

An Unsaturated Soil is postulated to have 4 phases

- 1.) Solids
- 2.) Air
- 3.) Water
- 4.) Contractile Skin (Air-Water Interface)



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# TERMINOLOGY AND DEFINITIONS

Will use a MACROSCOPIC, PHENOMENOLOGICAL approach to Unsaturated Soil behavior

Additional terms required from Continuum Mechanics

- 1.) STATE - Non material variable required for the characterization of a system
- 2.) STRESS STATE VARIABLE - Variables required for the characterization of the stress state
- 3.) DEFORMATION STATE VARIABLES - Variables required for the characterization of the deformation conditions or deviations from an initial state
- 4.) CONSTITUTIVE RELATIONS - Single-valued equations expressing the relationship between state variables

Fung (1969) - are single-valued expressions which relate one state variable to one or more other state variables. They always incorporate the material properties

Examples are:

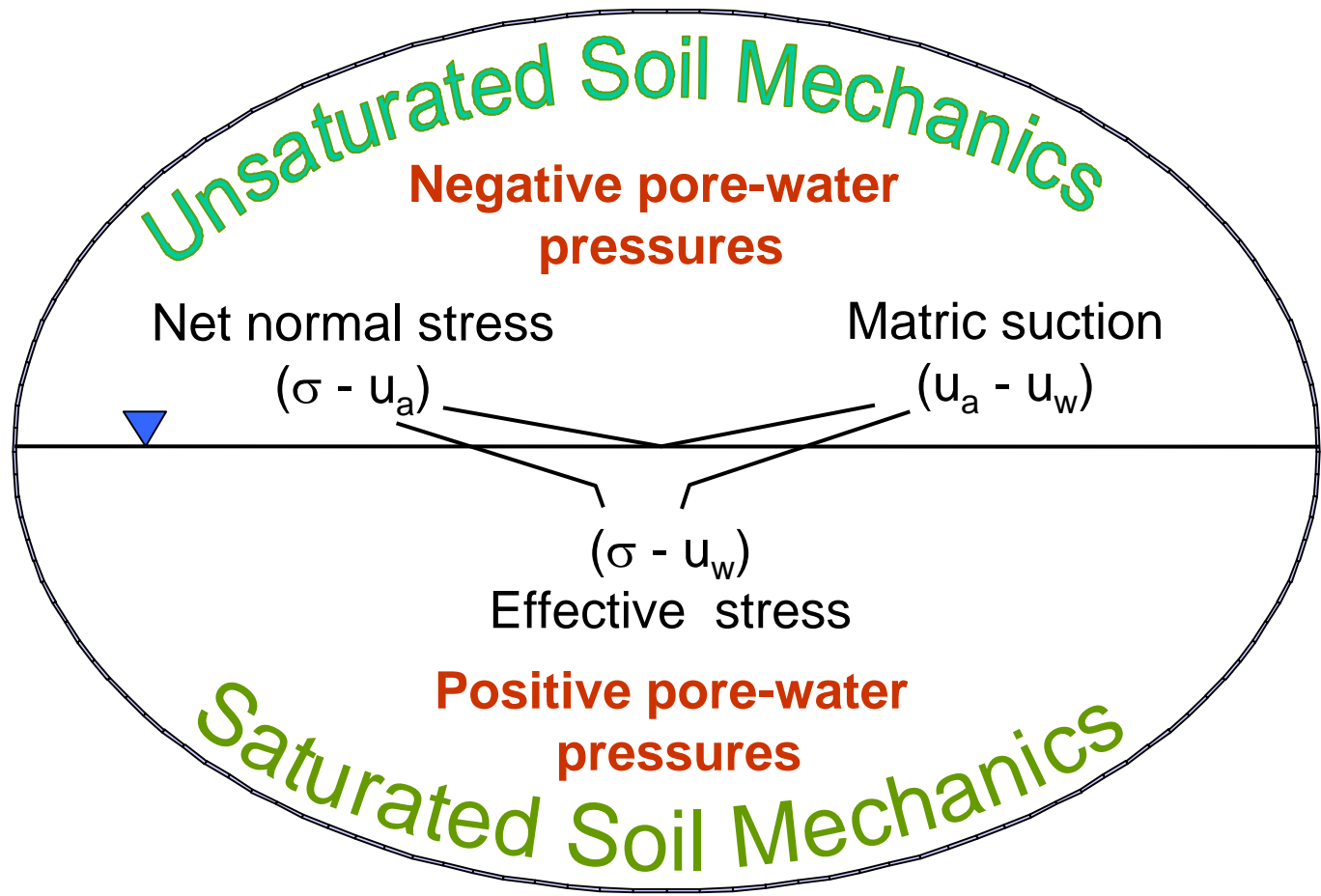
- Stress versus Strain Relations (  $E$ ,  $\mu$  )
- Ideal Gas Law, relates pressure to density and temperature. Gas constant is the property
- Shear Strength equation (  $c'$ ,  $\phi'$ ,  $\phi^b$  )
- Pore Pressure Parameter equation (  $S$ ,  $m_v$ ,  $\beta_w$  )



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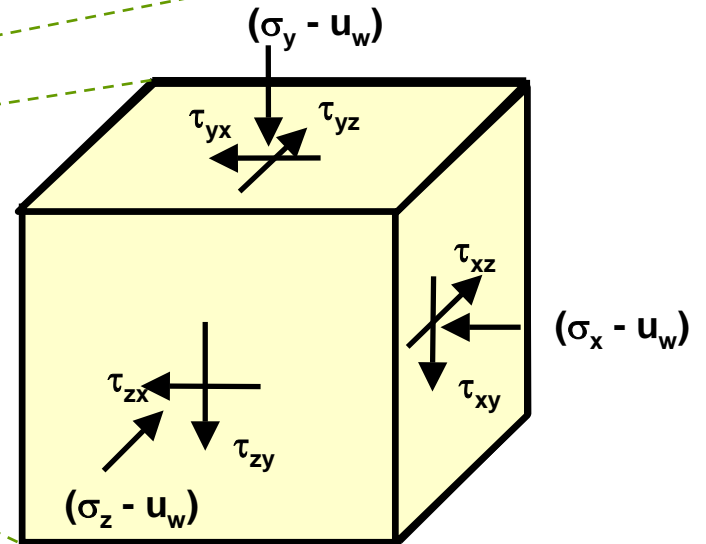
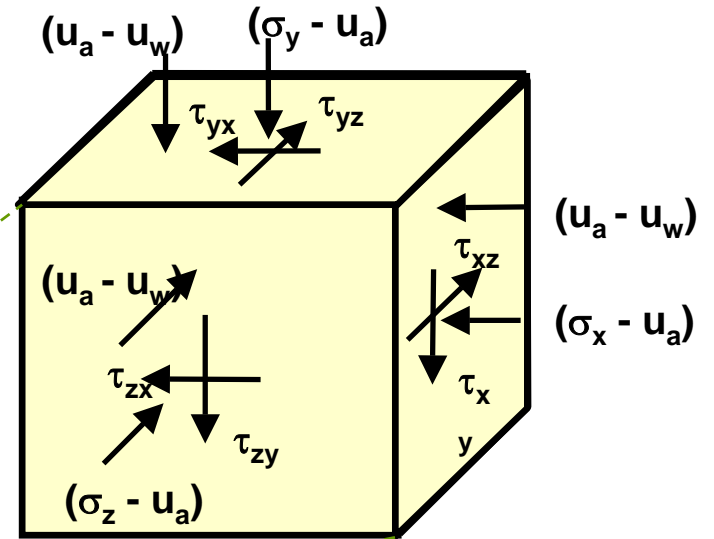
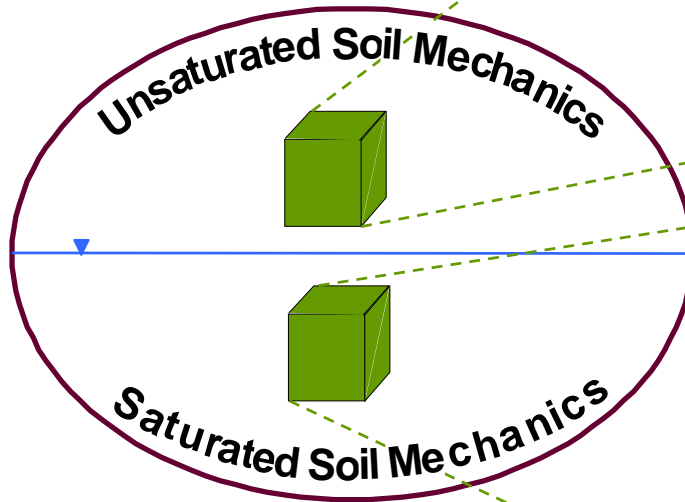
# Categorization of Soil Mechanics Based on Stress State Variables

