Development of Accurate Phase Unwrapping Algorithm for Noisy Images Obtained by Interferometer

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Introduction

Over the last few years increasing attention has been focused on phase unwrapping on account of the interest of this technique in various scientific branches: from optical shape reconstruction to medical analysis and geological surveys [1]. Two-dimensional (2D) phase unwrapping denotes the retrieval of the original (unwrapped) phase \( \varphi \) starting from the corresponding restricted (wrapped) phase \( \psi \) in the \((-\pi, \pi)\) interval. Phase unwrapping is not a straightforward step because of the possible presence of different error sources and problems such as:

- Low signal-to-noise ratio of the fringes caused by electronic noise, speckle noise, or a low fringe modulation.
- Violation of Shannon’s theorem.
- Object discontinuities [2].

As a result, many phase unwrapping algorithms have been developed in an attempt to solve these problems [1-5]. However, the variety of forms, shapes and densities of noise that might be found in real wrapped phase maps makes the problem of phase unwrapping complex and difficult to solve, even given the significant amount of research effort expended to date and the large number of existing phase unwrapping algorithms. So, phase unwrapping algorithms were developed to overcome the pixels affected by noise. However, there is no common technique can be applied accurately in all applications of wrapped noisy phase data among the existing algorithms. For this purpose, we introduce a new phase unwrapping approach namely rotational compensator phase unwrapping with virtual singular point (RCPU-VSP) for wrapped noisy images. The new approach can compensate the phase singularity in a continuous manner with its vicinity. Its applicability is demonstrated by simulated data compared to existing methods.

Rotational Compensators

The basic idea of the proposed algorithm depends on that the residue in each loop, which consists of four adjacent pixels surrounding the point \( \vec{r} \) in the wrapped phase image, has the first order singularity, as proven in previous study [6]. Residues, which are called singular points, are local inconsistencies, which mark the beginning and end of \( 2\pi \) discontinuities. The singularity \( S(\vec{r}) \) in Two-dimensional (2-D) phase data can be calculated and evaluated as follows:

\[
S(\vec{r}) \approx \sum_{i=1}^{4} W\{\Delta \psi_i(\vec{r})\}
\]

where \( W \) is wrapped operator, and \( \Delta \psi_i(\vec{r}) \) is difference wrapped phase between two adjacent pixels in the phase image.

Residue in the wrapped phase data is preventing straightforward unwrapping. Moreover, the existence of residue causes the phase unwrapping process to be path dependent. Hence, it should be canceled the effect of singular point before start the phase unwrapping process.

Fig. 1 Sketch indicates singular function and configuration to compute rotational compensator
Figure 1 shows $\Omega$ domain has singular point $\vec{r}_s$ surrounding by closed boundary $\Gamma$. It is assumed another domain does not contain the singular point surrounding by closed boundary and has the same area as $\Omega$ domain. If singular function can be found the singularity of the phase is canceled by subtracting the phase function and this singular function. Equation 2 shows one example of the singular functions. The difference vectors, $\vec{r}_{\Delta i} - \vec{r}_s$ where $i = 1, 2$ are collinear to the difference vectors, $\vec{n}_i - \vec{r}_s$, respectively as shown in Fig. 1. Taking the path of integral $\vec{r}_1 \rightarrow \vec{r}_2 \rightarrow \vec{r}_{\Delta i} \rightarrow \vec{r}_s$, since the domain surrounded by the path is regular; the integral along the path is 0. Furthermore, the normal vector along path between $\vec{n}_i$ and $\vec{r}_{\Delta i}$ is perpendicular to $\vec{n}_i - \vec{r}_s$; therefore, the integral over this section becomes 0, as described in Eq. 3.

$$\vec{f}(\vec{r}; \vec{r}_s) = \frac{\vec{r} - \vec{r}_s}{2\pi |\vec{r} - \vec{r}_s|^2}$$

$$\oint \vec{f} \cdot \vec{n} \, d\Gamma = \begin{cases} 0 & \text{if } \vec{r}_s \text{ is outside of } \Omega \\ 1 & \text{if } \vec{r}_s \text{ is inside of } \Omega \end{cases}$$

where $\vec{f}(\vec{r}; \vec{r}_s)$ is function shows first order singularity at $\vec{r} = \vec{r}_s$.

