Practical Evaluation for Interactive Behavior between Structure and Two-dimensional Frost Heave

Hao ZHENG
Candidate for the Degree of Master
Supervisor: Prof. Shunji KANIE
Division of Construction Engineering for Cold Regional Environment

Introduction

This research is initiated to investigate the interactive behavior between chilled gas pipe and frost heave. At the same time, verify the possibility of applying Takashi’s equation into Two-Dimensional Frost Heave. And then confirm the calculation results with the Alaska field observed data. Based on the model and parameters, the influence of each factor to the frost heave is evaluated by the FEM simulation program. Finally some conclusions are given based on our model and calculation.

In this paper, three different frost heave models are introduced and then they are compared. After that, Takashi’s model is adopted. Takashi’s equation is originally derived from numerous one dimensional experiments. Here it has been extended to solve two dimensional problems based on the anisotropic parameter which can distribute the frost heave ratio in different directions. This is one of our original research motivations.

Takashi’s Equation

Takashi’s equation is an experimental equation which is originally derived from one dimensional experiment. It is adopted by JGS and has a complete set of experimental standard.

Though it is originally applied only in one-dimensional problem, the applications for multi-dimensional problems are tried these days. Takashi’s equation relates the frost heave ratio with the constraining stress and freezing rate as Eq. 1.

\[
\xi = \xi_0 + \frac{\sigma_0}{\sigma} \left( 1 + \sqrt{\frac{U_0}{U}} \right)
\]  

(1)

\(\xi\): frost heave ratio, \(\sigma\): constraining stress in the freezing direction, \(U\): freezing rate. \(\xi_0, \sigma_0\) and \(U_0\) are constants for the material obtained by experiment regulated by JGS. \(U\) is obtained by heat transfer analysis and \(\sigma\) is obtained by mechanical analysis. Frost heave can be obtained by using frost heave ratio to multiply height.

In two-dimensional situation, we have to consider how to distribute the frost heave ratio into two directions; accordingly, we introduce the anisotropic parameter \(\beta\) to solve this issue. \(\xi_v\) is the frost heave ratio in heat flux direction and \(\xi_h\) is the frost heave ratio in direction perpendicular to heat flux direction.

\[
\xi_v = \frac{1}{1 + \beta} \xi, \quad \xi_h = \frac{\beta}{1 + \beta} \xi
\]

(2)

\(\beta = 0\), it means the material is extremely anisotropic.

\(\beta = 1\), it means that the material is isotropic.

Based on this assumption, we can expand Takashi’s equation into two dimensional problems.
The experiment in Alaska

The primary purposes of the test facility in Alaska were to evaluate the effects of differential frost heave and associated induced stresses in the pipe across a permafrost-non-permafrost boundary. The experiments were conducted in December 1999. A 0.914 m diameter, 105 m long chilled pipeline with X65 grade and 9 mm wall thickness was constructed and activated near 3.8 km Chena Hot Springs Road, Fairbanks, Alaska. But here only the cross section of this experiment is used in our research.

![Pipe Installation Diagram](image)

**Fig 1** The side view of the installment of the pipeline in Alaska

Our aim is to evaluate the interactive behavior between the chilled gas pipe and the soil around it. This process can be divided into two parts: heat transfer analysis and mechanical analysis. Before any calculation, we need to analyze the initial condition, and boundary condition. After that the distribution of temperature in the soil should be known in every step. One important thing in this process is that we need to consider the latent heat during freezing. Based on the temperature distribution, the freezing direction and freezing rate can be obtained. These are two critical parameters for the evaluation of frost heave ratio. With the help of freezing direction and the initial stress analysis, we can get the constraining stress in freezing direction. Until now, we have all the parameters used in Takashi’s model. Consequently, applying the Takashi’s equation, we can get the frost heave ratio. Because that this is the multi-dimensional problem, anisotropic parameter $\beta$ is introduced to distribute the frost heave ratio in each direction. This is the whole idea for our program to evaluate the interactive behavior of pipe.

**Numerical results and discussions**

In this research, Takashi’s equation is expanded into two dimensional problems and the calculation results are compared with the observed data. After that the influence of some parameters in this research are discussed.

In Fig.3, we can see that, as the increasing of $\beta$, the value of calculation result becomes smaller. And when $\beta=0.75$, the simulation result is very close to the observed data from the Alaska field data. $\beta=0.75$ means that most of the frost heave ratio was distributed to the vertical direction (4/7 of the frost heave ratio is distributed to vertical direction and 3/7 is given to horizontal direction). That also expresses that the soil is kind of anisotropic material.

**Analytical method**

In this part, we introduce our idea of coupling analysis.
Fig 3 The comparison of different $\beta$ values and the observed data (the frost heave under the pipe)

And Fig. 4 indicates the relationship between $\beta$ and frost heave under the pipe. We can see that the vertical frost heave decreases when the value of $\beta$ increases.

Fig 4 the relationship between $\beta$ and frost heave under the pipe

In Fig. 5, we find that the relationship between thermal conductivity and the maximum of frost heave is not linear. And the $k$ value between 1.0 and 1.1 may be a critical point of inflexion.

Fig 5 Thermal conductivity and the maximum of frost heave

In Fig. 6, we can see the influence of thermal conductivity to the size of frost bulb. As the increasing of $k$, the size of frost bulb also becomes larger.

Fig 6 The radius of frost bulb at different thermal conductivity

Here we advance a new parameter, using the frost heave to divide frost bulb radius:

$$\frac{F}{R} = \frac{\text{Frost heave}}{\text{Radius of frost bulb}}$$  \hspace{1cm} (3)
In Fig. 8, case 1 and case 2 mean two different initial temperature distributions, and we cannot tell any difference between case 1 and case 2. In fact the frost heave of case 2 is only 0.2mm bigger than that of case 1. We can regard them as the same. That demonstrates that our assumption of the depth of active layer in case 1 is right. Maybe after a long period of calculation, 5 years or even longer, the difference would be more obvious.

Conclusion

During the building of numerical model and the expanding of Takashi’s equation, we get some new understandings. The Takashi’s equation can be expanded to two dimensional and in this situation the appropriate value for $\beta$ is 0.75. In addition, we also know that the thermal conductivity $k$ has a complex influence to the frost heave. A big $k$ can bring a large size of frost bulb, a big freezing rate, and a small frost heave ratio. At last the influence of initial temperature distribution, a longer calculation period needs a longer observation period.

Besides these findings, there are also some questions confuse me and would be discussed in the future work. Such as the factors that can affect the freezing rate and the application of Takashi’s equation into three dimensional problem. In fact there are some huge challenges which may lead to some modifications to the equation itself.