Visualization  
of the World  
of Saturated-  
Unsaturated  
Soil  
Mechanics

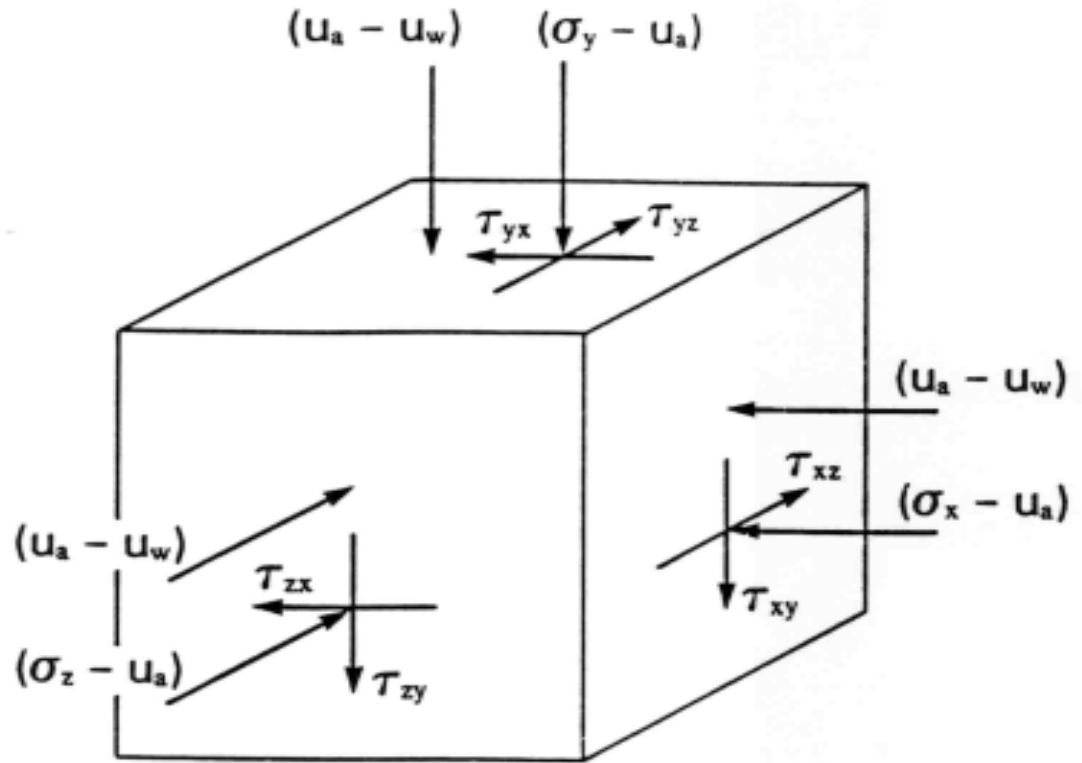
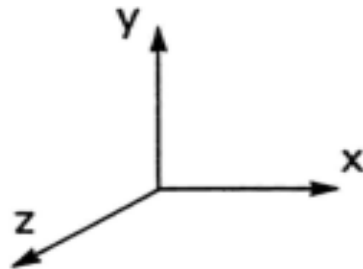


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**Stress State  
at a Point in an  
Unsaturated Soil  
and in a  
Saturated Soil**



**Summary of the description of the Stress State for an Unsaturated Soil Element with the Preferred Stress State Variables**



The stress state variables for an unsaturated soil using the combination of  $(\sigma - u_a)$  and  $(u_a - u_w)$



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# Stress State Variables (Unsaturated Soils)

## Net Total Stress Tensor

- **Stress Tensors form the basis for a Science because we live in a 3-D Cartesian coordinate world**

$$\begin{bmatrix} (\sigma_x - u_a) & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & (\sigma_y - u_a) & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & (\sigma_z - u_a) \\ (u_a - u_w) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (u_a - u_w) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & (u_a - u_w) \end{bmatrix}$$

## Matric Suction Stress Tensor



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## Variations in Stress State Description

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w)$$

$\sigma'$  = effective stress

$\chi$  = parameter related to saturation

$$\sigma^{*}_{ij} = \sigma_{ij} - [S u_w + (1 - S) u_a] \delta_{ij}$$

$\sigma_{ij}$  = total stress tensor,

$\delta_{ij}$  = Kroneker delta or substitution tensor,

$\sigma^{*}_{ij}$  = Bishop's soil skeleton stress (Jommi 2000)