In order to cancel the singularity we introduce compensator by using integral with opposite sign of residue, as clarified in Eqs. 4. When the image contains several singular points, total compensator is estimated as summation of them, as shown in Eq. 5. The phase unwrapping process can be applied straightforward as shown in Eq. 6.

$$\sum_{i=1}^{N_s} Q_i \left( \theta_i - \theta_{i+1} \right) = 0$$

$$C_s = \sum_{i} \frac{Q_i \left( \theta_i - \theta_{i+1} \right)}{2\pi}$$

where $Q_i$ is the normalized residue at the singular point $\vec{r}_{s,i}$, $\theta$ denotes the unbounded azimuthal angle from the x-axis and also $\theta_{i+1} > \theta_i$, $i = 1, 2, ..., N_s$ as seen in Fig. 1.

We call this approach rotational compensator phase unwrapping (RCPU) algorithm.

**Virtual Singular Points**

It was found that distribution of SPs in the image affects the unwrapping process. Depending on SPs distribution two cases are found. First case is dipole residues which exist in pairs of two opposite polarity states. Second case is monopole residues or isolated SPs, which are single value residues without corresponding opposite-sign partner. The isolated SPs spread error in the whole image. Hence, it is needed that the number of positive SPs and that of negative one to be balanced to reduce the error by appending SPs outside the border of
data have opposite polarity to conform pairs which are called virtual SPs (VSPs).

The process of appending VSPs is as following:

- **Preparation:** Mark all SPs as isolated. For each isolated SP, search the nearest border point and determine the symmetrical point to nearest border point. This symmetrical point is considered as virtual SP candidate has opposite sign. The VSP candidates are shown as the end of arrow in Fig. 2 (a).

- **Dipole determination:** For every isolated SP search the nearest SP with opposite sign among SPs marked with isolated and that one marked with virtual SP candidate. If the chosen pair is original SPs so they called internal dipole and remove the corresponding virtual SP candidate. And if one of the chosen pair is virtual SP candidate so they called dipole with virtual and the corresponding virtual SP candidate called virtual SP. The pair of them is shown as an encircled pair with single headed arrow shown in Fig. 2 (b).

- **Repeating:** Repeat procedure 2, until any SPs marked with isolated is not found. See Fig. 2 (c).

By applying our algorithm, the phase inconsistency is canceled. Then, we can unwrap the phase data simply by summing the phase differences between neighboring pixels. We call this modified approach rotational compensator phase unwrapping with virtual singular point (RCPU-VSP) algorithm.

**Results**

The validity of the proposed algorithm is examined by generated simulation phase data. The original data with noise shown in Fig. 3(a) is wrapped in the range between $-\pi$ to $+\pi$, as shown in Fig. 3(b). This wrapped phase data corresponds to the input of phase unwrapping algorithms. Figure 3(c) shows the distribution of singular points that are caused by the noise which appended into the original data. To compare the characteristics several algorithms Goldstein’s path-following method [3], DCT [4], SSPU [5], and the
The results of the unwrapped phase maps for each algorithm are shown from Fig. 3(e) to 3(h). Some continuous phase gaps are found in the unwrapped phase map obtained by the Goldstein’s method in Fig 3(e). It can be noticed from Fig. 3 that it is difficult to compare the accuracy of the other studied algorithms by visualized results only. Hence, a quantitative comparison, gradients of the unwrapped phase maps is presented in Fig. 3(d) to clarify the correlation between each of them and the original data. From this comparison, it can be inferred that the proposed algorithm gives better correlation to the original data compared to the studied algorithms.

Reference


