A New Method to Characterize Cross-anisotropic Stiffness of Saturated Clay at Small Strain Level using Pore Water Pressure Behavior

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Introduction

Soil mechanics theories generally assume soil is isotropic. However, anisotropic characteristics have been observed from several natural soils. Their anisotropy were greatly influenced by the manner of how they were formed. Even so, isotropic linear elasticity model is still commonly adopted for practical geomechanics since complete characterization of the soil’s behavior using this model would only require the determination of two strength parameters.

Recent studies have shown the importance of considering the stiffness anisotropy of soils, especially in obtaining more reliable predictions of ground deformations. However, fully characterizing soils’ anisotropy in the laboratory remains to be a complex task. Proposed methods [1, 2] in deriving the five-independent cross-anisotropic elastic parameters involve conducting both drained axial loading with constant radial stress (“drained axial probe”) and radial loading with constant axial stress (“drained radial probe”) with local sensors, coupled with shear wave velocity measurement. Nishimura [3] showed that obtaining all stiffness parameters was possible for saturated soils without measuring the radial deformation, by interrelating the undrained Young’s modulus to the drained elastic parameters. This study is a further simplification of Nishimura’s method, in which the pore water pressure measurement during undrained axial probing was utilized to deduce anisotropy by exploiting its theoretical relation to undrained and drained elastic parameters (see Fig. 1). The new approach obviates the need for an ultra-precise cell pressure controlling device since drained radial probe becomes unnecessary and avoids the complex radial instrumentation, making full anisotropic stiffness characterization possible for less-equipped laboratories.

Cross-anisotropic equations

Cross-anisotropy is the most prevalent type of anisotropy observed in soils. The cross-anisotropic constitutive equation containing eight elastic stiffness parameters is expressed in Equation 1 where the prime (‘) indicates that these parameters are obtained under drained conditions (the z-axis is taken as the axis of symmetry, usually being the vertical direction). Equations 2, 3 and 4 are obtained following the theory of cross-anisotropy and thermodynamic requirement. Hence, only five out of eight parameters are independent.

Under undrained isotropic state, Skempton’s coefficient of pore water pressure, \(B\) is expressed as the ratio between the change in pore water pressure to the change in cell pressure, or \(B = \Delta u / \Delta \sigma_c\). By using \(B\), the pore water pressure change and Young’s modulus against undrained vertical uniaxial compression, \(E'_u\). Equations 5 and 6 are obtained. Equation 5 shows the ratio of the change in pore water pressure \(\Delta u\) to the change in the vertical strain \(\Delta \sigma_v\). These two equations that relate the pore water behavior to the elastic parameters are then used with the pore water measurement to fully obtain all anisotropic elastic parameters in Equation 1. Note that the drained axial probe and multi-directional bender elements are still necessary along with them.
Soils tested, test setup and probing scheme

A natural sample of Higashi Osaka clay and reconstituted samples of Kasaoka clay, Izumi clay and Kaolin clay were tested on this study. Although the tests on Higashi Osaka and Kasaoka clay samples were unsuccessful, various factors to further improve the laboratory practice were noted. For example, it was observed that large temperature fluctuations affected the stability of radial sensor readings. This was solved by insulating the triaxial cell using a box made of polyethylene foam. Stress tolerances when doing small-strain probe loadings were also explored and pause intervals before stress reversals were applied to account for any delay on pore water pressure response during undrained probe loading, while ensuring that the total radial stress was held constant. Results from Izumi clay were generally better than the earlier methods employing it. The adopted small-strain probe loading routine typically involved; (i) six cycles of drained axial probe with axial strain rate, $\dot{\varepsilon}_a = 0.001\%/\text{hour}$, (ii) six cycles of drained radial probe with radial strain rate $\dot{\varepsilon}_r = 0.001\%/\text{hour}$, (iii) six cycles of undrained axial probe with axial strain rate of 0.02%/hour, and (iv) bender element probes. Settings of the probe loading test were sometimes adjusted while ensuring that the soil is still within the linear elastic region, which is at strains smaller than 0.001% for uncremented soils. Both Kaolin clay samples were re-consolidated at an effective isotropic stress of 70 kPa inside the triaxial cell before subjected to small-strain probe loadings. The test on the horizontally cut specimen was conducted to further verify the test results obtained from the vertically cut specimen.

Test results

Vertically cut Kaolin sample

Examples of one cycle stress-strain and strain-strain relationships are shown in Fig. 3, while samples of received waves during bender element test and the interpreted point of arrival are shown in Fig. 4. The behavior of the pore water pressure response during undrained testing is shown in Fig. 5. The measured and derived small strain stiffness parameters are summarized in Table 1.
This sample was saturated inside the triaxial cell using the Dry Setting Method. The measured B-value at a back pressure of 250 kPa was 94.4%. Though generally, several factors affect B-value measurements i.e. stiffness of sample and system compliance, it can be said that the applied method was effective in increasing the degree of saturation of the sample. A good linearity among the stress-strain relationships were observed from the small strain probe loadings. From Fig. 3c, the obtained value of undrained Poisson’s ratio, \( \nu_{uh} \), measured as \( -\Delta \varepsilon / \Delta \varepsilon \) (ratio of radial to axial deformation) was 0.44. This shows good agreement with the ideal value of 0.50, indicating that careful end lubrication prevented any end effects arising from a relatively small height-to-diameter ratio. In doing the undrained test, any delay on the pore water pressure response was not observed and the total radial stress was held constant as shown in Fig. 5. It is important to note that Eq. 5 and Eq. 6 were derived considering that the total radial stress was constant. In Table 1, the non-italic values (referred here as ‘measured parameters’) denote that these parameters were directly obtained from the probe loadings and bender element tests while the italic values (referred here as ‘derived parameters’) were calculated using Eqns. 1–6.