*Above proposed equations are constitutive relations*

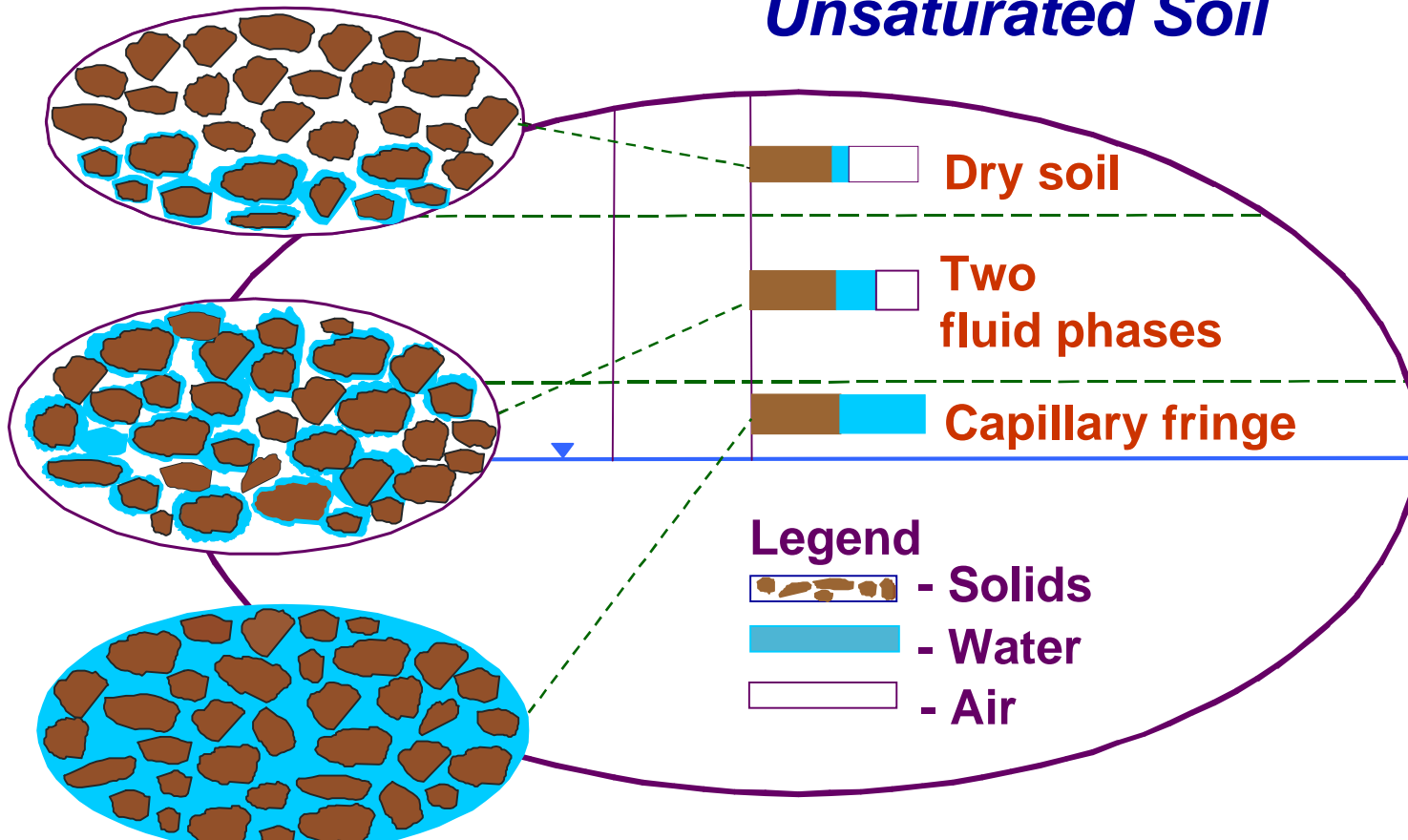


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# Separation of the Zones above the Water Table

## Unsaturated Soil



Occluded air bubbles

## Saturated Soil



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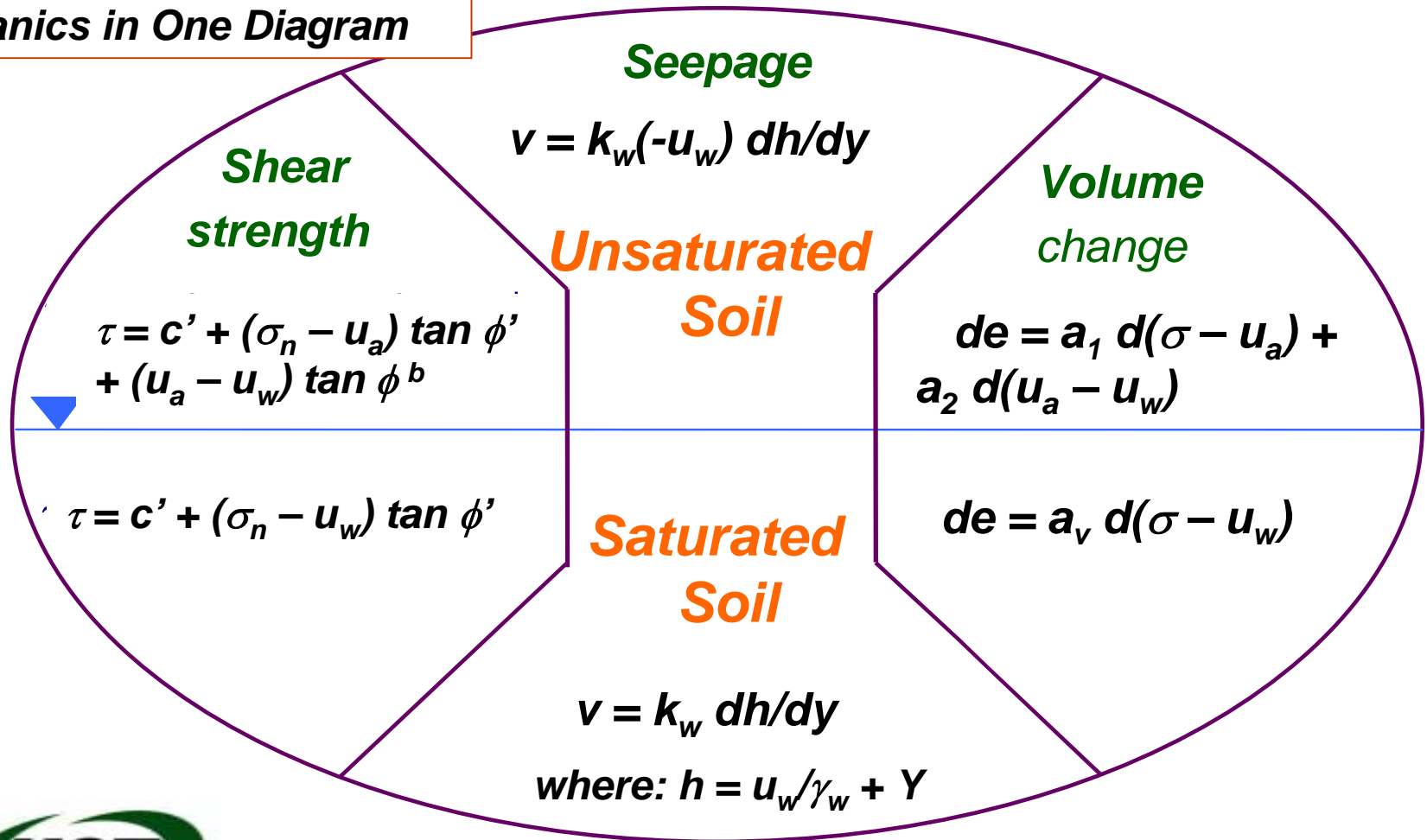
# **Constitutive Relations for Classic Problems in Unsaturated Soils Mechanics**

- ❑ ***Empirical, semi-empirical and theoretical relationships are proposed and verified***
  - ***Volume change (Stress versus Strain)***
  - ***Shear strength (Stress versus Stress)***
  - ***Flow (Velocity versus Stress)***
- ❑ ***Demanding laboratory experiments***
  - ***Careful experiments required for uniqueness studies***
  - ***May alter test procedures for economic reasons***



# Constitutive Relations for Saturated/Unsaturated Soil Mechanics

Fundamentals of Soil Mechanics in One Diagram



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## **Selected Reading Materials**

- **Fredlund, D.G. (2000). “The 1999 R.M. Hardy Lecture: The Implementation of Unsaturated Soil Mechanics into Geotechnical Engineering”, *Canadian Geotechnical Journal*, 37 (5), 963-986**
  - **Fredlund, D.G. (2002). “Use of the Soil-Water Characteristic Curve in the Implementation of Unsaturated Soil Mechanics”, *Keynote Address, March 10-13, UNSAT 2002, Recife, Brazil, Vol. 3.***
  - **Ha, T.V. Pham, and Fredlund, D. G. (2002). “The Application of Dynamic Programming to Slope Stability Analysis”, *Canadian Geotechnical Journal*, pp. 830-847.**
- 
- **Gitirana, Gilson, and Fredlund, D. G., (2003). “From Experimental Evidence Towards the Assessment of Weather-Related Railway Embankment Hazards” *Proc. Of the Conf. on “From Experimental Evidence Towards Numerical Modelling of Unsaturated Soils”, Sept. 18-19.***



