Computation of Flow and Bed Morphology at River Confluences

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Introduction

River channel confluences are commonly encountered in nature as well as in engineering. These are critical interfaces where intense changes in physical processes within drainage or fluvial networks occur. These changes influence both the local and downstream characteristics of flow dynamics and the bed morphology. Flow features in these regions are often known to be complex and highly three-dimensional. Although there are a number of studies on flow at these regions conducted in the past, there are still gaps to be studied for this complex flow. Furthermore, behavior of bed morphology at river confluences has not still been clearly understood. Besides, although local configuration of the confluent river mouths may influence flow and bed morphology, this aspect has not been paid attention at all. In addition, effect of bank vegetation and bar vegetation on bed morphology within these regions has not been obviously understood.

Besides the above problems, numerical modeling of flow dynamics and bed morphology at river confluences still requires much more attempts. The 3D nonlinear k - ε model are proved to be successful in simulating complex flow. However, this model has not been developed for river confluence problems. Although a 3D model is broadly adopted to simulate flow at a small part of river course, it is difficult to predict flows and sediment transports during floods from the upper region of a river to the river mouth. In addition, at river confluences, adding a sediment-transport module to a 3D flow model remains a challenge due to difficulties in designing a suitable numerical mesh. Furthermore, 3D modeling is still a time-consuming and costly work. Therefore, development of depth-averaged 2D models, which are corrected for effect of secondary current, are still a reasonable compromise for the problems of flow dynamics and bed morphology up to now.

The purpose of this PhD thesis is to address the above problems through computations of flow, bed morphology at these regions.

This paper summarizes the main contents of this thesis.

Computation of flow at an open-channel confluence with depth-averaged 2d models

- Governing equations

The governing equations used in this study are depth-averaged 2D shallow water flow equations described in the Cartesian coordinate as follows:

Continuity equation:
\[
\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0
\]  

Momentum equations:
\[
\frac{\partial M}{\partial t} + \frac{\partial uM}{\partial x} + \frac{\partial vM}{\partial y} + gh \frac{\partial (h + z_a)}{\partial x} = gh \sin \beta_a - \frac{\tau_{ux}}{\rho}
\]  

\[
\frac{\partial -u^2h}{\partial x} + \frac{\partial -u^2v}{\partial y} + v \left( \frac{\partial}{\partial x} \left( h \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \frac{\partial u}{\partial y} \right) \right) + S_x
\]  

\[
\frac{\partial N}{\partial t} + \frac{\partial uN}{\partial x} + \frac{\partial vN}{\partial y} + gh \frac{\partial (h + z_a)}{\partial y} = \frac{\tau_{uy}}{\rho} - \frac{\partial -v^2h}{\partial x}
\]  

\[
\frac{\partial}{\partial x} \left( h \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( h \frac{\partial v}{\partial y} \right) \right) + S_y
\]  

where (x, y): spatial coordinate, (u, v): depth-averaged velocity components in (x, y) directions, t: time, h: water depth, (M, N): discharge fluxes in (x, y) directions defined as (hu, hv) respectively, g: gravity acceleration, (u’, v’): turbulence velocities in (x, y) directions, z_b: bed level, (\tau_{ux}, \tau_{uy}): bottom shear stress vectors, v: dynamic viscosity coefficient, sin\beta_b: bed slope, f: friction coefficient (function of Reynolds number), \rho: water density, \beta_u: momentum coefficient, \tau_{ux}, \tau_{uy}, \tau_{uv}: depth-averaged Reynolds stress tensors, and S_x, S_y: additional terms caused by secondary currents and defined later.

The depth-averaged Reynolds stress tensors are evaluated based on the zero-equation turbulence model. For linear model, this term is evaluated as
\[
\frac{\partial -\bar{u}\bar{v}}{\partial x} = (D_h + v) S_{xy} - \frac{2}{3} k \delta_{ij}
\]  

but for nonlinear one, a non-linear term is added to the Reynolds stress tensor as
\[
\frac{\partial -\bar{u}\bar{v}}{\partial x} = (D_h + v) S_{xy} - \frac{2}{3} k \delta_{ij}
\]  

\[
\frac{1}{2} \lambda_p h u^2
\]

\[
\frac{1}{2} \lambda_p h u^2
\]  

where D_h is eddy viscosity and is evaluated based on an 0-equation turbulence model with considering reduction of eddy viscosity near the wall. In Eq. (4), D_h is evaluated
\[
D_h = f_p \rho u^2
du
\]  

while D_h in Eq.(5) considers contribution of strain and spin and is evaluated as
\[
D_h = f_p \rho c_d u^2
du
\]  

k is depth-averaged turbulent kinetic energy evaluated by the empirical formula as follows:
\[
k = 2.07 u^3
\]  

