Bed Instability in Suspended Load-Dominated Environments

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Introduction

Bed wave formation on the bed and on the floor of the river and ocean is one of the most interesting subjects of morphology. In this study, we investigate the bed instability of the two configurations, the stratified open channel flows and the turbidity currents.

In an open channel flow, the bed and the flow are subject to instability. This instability is due to the interaction between the flowing water and the bed morphology. Under proper conditions, the bed evolves into a train of boundary waves, such as ripples, dunes, and antidunes.

The bed wave formation was investigated by linear stability analyses by Engelund (1970), Fredsoe (1974), Richards (1980), Colombini (2004). The results of the analyses are able to predict the unstable region which is reasonable by comparing to the experimental results. However, these analyses do not shed further light on the formation of the bed wave under the suspended sediment being dominated condition. In this regards, we perform linear stability analysis of bed instability due to suspended sediment of an open channel flow.

In this study, we consider an open channel the flow of which is assumed to be under the upper flow regime condition. Under this consideration the suspended load is dominant and the bedload is negligible. The suspended sediment particle is assumed to completely follow the velocity of the flow except it sinks at its settling velocity in the vertical direction. As far as suspended sediment is concerned, the flow is subject to density stratification induced by the presence of suspended sediment. In the case of suspended load being dominant, the effect of density stratification is important in the way it affects the flow velocity, suspended sediment concentration, the turbulent kinetic energy, and the dissipation rate of turbulent kinetic energy. In addition, these changes influence the formation of the bed waves. Therefore, it is necessary to investigate the effect of density stratification to the formation of the bed wave. In order to investigate this effect, the sediment transport model is assumed to be solely suspended sediment. For the turbulent closure, the simple mixing length model and the two-equation turbulent closure (the standard $k-\varepsilon$ model) which includes the turbulent kinetic energy $k$ and the dissipation rate $\varepsilon$.

Another setting of stratified flow is the turbidity currents which occur naturally in the ocean, lake, reservoir, and dam, etc. Turbidity current is the current driven by the gravitational force on the suspended sediment in the flow. The sediment is considered to be suspended by the fluid turbulence. This kind of current is different from other density currents by the tendency to settle down of the suspended particles which result in a profile biased towards the bed. The turbidity current is an important agent that distributes and transports much amount of sediment in the subaqueous environments. The sediments transported are rich in organic matter and mineral which are the main sources of petroleum and methane hydrate.

In the case of saline or thermal density flow, salt concentration or temperature as a driving force is diluted due to diffusion as it flows down, and therefore, it cannot travel to far distance from its source of origin without any external forces acting on it. However, there are observational evidences which show that turbidity current is able to travel hundreds of kilometers at the speed range of tens of meters per second before total dissipation and deposition on the seabed. This mechanism is proved by the theoretical work of Bagnold (1962) and Parker et al. (1986).

Under auto-suspending and self-accelerating condition, it is believed that the turbidity currents are the mechanism that transports turbidite several kilometers and generates various topographical features in the submarine environments including subaqueous abyssal fans and deltas, cyclic step, submarine canyons and gullies, etc. This suggests that the turbidity currents possess equilibrium condition at least at the relatively thin layer near the bottom. In this study, we perform linear stability analysis to investigate the bed instability generated by turbidity current.

Formulation

Open channel flow

The conceptual framework of the problem is an open channel with sufficiently large width to ignore the effect of the bank. The flow is considered to be steady uniform in which the time derivative term is dropped from the equation. The slope of the bed is assumed to be constant. Under quasi-steady approximation, the flow is described by two-dimensional Reynolds-averaged Navier-Stokes equations. In addition, the stream function is used.

Suspended sediment is transported and described by the diffusion/dispersion equation. The particle completely follows the fluid except it sinks at its settling velocity in the depth direction.

For the turbulent closure, we employed two turbulent models in this study, the simple mixing length model and the standard $k-\varepsilon$ model. In mixing length model, the eddy viscosity is related to a mixing length scale. The eddy viscosity vanishes at the reference level, at which the velocity vanishes in the logarithmic distribution, and at the water surface. In standard $k-\varepsilon$ model, the turbulent kinetic energy
and dissipation rate are described by two different transport equations. The eddy viscosity however does not necessarily goes to zero at the water surface. It is known that the standard $k$-$\varepsilon$ model cannot describe well the turbulence very near to the surface. In line with this, the bottom boundary conditions are evaluated at a location slightly shifted over the bed.

At the bottom, the normal and tangential velocity components vanish in mixing length model. However, in $k$-$\varepsilon$ model, the bottom boundary is assumed to be located above the bed, namely top of bedload layer. Therefore, the bottom boundary condition at the top of bedload layer is the logarithmic velocity evaluated at the top of bedload layer. In addition, the dissipation rate and the production due to shear are in balance, and the eddy viscosity should be continuous at this location.