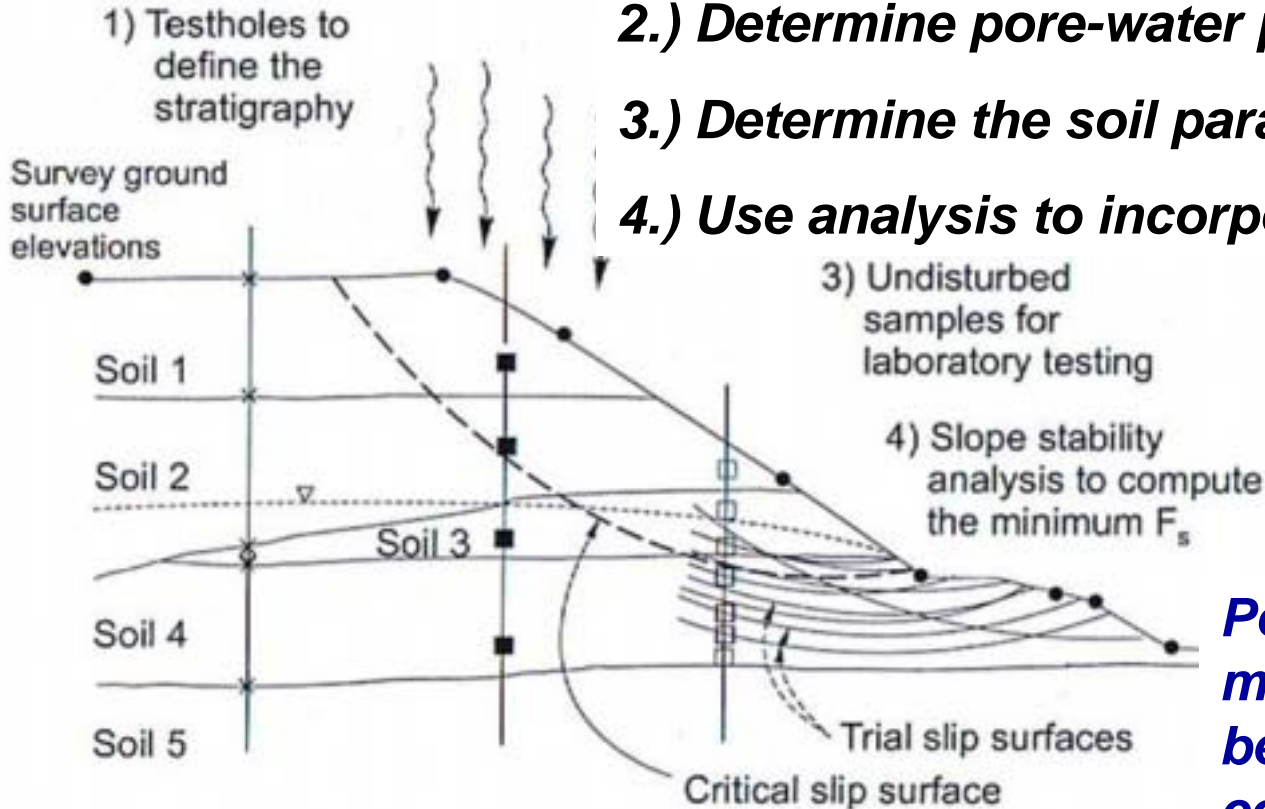
# Review of Procedure Typical to **Saturated Soil** Mechanics

1.) Define geometry of the problem (Surface & Stratigraphy)

2.) Determine pore-water pressures

3.) Determine the soil parameters ( $c'$  &  $\phi'$ )

4.) Use analysis to incorporate total stresses



**Pore-water pressures must be assessed because behavior is controlled by  $(\sigma - u_w)$**



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## ***Additional Features to Accommodate Rainfall Induced Landslides***

- ***Slope must be visualized as a **transient** analysis on a **saturated-unsaturated soil profile*****
- ***Unsaturated soil has **water storage capabilities*****
- ***Unsaturated soil has highly varying **coefficient of permeability and infiltration conditions*****
- *****Shear strength of the unsaturated soil must be taken into account*****
- ***Actual (or real-time) **flux moisture conditions** (i.e., rainfall) must be taken into account***
- *****Calculation of factor of safety** must account for unsaturated soil behavior***



## Analysis Can Be Viewed as a **Combination** of the Following Elements of Physics

- **Saturated-unsaturated seepage analysis** (Permeability and Storage)

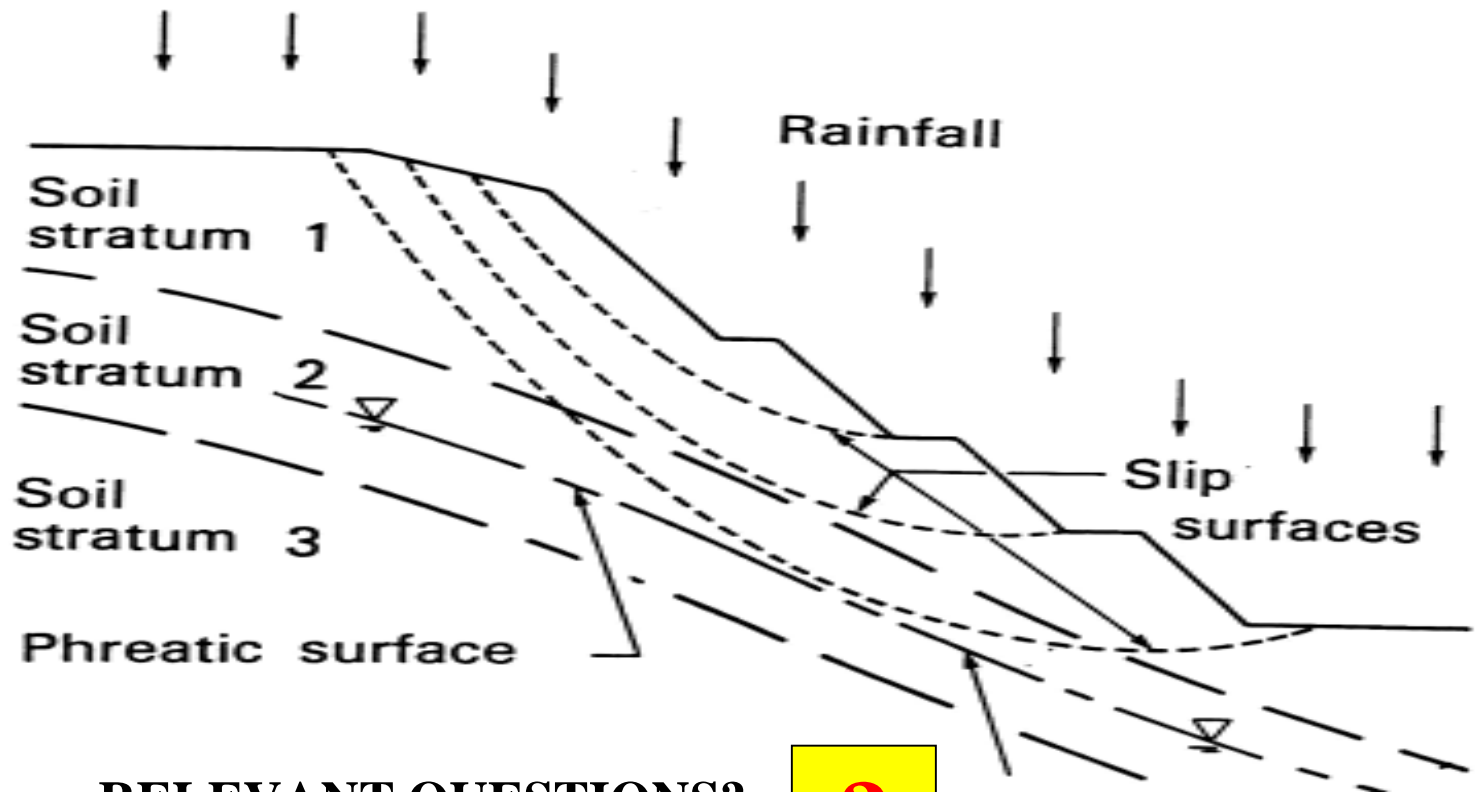
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- **Stress analysis** for the shear and normal forces (Method of Slices or Stress Analysis)
- **Shear strength** evaluation of the unsaturated soil (angle  $\phi^b$ )
- Evaluation of **surface moisture flux** conditions (Percentage of  $k_{sat}$ )
- Calculation of **factor of safety,  $F_s$**



# NATURAL SLOPES SUBJECTED TO ENVIRONMENTAL CHANGES

An example of the effect of excavations on a natural slope subjected to environmental changes