Here, u_0 is local friction velocity; \alpha is calibrated constant (\alpha = 0.80 is used in this study).

\[
c_p = \frac{1 + c_{d1} S^2 + c_{d2} \Omega^2}{1 + c_{d3} S^2 + c_{d4} \Omega^2 + c_{d5} S \Omega + c_{d6} S^2 \Omega + c_{d7} S^3 \Omega + c_{d8} S^4 \Omega + c_{d9} S^5 \Omega}
\]  

where S and \Omega are strain and rotation parameters.
$c_{p}$ is the coefficient of the non-linear quadratic term and evaluated as}

$$c_{p} = c_{pn} \frac{1}{1 + m_{cn}S_{n}^{2} + m_{dn}\Omega_{n}}$$  \hspace{1cm} (10)

**Secondary current model**

The additional terms expressing effects of secondary currents, $S_{sx}$ and $S_{sy}$, in Eq. (2) and Eq. (3) are defined as

$$S_{sx} = C_{as} \left[ \frac{\partial \bar{u}_{s}A_{h}\sin 2\theta}{\partial x} - \frac{\partial \bar{u}_{s}A_{h}\cos 2\theta}{\partial y} \right]$$

$$+ C_{as2} \left[ - \frac{\partial A_{sh}^{2}\sin^{2}\theta}{\partial x} + \frac{\partial A_{sh}^{2}\cos^{2}\theta}{\partial y} \right]$$

$$S_{sy} = C_{as} \left[ \frac{\partial \bar{u}_{s}A_{h}\cos 2\theta}{\partial x} + \frac{\bar{u}_{s}A_{h}\sin 2\theta}{\partial y} \right]$$

$$+ C_{as2} \left[ \frac{\partial A_{sh}^{2}\sin^{2}\theta}{\partial x} - \frac{\partial A_{sh}^{2}\cos^{2}\theta}{\partial y} \right]$$  \hspace{1cm} (11a)

$$+ C_{as3} \left[ \frac{\partial \bar{u}_{s}A_{h}\sin^{2}\theta}{\partial x} + \frac{\partial \bar{u}_{s}A_{h}\cos^{2}\theta}{\partial y} \right]$$

Here $\theta$ is angle between a streamline and x-axis. $C_{as}$ and $C_{as2}$ are model coefficients defined by Eq. (12) using the similarity functions of velocity profile in the stream-wise and transverse directions $f_{s}$ and $f_{t}$, respectively.

$\bar{u}_{s}$ is depth-average velocity in the stream-wise direction and is defined by Eq. (12).

$$C_{as} = \int_{0}^{1} f_{s}(\xi) f_{t}(\xi)\ d\xi \cdot C_{as2} = \int_{0}^{1} f_{n}(\xi)\ d\xi$$  \hspace{1cm} (12)

$u_{s}(\xi)$ is the stream-wise velocity profile in the vertical direction.

The coefficient $A_{n}$ in Eq. (11a) and Eq. (11b) means the magnitude of the secondary current, which is induced by streamline curvature, and is defined as

$$A_{n}(\xi) = u_{s}(\xi) \cdot \zeta = \frac{z}{h}$$  \hspace{1cm} (13)

Here $u_{s}(\xi)$ is the transverse velocity profile in the vertical direction and $z$ is the direction perpendicular to the bottom bed.

In this chapter as well as throughout this thesis, four types of depth-averaged 2D models are proposed to simulate the flow at an open-channel confluence. Model 1 is a conventional 2D model using a non-linear 0-equation turbulence model without effects of secondary currents; Model 2 is also a 2D model with effects of secondary current without consideration of lag between the streamline curvature and development of secondary current; Model 3 is a 2D model with effects of secondary currents and lag between a streamline curvature and development of secondary current; and Model 4 is a 2D model that consider effects of secondary currents, lag between a streamline curvature and development of secondary currents as well as change of mainstream velocity profile influenced by secondary currents.