At the water surface, normal and tangential stress components vanish. The symmetry boundary condition for turbulent kinetic energy and dissipation rate are assumed.

**Turbidity current**

The turbidity currents are described by two-dimensional Reynolds-averaged Navier-Stokes equation with the use of Boussinesq approximation as well as the continuity equation. The driving force of the turbidity current is the gravity force acting on the sediment particle. Therefore, the suspended sediment concentration is included in the governing equation.

The suspended sediment is considered to be sufficiently small that follows completely the flow except its settling velocity in the vertical direction. The settling velocity is the parameter which induces the self-stratification in the turbidity currents. In addition, the hindered settling effect is also neglected.

For the turbulent closure, the simple mixing length model is employed. The eddy viscosity is proportional to the mixing length scale which becomes zero at the bottom and upper boundary.

In this study, we assume that the turbidity current possesses an equilibrium layer near to the bottom which allows the turbidity current to travel long distance from its source. We perform linear stability analysis to investigate the formation of the bed waves by the turbidity current.

**Continuity equation of the bed**

In the upper flow regime, the suspended load is dominant. Thus, we totally ignore the bedload. In this regards, the bed elevation change is proportional to the difference between the amount of suspended sediment being entrained into suspension and the amount of suspended sediment deposit on the bed. In order to determine the amount of entrainment rate of suspended sediment, we employed Garcia and Parker (1991) formulation. It is noted that the entrainment rate increases as the shear velocity increase for the same sediment size condition.

**Base state condition**

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In the base state, the derivatives in the streamwise direction of the variables vanish, except for the vertical variation of the variables. The flow depth is assumed to be constant. The governing equations are described by second order ordinary differential equation.

The base state solution does not admit to any analytical solutions. It is required a numerical method to solve the problem. In this study, finite volume method is used.

In order to investigate the effect of density stratification on the variables, we solved the governing equations with two conditions, with and without the effect of density stratification. We obtain the results of the case not including the effect of density stratification by ignoring all the terms associated with the density gradient.

It is found that the effect of density stratification generally increases the velocity particularly in the upper part and deviates from the logarithmic velocity profile predicted by the mixing length model. In the case of suspended sediment concentration, the effect of density stratification is rather small, and it increases slightly in the lower part and decreases slightly in the upper part due to the effect of density stratification. The concentration is relatively more uniform in the mixing length model. These results can be explained in terms of the eddy viscosity.

The density stratification generally suppresses the turbulent mixing, and therefore, the diffusion of momentum and suspended sediment is reduced. This effect is reflected by the decreases of eddy viscosity due to the density stratification. Because of the reduction of turbulent mixing, relatively large momentum and small suspended sediment concentration in the upper are not reduced by the effect of small momentum and large sediment concentration near the bed.

**Turbidity current**

In the analysis of the turbidity current, we have a dimensionless settling velocity parameter. The effect of the settling velocity on the flow velocity and the suspended of the turbidity currents are discussed. The flow velocity and suspended sediment concentration are evaluated for various values of settling velocity. For the same friction velocity, smaller settling velocity represents smaller sediment size.

In the case of settling velocity equals to 0.01, the suspended sediment is almost uniform in the depth direction, and the flow velocity becomes larger particularly in the upper part. Due to the tendency of the sediment particles to settle down, the suspended sediment concentration profile deviates from the uniform profile under the condition of increasing settling velocity, and it is found that the suspended sediment concentration increases in the region near to the bottom and decreases in the upper part.

In addition, when the settling velocity is bigger than 0.08, the suspended sediment concentration at the density interface becomes 0. This implies that the assumption that there exists
a density interface no longer holds when settling velocity is rather large.

**Linear stability analysis**

Small perturbations are introduced to the bed elevation in order to investigate the temporal growth of the bed elevation. Assuming that the flow adapts rapidly to the perturbation of the bed, other variables are also perturbed with the same wavelength as of bed elevation. In the scheme of linear stability analysis, the amplitude of the perturbations is assumed to be infinitesimally small, such that the terms with second order or higher order of the amplitude parameter vanish. The perturbation problem do not subject to analytical solution, therefore, the perturbation problem is solved by employing the spectral collocation method incorporated with the Chebyshev polynomials.

The conditions for the instability of the bed to occur are studied with three dimensionless parameters, namely Froude number, wavenumber, and settling velocity.

**Open channel flow**

According to the results of the analysis, an instability region is predicted at the upper flow region. This instability corresponds to the formation of the antidunes. The instability region predicted by the analysis is fairly reasonable as the experimental results (Kennedy 1961) fall almost all in the instability regions.

Under the density stratification effect, the instability region of the antidunes is shifted to the range of smaller critical Froude number. In the vicinity of the critical Froude number, the range of unstable wavenumber is shifted to the smaller range, which is corresponding to the range of longer wavelength. Moreover, the results predicted by the mixing model is consistent with the result of the standard k-ε model without density stratification effect such that the critical Froude number is higher and the instability expands towards the range of bigger wavenumbers.