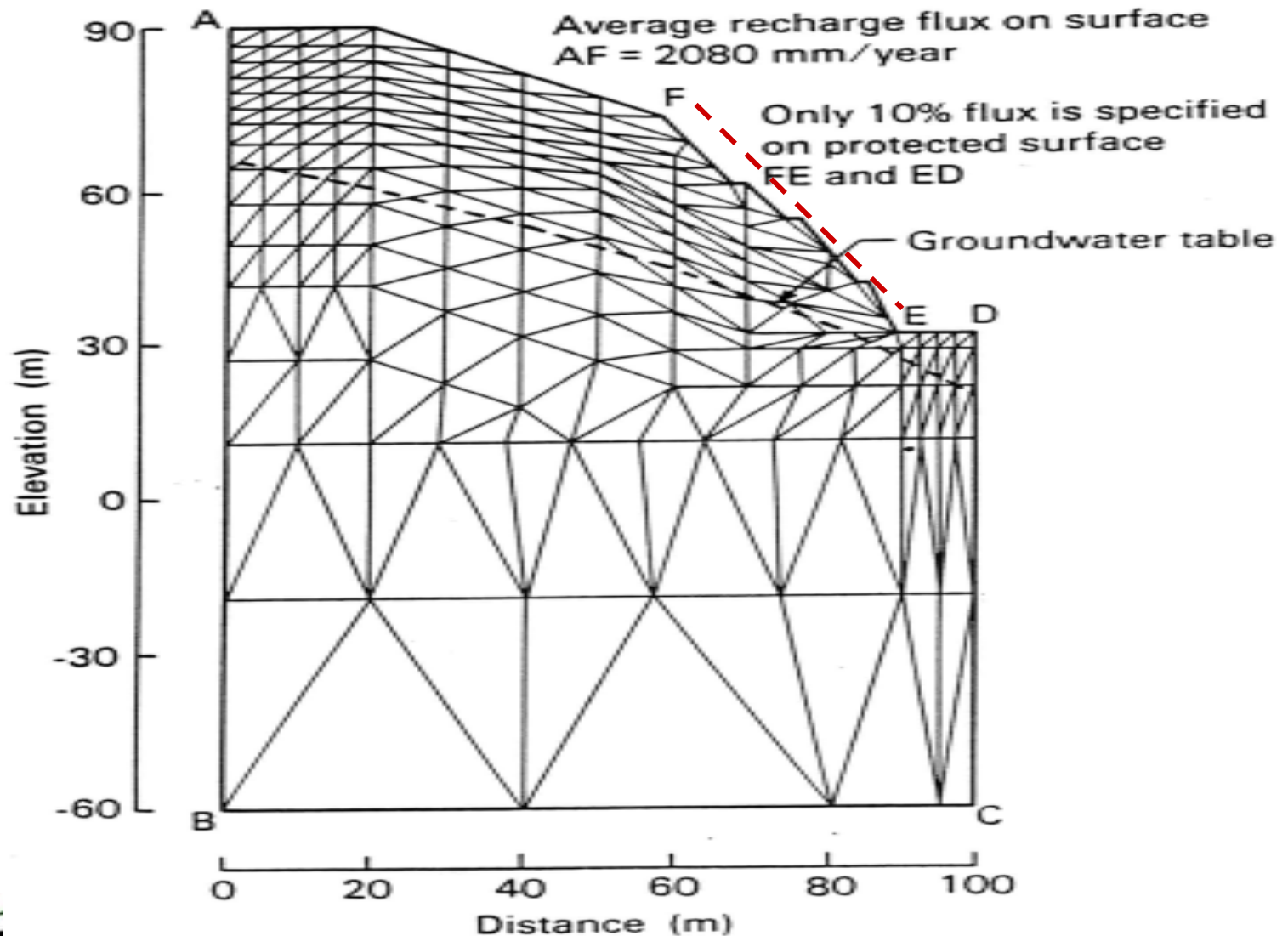
RELEVANT QUESTIONS?

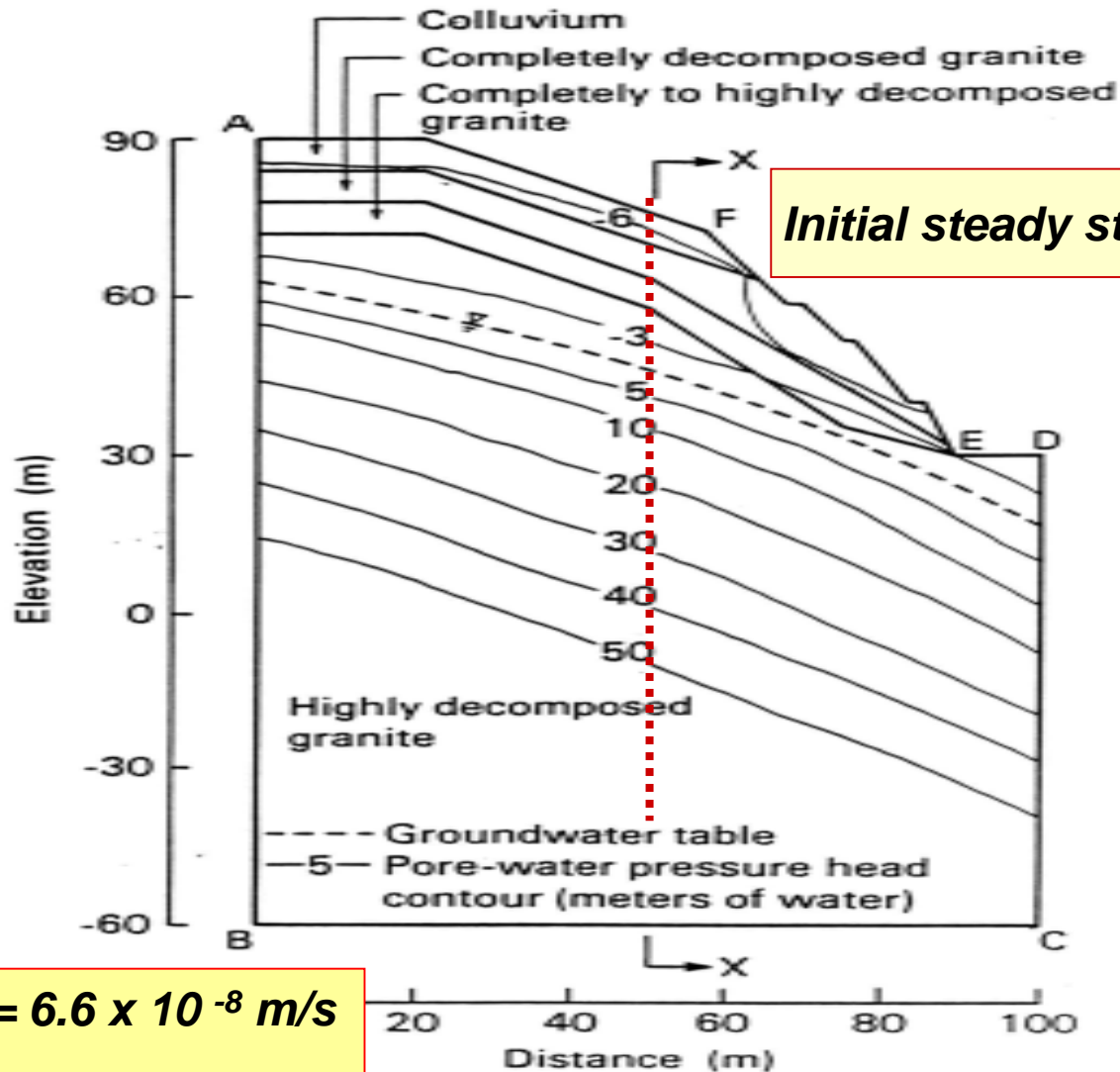


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## Initial conditions for the seepage analysis