When Model 2 and Model 3 are used, $A_{n}$ is evaluated by Engelund (1974) and Hosoda et al. (2001), respectively, while the similarity functions $f_{s}$ and $f_{t}$ are those proposed by Engelund (1974). However, when Model 4 is employed, the similarity functions are those derived by Onda et al. (2006) and the strength of secondary current is evaluated using Hosoda et al. (2001).

The fundamental equations are solved numerically using the finite volume method with a full staggered grid including conservativeness of physical quantities and computational stability. The QUICK scheme with second order accuracy in space is employed for convective inertia terms. The Adams Bashforth method with second order accuracy in time is used for time integration.

To solve the flow equations, discharge and flow velocity at the upstream end of the main channel and the branch channel, and water level downstream are specified as boundary conditions.

Computational results are compared to the experimental results of Weber et al. (2001). It should note that attempts to apply Model 2 are done during the process of implementation of this study, but are not successful. The reason for this may be attributed to very sharp streamline curvature of flow downstream of the junction causing strong instability of this model. Therefore, in the following, only the results obtained from the models 1, 3 and 4 are reported.

**Computational results and discussions**

Comparison of performance of Model 1 with the linear and nonlinear turbulence models carried out shows that the nonlinear is suitable for further simulations in this study. The followings are some of the main results obtained.

Figs. 1 and 2 show the results predicted by the models in comparison with the experimental one for the case of low discharge ratio ($q^*$). As seen from Figs. 1 and 2, all three models reproduce important flow patterns at the vicinity of the junction, that is, a separation zone immediately downstream of the junction and a contracted flow with higher velocity. However, it can be seen that there is a difference in separation generated by the models for $q^* = 0.25$ in which secondary current is strong (Fig. 1). Model 1 under-predicts the length of this region, while both Model 3 and Model 4 fairly well reproduce it.

In contrast, for a high $q^*$, that is, weak secondary current, all the models generate the results (not be shown here) agreeing well with the measured one and there is no significant difference in the results by the models.
Using the models with effect of secondary current, this flow pattern is able to be visualized as shown in Fig. 3. This also explains why the results with the models with secondary current effect (Model 3 and Model 4) are improved in comparison with that by Model 1.

**Computation of flow at an open-channel confluence with 3D models**

- **Governing equations**

In regions of open-channel flow where its solution domain changes in time due to the movement of boundaries, for example, in free-surface flows, this movement must be calculated as part of the solution. In these cases the grid has to move with the boundary. In this study, the unsteady Reynolds-Averaged Navier-Stokes equations are used and basic equations are written in a moving boundary-fitted coordinate system using the covariant derivative instead of the partial differential as follows:

**Continuity equation:**

\[
\frac{1}{\sqrt{G}} \frac{\partial \sqrt{G} \rho}{\partial \xi} = 0
\]  

**Momentum equation:**

\[
\frac{\partial \rho V'}{\partial t} + \nabla \left[ V' \left( V' - V'' \right) \right] + \rho V' \nabla V'' = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \nabla \rho \frac{\partial \rho}{\partial y} I + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
-\nabla \cdot \left[ \kappa \nabla V' \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
\frac{\partial \rho V'}{\partial t} + \nabla \left[ V' \left( V' - V'' \right) \right] + \rho V' \nabla V'' = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \nabla \rho \frac{\partial \rho}{\partial y} I + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
-\nabla \cdot \left[ \kappa \nabla V' \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
\frac{\partial V'}{\partial t} + \nabla \left[ V' \left( V' - V'' \right) \right] + V' \nabla V'' = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \nabla \rho \frac{\partial \rho}{\partial y} I + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
-\nabla \cdot \left[ \kappa \nabla V' \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
\frac{\partial V'}{\partial t} + \nabla \left[ V' \left( V' - V'' \right) \right] + V' \nabla V'' = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \nabla \rho \frac{\partial \rho}{\partial y} I + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
-\nabla \cdot \left[ \kappa \nabla V' \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
\frac{\partial V'}{\partial t} + \nabla \left[ V' \left( V' - V'' \right) \right] + V' \nabla V'' = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \nabla \rho \frac{\partial \rho}{\partial y} I + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

\[
-\nabla \cdot \left[ \kappa \nabla V' \right] = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \rho g \frac{\partial y}{\partial x} + \frac{1}{2} \frac{\partial \rho}{\partial x} I - \frac{\partial \rho}{\partial x} \frac{\partial \rho}{\partial y}
\]