In addition, this analysis also shed light on the migration mechanism of the antidunes. The model predicts that the antidunes could migrate in both the upstream and downstream directions. The mechanism of the migration is explained by the phase shift between the bed elevation and the net erosion rate. For the upstream migrating antidunes, the net erosion rate reaches maximum (minimum) slightly upstream of the trough (crest) of the bed waves. This process implies that the wave amplitude increases and the waves migrate in the upstream direction. In the case of downstream migrating antidunes, the increase of the amplitude of the bed waves is governed by the same process as that of the upstream migrating antidunes, however, the maximum (minimum) of the erosion rate takes place slightly downstream of the trough (crest) of the bed waves. Thus, the bed waves migrate in the downstream direction.

We also investigate the effect of parameter settling velocity to the stability of the bed. Smaller settling velocity corresponds to the less stratified condition. Here, we compare two cases.

0.2. According to the result, the unstable region expands in the direction of larger wavenumber in the vicinity of critical Froude number for the case of smaller settling velocity.

**Turbidity current**

As a result of the linear stability analysis of bed instability under turbidity currents, it is found that the flat bed becomes unstable to evolve into a bed covered with bed waves in the range of densimetric Froude number larger than approximately 0.4. For the case of settling velocity is 0.01, the instability is within the range of unstable wavenumber 1–1.5 in the vicinity of the critical Froude number. While the Froude number increases, the range of unstable wavenumber is 0.1–1.

In the condition of small settling velocities the instability region in the case of turbidity currents resembles that in open channel flow. In addition, the instability region is affected also by the settling velocity non-dimensionalized by the friction velocity in the base state. For the non-dimensional settling velocity larger than 0.08, the instability region show a strange shape. It is suggested that turbidity currents do not have normal flow conditions under the condition of sufficiently coarse suspended sediment.

**Conclusion**

In this study, we performed linear stability analysis with the concept of varying viscosity on the bed instability of an open channel flow and that of the turbidity currents.

For the analysis of an open channel flow, the mixing length model and the standard k-ε model are used for the turbulent closure. In the latter model, the effect of density stratification due to suspended sediment is investigated.

The Reynolds-averaged Navier-Stokes equation and dispersion/diffusion equation of suspended sediment coupled with the turbulent closure of both the mixing length model and k-ε model are found to be capable of predicting the bed instability. The unstable region predicted by the analysis is in fairly good agreement with the experimental results. The formation of antidunes is observed in the upper flow regime. The model explains physical process of instability and the migration direction of the antidunes.

In k-ε model, the transport equations of turbulent kinetic energy and the dissipation rate include the buoyancy term to study the effect of density stratification. In the base state, the effect of density stratification increases the flow velocity, and decreases the suspended sediment concentration in the upper part and increases near the bed. These results are caused by the general decrease in the eddy viscosity due to the suppression of turbulent mixing by the density stratification.

In linear stability analysis, an unstable region appears at the upper flow regime which corresponds to the formation of the antidunes when only suspended load is considered. The model is in fair agreement with the experimental results. In addition, the model reveals the migration mechanism of the antidunes in both upstream and downstream directions. The long wavelength antidunes usually migrate in the upstream
One is the case which the settling velocity equals to 0.1 and direction, whereas shorter wavelength antidunes migrate in the downstream direction. Moreover, it is found that the effect of density stratification decreases the critical Froude number. In addition, density stratification stabilizes the bed in the range of large wavenumbers in the vicinity of the critical Froude number.

Another setting of the stratified flow is the turbidity current. The turbidity currents possess equilibrium condition at least at the layer near to the bottom which is independent from the upper diluted layer. In this context, we perform linear stability analysis to investigate the instability of the bed generated by the turbidity current.

In the base state condition, the decrease of the settling velocity increases the suspended sediment concentration at the density interface. As the driving force is generated by the suspended sediment increases, the flow velocity at the density interface also increases.

However, in the equilibrium state, the suspended sediment decrease to zero at the density interface as the settling velocity is greater than 0.08. This result is physically impossible. In line with this, it is suggested that equilibrium state of the turbidity currents does not exist in the case of large sediment particle.

According to the results of the stability analysis, the plane bed becomes unstable when the densimetric Froude number is greater than 0.4. In addition, at the vicinity of the critical Froude number, the wavenumber of the unstable bed is in the range of 1.0-1.5, whereas when Froude number increases, the range of unstable wavenumber reduces to the range of 0.1-1.0.

Comparing the instability diagrams of the turbidity current and open channel flow when in the range of small settling velocity, it is found that the unstable region of the two cases are very similar. It is convenient to explain the similarity of these results from the perspective of the driving force of the flow. Gravity force is acting on the water in open channel flow, and acting on suspended sediment in the turbidity current. While the driving force is almost uniform in the depth direction when the suspended sediment is small for the two flow configurations, it is therefore reasonable to obtain the similar results.

**Bibliography**