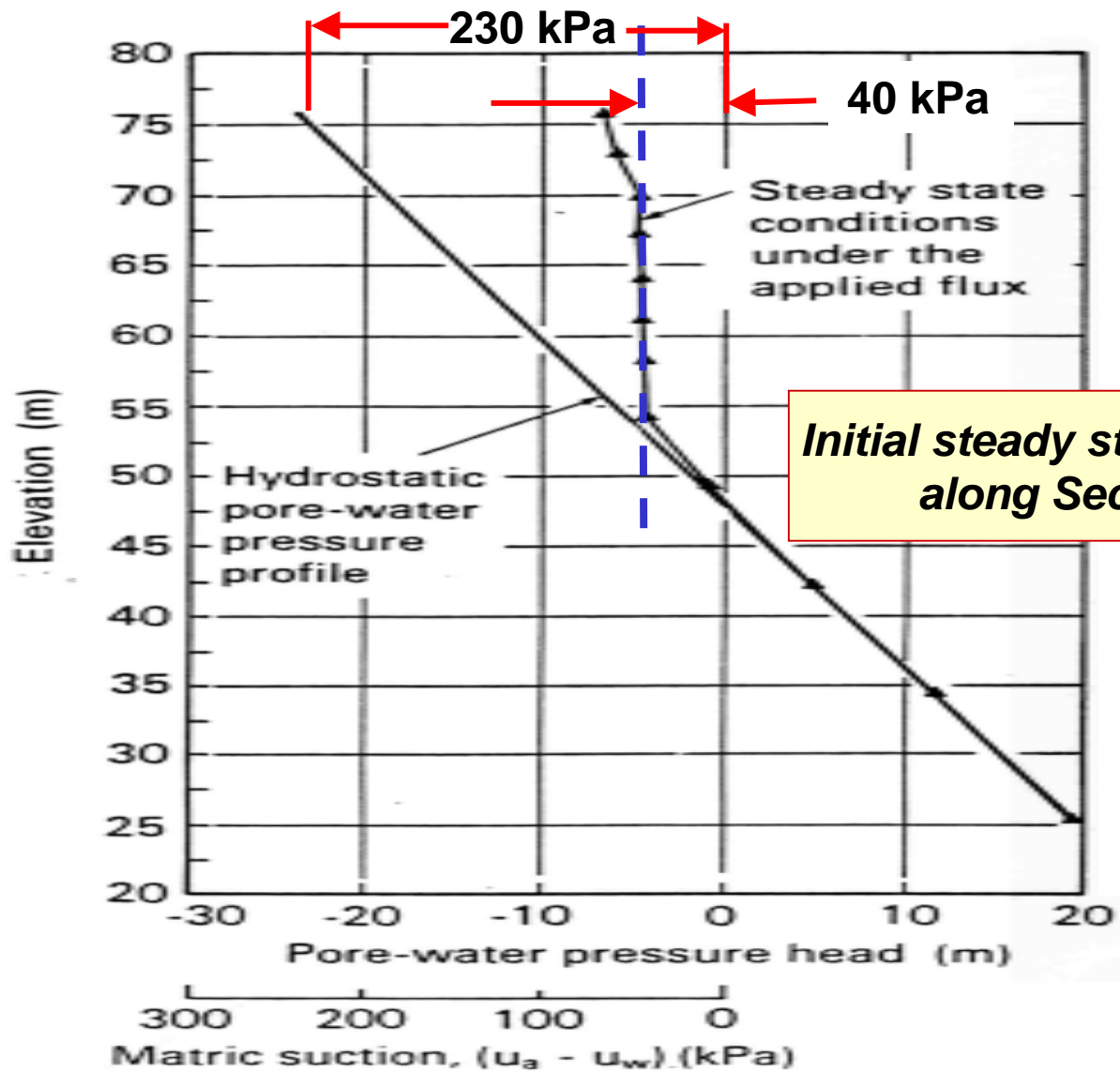
$2080 \text{ mm/year} = 6.6 \times 10^{-8} \text{ m/s}$

Quite an extreme condition



Initial groundwater condition and pore-water pressure head contours

Geoteknology



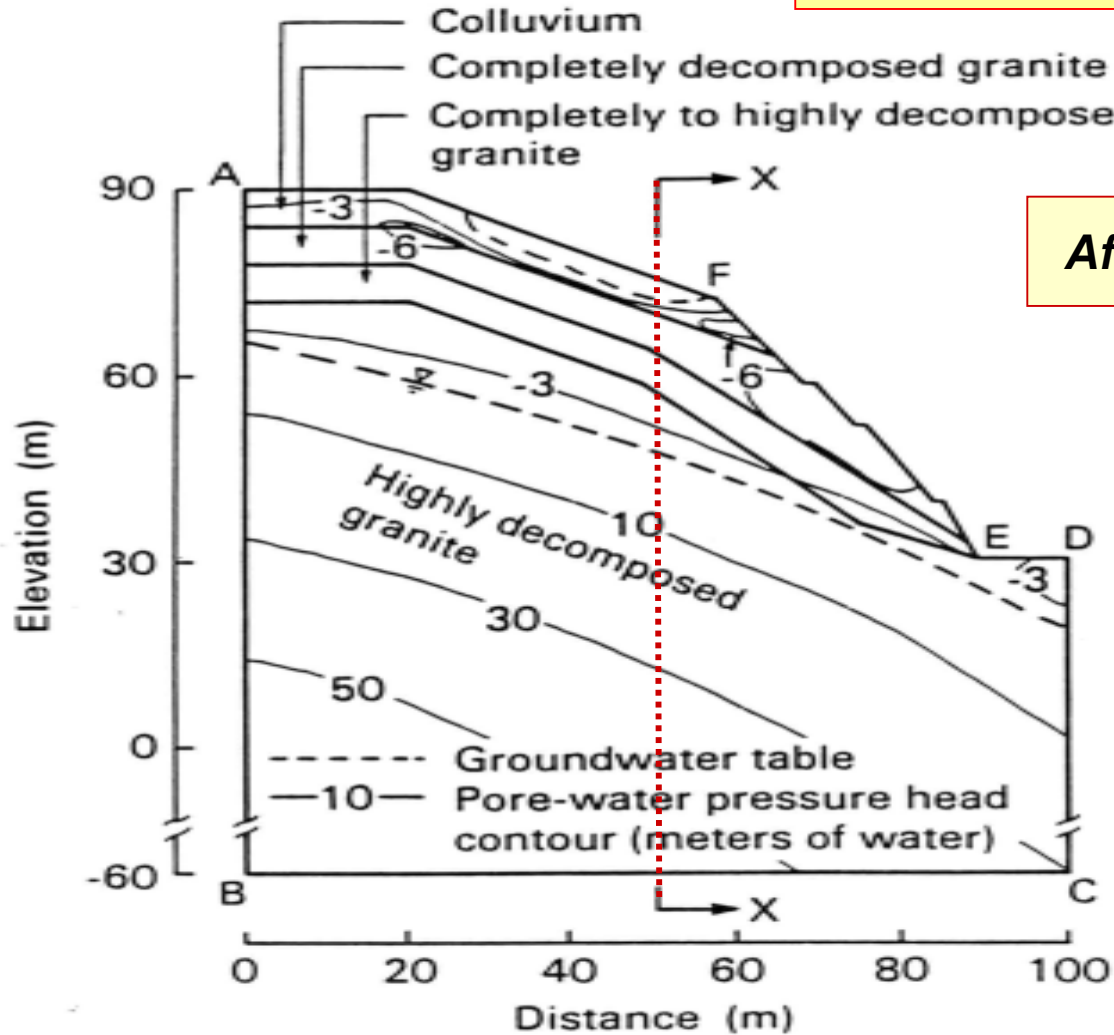
Matric suction profiles for section X-X under steady state flux conditions



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# Seepage and slope stability results under high intensity rainfall conditions