**Kinematic boundary condition**

For the junction flow in this study, the free-surface elevation changes apparently, increasing before and decreasing after the junction. Therefore, a treatment of the free-surface which uses the kinematic boundary condition is employed.

\[
\frac{\partial H}{\partial t} + V^1 \frac{\partial H}{\partial \xi^1} + V^2 \frac{\partial H}{\partial \xi^2} = 0
\]

where \( H \) is the height of the free surface; \( t \) is time; \( V^1 \) and \( V^2 \) are contra-variant components of flow velocity in \( \xi^1 \) and \( \xi^2 \) directions, respectively; and \( w \) is the Cartesian component of velocity in the \( z \)-direction. The superscripts \( l \) and \( 2 \) indicate the directions \( \xi \) and \( \eta \), respectively.

**Dynamic boundary condition**

In the previous studies, the hydrostatic pressure is often assumed as considering dynamic boundary condition for the cases with insignificant deformation of free surface. However, for the cases where the curvature of free surface is large, surface tension may affect pressure gradient in the interfacial region. Therefore, non-hydrostatic pressure is used in the present work with effect of the surface tension considered. The approach employed calculates the dimensional dynamic boundary condition as Eq. (25).

\[
DP = 2\rho \nabla V^1 = 2McN
\]
where $DP$ is dynamic pressure; $M_\gamma$ is the mean curvature of the free surface and is defined for the $\gamma^2$ surface; $\gamma$ is the surface tension; and $\nabla V^3$ is covariant derivative of the contra-variant velocity component, $V'$ with respect to the $\gamma^2$ direction and the superscript $\gamma$ indicates the vertical coordinate.

- **Numerical procedure**

The momentum equations and transport equations of $k$ and $\varepsilon$ are solved with the conservative finite-volume method based on a fully-staggered grid system. A TVD-MUSCL scheme is applied to the convective terms in the momentum equations and the transport equations of $k$ and $\varepsilon$. The Adams-Bashforth scheme with second-order accuracy in time is used for time integration in each equation. Calculation of pressure field is implemented using a fractional step method coupling with the Highly Simplified Mark and Cell (HSMAC) method.

- **Computational results and discussions**

One case of the experiment of Weber et al. (2001), $q^* = -0.25$, is used for validation of the linear and nonlinear models proposed above.

**Kinetic turbulence energy:**

In the literature, there has been no validation of turbulence kinetic energy at an open-channel confluence found so far. In this study, this quantity is first validated with the present 3D model using the linear and nonlinear turbulence models. The result is illustrated in Fig. 4. It can be seen that the nonlinear model reproduces turbulence distribution better than the linear does in comparison with the experiment.

**Water surface elevation:**

Fig. 5 displays water surface contours measured and predicted by the linear and nonlinear turbulence models, respectively. It can be observed that the predicted results agree well with the measured ones.

**Secondary current:**

Secondary current predicted by the linear and nonlinear 3D models and the measured one at the section of $x/W = -2.00$ are shown in Fig. 6. The important points which can be observed from Fig. 6 are as follows: i) both the linear and nonlinear models can reproduce the large vortex near the outer bank well; and ii) the nonlinear predicts correctly the small vortex in terms of its position and rotation direction, thus the distortion of the separation zone boundary, while the linear incorrectly does. This great difference is attributed to consideration of anisotropy of Reynolds stresses in the nonlinear model.

- **Effect of discharge ratio on secondary currents, bed shear stress, and flow constriction**

The nonlinear 3D model is used for this study purpose. The total discharge is kept constant as 0.17m$^2$/s, while only discharge ratio changes. The brief results realized as follows (figures are not shown here).

Strength of secondary currents decreases as discharge ratio increases. There is a change in the rotation direction of the large vortex as discharge ratio is larger than 0.50.

As discharge ratio increases, bed shear stresses decreases. There is a change in the direction of the transverse bed shear stress component, thus potential tendency of sediment transport as discharge ratio exceeds 0.50.

In the current study, a more accurate approach based on an effective area is presented to estimate coefficient of flow contraction at an open-channel confluence. This approach generates the results agreeing well with the experimental one, while the conventional approach which uses the concept of the effective width over-predicts this...
quantity. As discharge ratio increases, flow constriction substantially decreases.