One peculiar thing about the analysis of the test results is the selection of the shear wave arrival points shown in Fig. 4. The second zero cross-up point was selected for this sample. For the HH-direction, Fig. 4b, this is likely the case since the first wave was insignificantly comparable to the next received waves. But for the VH-direction, Fig. 4a, the same cannot be said. The derivation procedures of the small strain cross-anisotropic stiffness are dependent on the measured value of \( G_{hh} \), while \( G_{vh} \) can be considered independent from these procedures as shown in the various derivation flow paths in Fig. 1. Hence, for consistency, the second zero

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**Table 1. Summary of elastic parameters (vertically cut Kaolin sample)**

<table>
<thead>
<tr>
<th>Method</th>
<th>( G_{hh} )</th>
<th>( G_{vh} = G_{hv} )</th>
<th>( E_{vh}^\prime )</th>
<th>( E_{hv}^\prime )</th>
<th>( \nu_{vh}^\prime )</th>
<th>( \nu_{hv}^\prime )</th>
<th>( E_{vh}^\prime )</th>
<th>( E_{hv}^\prime )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuwano et al. [1] &amp; Lings et al. [2]</td>
<td>Flow 1</td>
<td>42.2</td>
<td>38.9</td>
<td>71.9</td>
<td>98.1</td>
<td>0.162</td>
<td>0.162</td>
<td>0.118</td>
</tr>
<tr>
<td>Nishimura [3]</td>
<td>Flow 3</td>
<td>99.1</td>
<td>0.174</td>
<td>0.139</td>
<td>0.101</td>
<td>110.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed method</td>
<td>Flow 4</td>
<td>95.7</td>
<td>0.133</td>
<td>0.079</td>
<td>0.059</td>
<td>111.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Strength moduli in MPa*
cross-up point was also selected as the shear wave arrival point for VH-direction. The interpretation of shear wave arrival has been very subjective. But by using the above-mentioned shear wave arrival point, the derived stiffness parameters from this specimen using all the derivation methods were consistent as shown in Table 1. It will also be shown later that these derived parameters were comparable to the results from the horizontally cut specimen with high precision.

In Table 1, a good agreement amongst all the values can be seen regardless of which derivation procedure is followed. It therefore follows, that the proposed method, Flow 4 is effective in deriving the 8 cross-anisotropic stiffness parameters with a good accuracy. The results from this test can also be considered typical by considering the reduced cross-anisotropic elastic relationship proposed by Graham and Houlsby [4]: 

\[ E'_h / E'_v = (G_{th} / G_{eh})^2 \]

The values of the relationship \((E'_h / E'_v) / (G_{th} / G_{eh})^2\) from this test range from 1.13~1.17. As observed from the same table, \(E'_h\) is a derived parameter regardless of which derivation procedure is followed. To confirm this value, a test on the horizontally cut specimen was performed.

**Horizontally cut Kaolin sample**

Small strain probe loading were performed under the same stress conditions with the vertically cut specimen. By using the same saturation process, a good degree of saturation was obtained with a B-value of 96.3%. Four gap sensors were used in this specimen to measure the deformation along principal planes, see Fig. 2. By performing a test on this orientation, \(E'_h\) can be directly measured with higher reliability due to the fact that \(E'_h\) (or following this specimen’s orientation, becomes \(E'_v\)) was obtained with the use of LVDTs. Axial strains were calculated from a 70-mm initial length compared to radial strains which were calculated from about 37.5-mm initial length (equivalent to the radius of the specimen) and therefore, the former has twice resolution than the latter.

Stress-strain and strain-strain relationships from this test are shown in Fig. 6 while the stiffness parameters (average from all cycles) from these relationships are shown in Table 2. It should be noted that the notations of \(v\) and \(h\) in Table 2 reflects the original orientation during pre-consolidation, i.e. \(E'_h\) is calculated as \(\sigma'_u/\varepsilon'_a\) from Fig. 6a. The notation, \(h1\) indicates the originally horizontal direction turned to the axial.

The derived \(E'_h\) values from the vertically cut specimen are from 95.7~99.1 MPa, whereas the measured value from this test is 96.2 MPa, indicating a good degree of precision within 3% maximum difference. A good agreement can also be noted among the Poisson’s ratios, \(\nu'_{h1h2}\) and \(\nu'_{h1v}\). Reflected in Table 2 which were direct measurements from the local strain sensors with those that were shown in Table 1 which were all derived parameters. With the compatibility of the results from both specimens, it can be said that the interpreted shear wave arrival in the vertically cut specimen was more likely the actual point of arrival.

**Conclusion**

Measuring the pore water pressure behavior during undrained axial probe loading was easy and a consistently linear relationship between the pore water pressure and the axial strain was recorded for all samples. Given the notable degree of precision of the measured and derived stiffness parameters between the two tests on Kaolin with different cut-orientation, the proposed derivation method in quantifying the cross-anisotropic stiffness parameters at small strain level without the use of radial instrumentation and without performing drained radial probe loading was further validated.

**References**