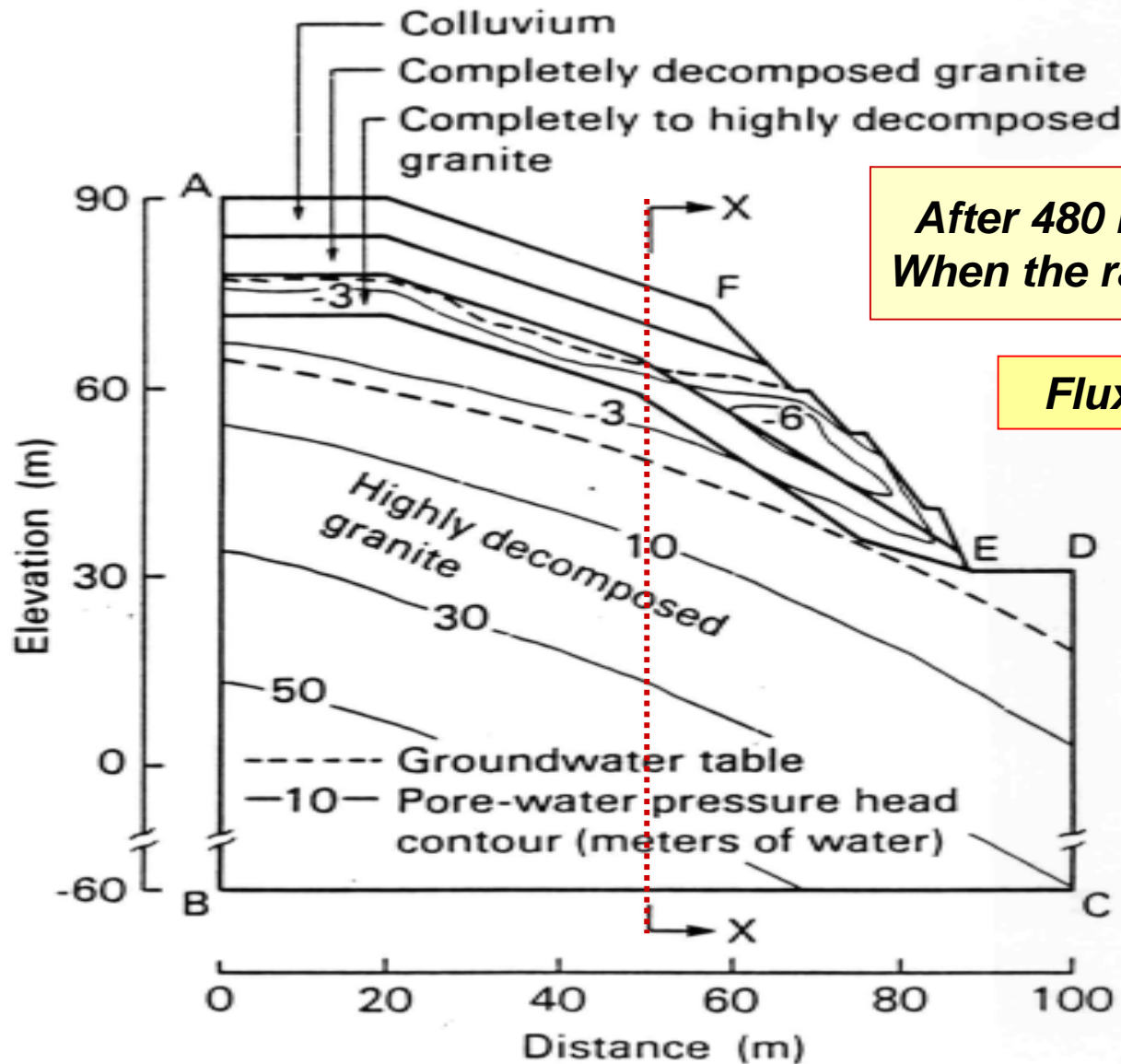
$Flux = 1.3 \times 10^{-5} \text{ m/s}$



*After 120 minutes*

Elapsed time = 120 min.





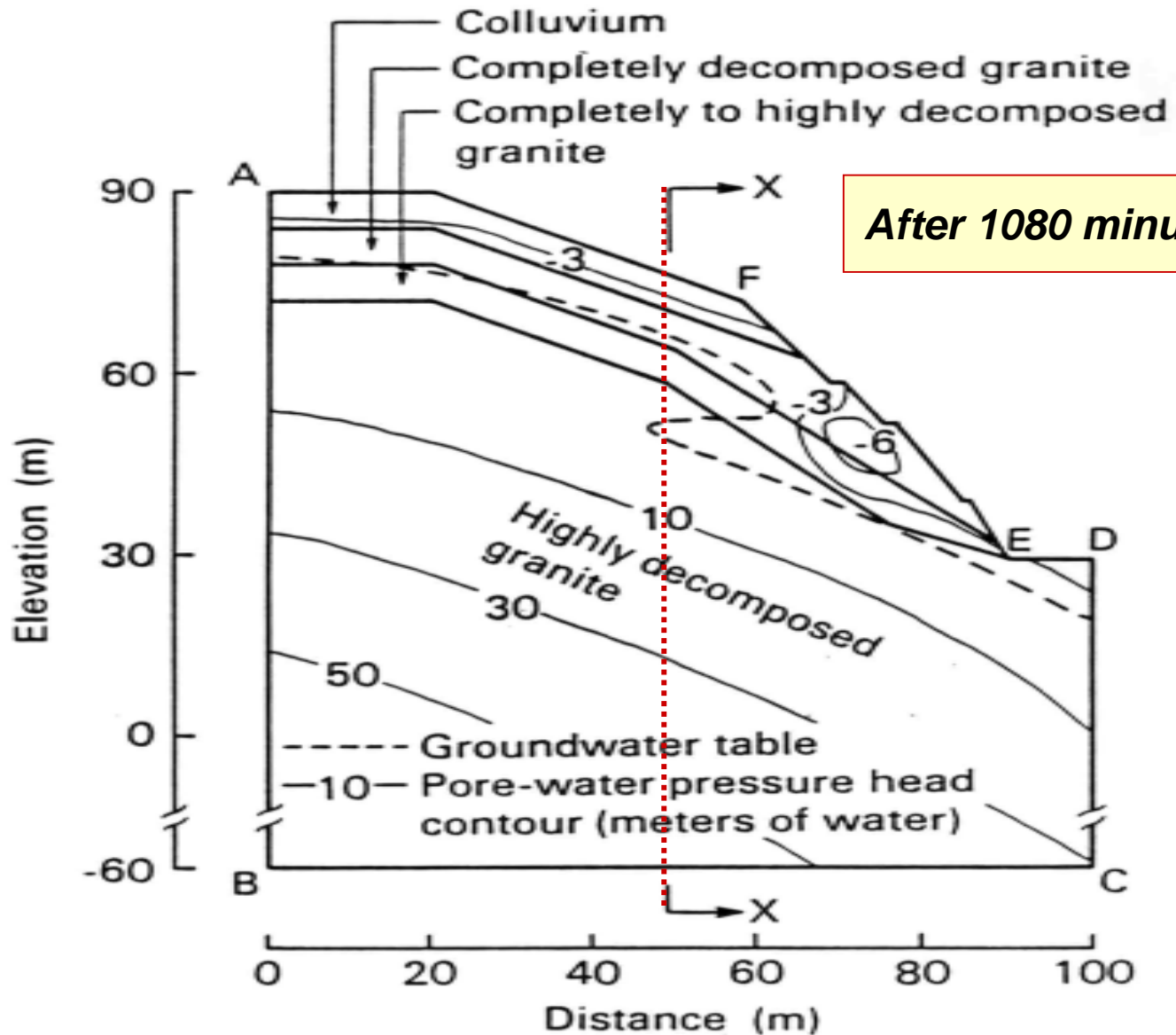
**After 480 minutes  
When the rain stops**

**Flux = 0.0**

Elapsed time = 480 min.