**Computation of bed morphology at a channel confluence with depth-averaged 2D models**

- **Flow model**

For the purpose of development of a model that is able to apply for various geometries as well as for real cases, depth-averaged equations are expressed in a general curvilinear coordinate as follows:

Continuity equation:

\[ \frac{1}{J} \frac{\partial}{\partial x} \left( J u \right) + \frac{1}{J} \frac{\partial}{\partial y} \left( J v \right) = 0 \]  

(26)

Momentum equations:

\[ \frac{\partial}{\partial x} \left( J M \right) + \frac{\partial}{\partial y} \left( J N \right) = \frac{\partial}{\partial x} \left( J \tau_{x} \right) + \frac{\partial}{\partial y} \left( J \tau_{y} \right) + J S + J \tau_{x} \frac{\partial}{\partial x} S + J \tau_{y} \frac{\partial}{\partial y} S \]  

(27)

\[ \frac{\partial}{\partial x} \left( J \tau_{x} \right) + \frac{\partial}{\partial y} \left( J \tau_{y} \right) = -\frac{\partial}{\partial x} \left( J \rho g \right) + \frac{\partial}{\partial x} \left( J u \right) + \frac{\partial}{\partial y} \left( J v \right) \]  

(28)

where \((x, y)\): spatial coordinate, \((u, v)\): velocity in \((x, y)\) direction, \((\xi, \eta)\): generalized curvilinear coordinate, \(t\): time, \(h\): water depth, \((\xi_{0}, \eta_{0})\): matrix, \(J\): Jacobian of coordinate transformation, \((M, N)\): discharge flux vectors, \((\hat{M}, \hat{N})\): contra-variant discharge flux vectors for unit width, \((U, V)\): contra-variant components of velocity vectors, \(g\): gravity acceleration, \(z_{i}\): water level, \((\tau_{x}, \tau_{y})\): contra-variant components of bottom shear stress vectors, \(\rho\): water density, and \(-u^{'2} - v^{'2} - \nu^{'2}\): Reynolds stress tensor.

The turbulence model and secondary current model are same as those used in Chapter 2 above. Depth-averaged 2D models are same as those in Chapter 2.

- **Sediment transport model**

Evolution of bed morphology is calculated using the equation of sediment continuity, the Exner equation. In a generalized curvilinear coordinate, this equation has the following form:

\[ \frac{\partial}{\partial t} (z_{b}) + \lambda \left[ \frac{\partial}{\partial x} \left( q_{x} \right) + \frac{\partial}{\partial y} \left( q_{y} \right) \right] = 0 \]  

(29)

where \(z_{b}\) is the time-dependent bed elevation; \(\xi\) and \(\eta\) are generalized curvilinear coordinates; \(\lambda\) is the porosity of the bed sediment; \(J\) is Jacobian of coordinate transformation; and \(q_{x}^{\xi}\) and \(q_{y}^{\eta}\), which are evaluated by Eqs. (30 – 31), are components of the bed-load vector of sediment flux in the \(\xi\) and \(\eta\) directions.

\[ q_{x}^{\xi} = \frac{\partial y}{\partial \eta} J q_{y}^{\eta} - \frac{\partial x}{\partial \eta} J q_{x}^{\eta}; \quad q_{y}^{\eta} = \frac{\partial x}{\partial \xi} J q_{x}^{\xi} - \frac{\partial y}{\partial \xi} J q_{y}^{\xi} \]  

(30)

\[ q_{x}^{\xi} = q_{x}^{\eta} \cos \theta - q_{y}^{\eta} \sin \theta; \quad q_{y}^{\eta} = q_{x}^{\eta} \sin \theta + q_{y}^{\eta} \cos \theta \]  

(31)

In the above relations, \(\theta\) is angle between a streamline and x-axis; \(q_{x}^{\xi}\) and \(q_{y}^{\eta}\) denote Cartesian components of the bed-load vector in the x- and y directions; and \(q_{x}^{\eta}\) and \(q_{y}^{\eta}\) are components of the bed-load vector in the stream-wise and transverse directions. \(q_{x}^{\eta}\) is calculated by Yamaguchi and Izumi (2006) after linearization, in which local slope in the \(s\)-direction is incorporated, while \(q_{y}^{\eta}\) is evaluated using Eq. by Hasegawa (1984).

- **Computational results and discussions**

Computational conditions are same as those in Shigeeda et al. (2009). Numerical procedures are same as used in Chapter 2 above. First, validation of the model is carried out. Then, effect of discharge ratio and confluence angle on bed morphology is investigated using Model 4.

To verify the models, bed morphology is assessed through deviation of bed elevation after an approximate equilibrium reached in comparison with the initial bed (flat bed). Fig. 7 shows bed deformation observed by Shigeeda et al. (2009) and predicted by the models. As seen from Fig. 7, only Model 4 can capture the important features of bed morphology at the present confluence including avalanche faces, scour hole, and scouring range downstream the junction, while Model 1 and Model 3 do not. The reason for this difference is because Model 4 reproduced distinct flow patterns including secondary currents. These secondary currents play an important role in preventing from sediment movement into the scour, hence maintaining the scour.

- **Effect of discharge ratio and confluence angle on bed morphology**

For these purposes, Model 4 is employed. The main findings are as follows. As discharge ratio increases, bed erosion within and downstream the confluence substantially decreases, while the penetration of the avalanche face into the confluence significantly increases.

Fig. 7 Comparison of bed deformation measured by Shigeeda et al. (2009) and predicted by the models

Effects of flow stages and vegetation on bed morphology at a real river confluence
**Flow model**

The flow model used in this study is similar to that presented in Chapter 4 above. However, in order to consider effect of vegetation, a new term which includes this effect is introduced into the right-hand side of momentum equations as follows:

Continuity equation:

\[
\frac{1}{\rho} \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t} + \frac{\partial (\rho \mathbf{v})}{\partial x} = 0
\]  

Momentum equations:

\[
\begin{align*}
\frac{\partial (\rho \mathbf{v} \cdot \mathbf{v})}{\partial t} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial x} &= -\rho \frac{\partial \mathbf{v}}{\partial x} \cdot \mathbf{S} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t} \\
\frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial x} &= -\rho \frac{\partial \mathbf{v}}{\partial x} \cdot \mathbf{S} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t} + \frac{\partial (\rho \mathbf{v} \cdot \mathbf{S})}{\partial t}
\end{align*}
\]

where \(\tau^e_x\) and \(\tau^e_y\) are contra-variant components of bottom shear stress induced by vegetation and are calculated by Eq. 35 below. Other symbols are the same as the ones in Chapter 4.

Sediment transport model is same as used in Chapter 4.

**Vegetation model**

\[
e^v = \rho C_P \lambda \min(h, H) \sqrt{|u'| + v'}  \quad \tau^v_i = \rho C_P \lambda \min(h, H) \sqrt{|u'^i + v'^i|}
\]

Here \(C_P\) is a bulk drag coefficient of vegetation (=1.0); \(\lambda\) is the projected plant area per unit volume; \(H\) is the height of trees. Vegetation is assumed to be stiff, cylindrical, and regularly distributed for simplicity in this study. Min-function is evaluated as

\[
\min(h, H) = \begin{cases} 
    h & \text{if } h < H, \text{ (emergent vegetation)} \\
    H & \text{if } h > H, \text{ (submerged vegetation)}
\end{cases}
\]

**Bank erosion model**

To calculate bank erosion, the simple approach proposed by Hasegawa (1984) is used in the present study.

The model proposed above is applied to the Tokachi-Toshibetsu River confluence locating in Eastern Hokkaido in Japan for investigation of response of bed morphology at this confluence to different flow stages and effect of vegetation and local configuration of the confluent river mouths on bed morphology.

**Computational results and discussions**

Fig. 8 shows the result of temporal evolution process of bed morphology at the study confluence predicted by Model 4. Analysis of this process in details indicates that response of bed morphology at the present confluence has a periodic tendency to different flow rate. In such a confluence, flow rate substantially influence flow dynamics and bed morphology here and discharge ratio seems less significant in this cases.

**Effect of local configuration of the confluent river mouths and vegetation on bed morphology**

The local configuration of the confluent river mouths is a major control on bed morphology within the confluence. This aspect is first identified through numerical simulation here.

Through assumed cases with vegetation growing over the confluence bar, the computational results demonstrate that bar vegetation greatly affect bed morphology within and downstream the confluence.

**References**