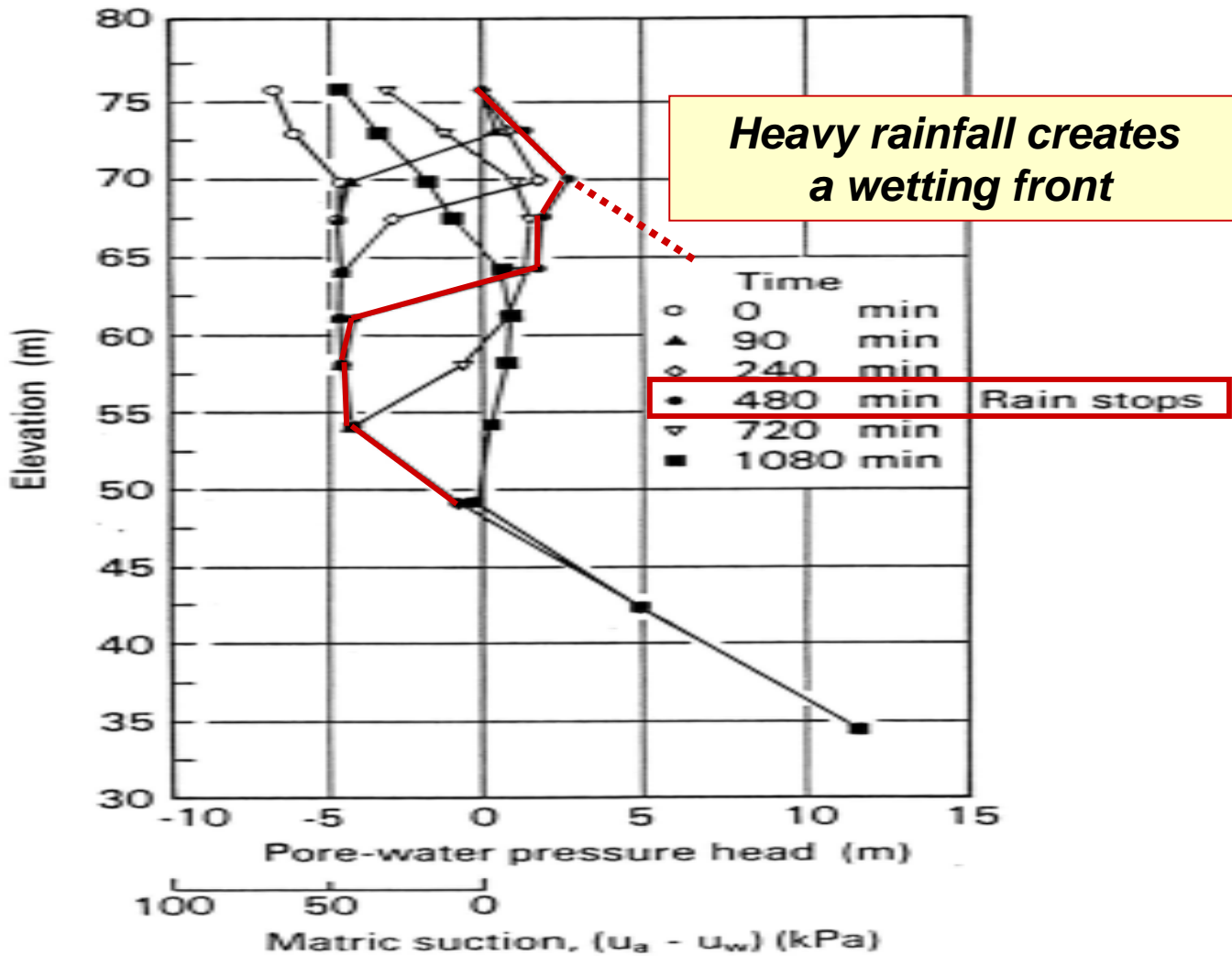
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Elapsed time = 1080 min.

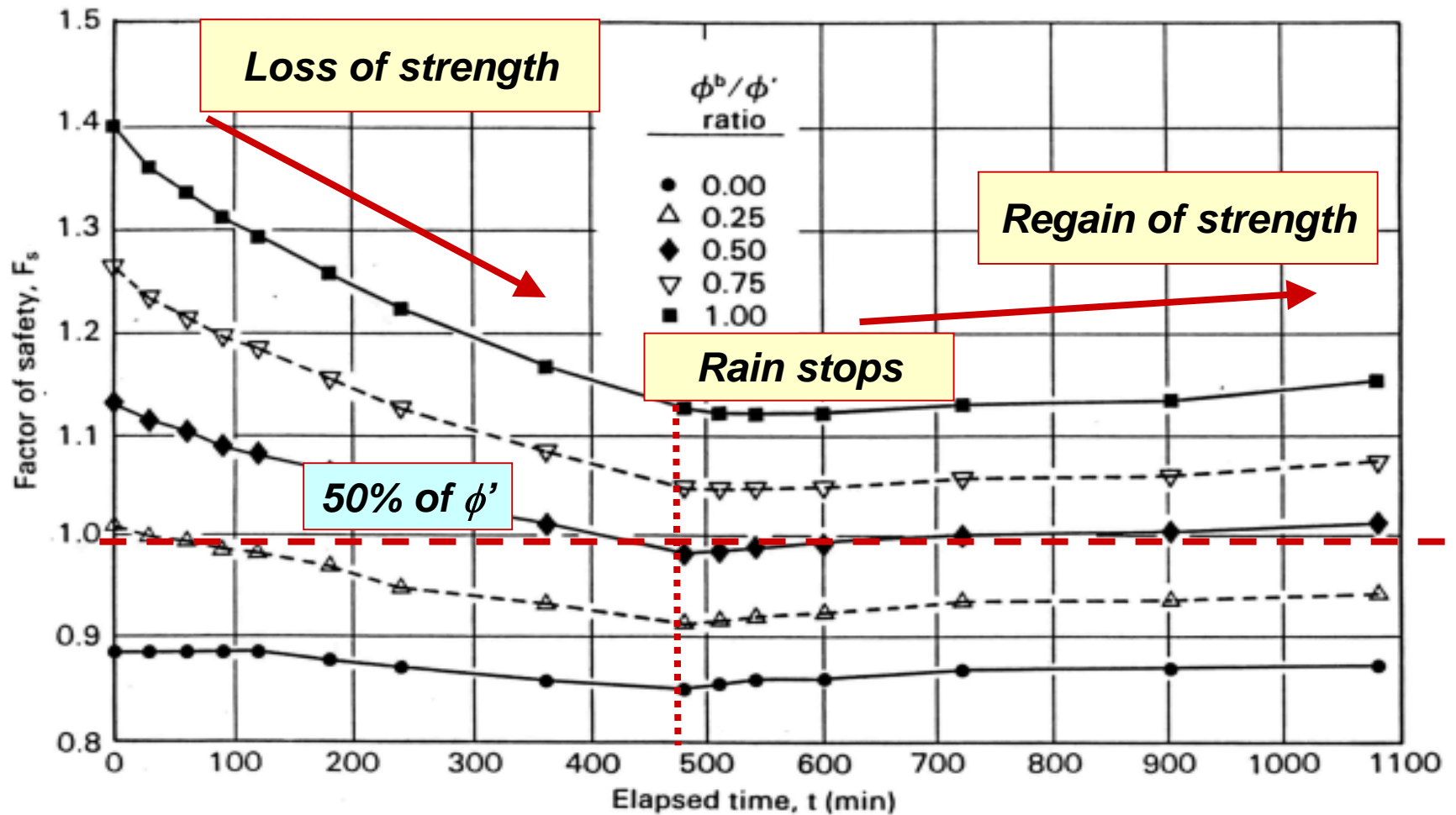


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Matric suction profiles for section X-X at various elapsed times





Factors of safety with respect to elapsed time from the beginning of rainfall



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