Introduction

Under erodible bed conditions, hydrodynamic forces drive the motion of water and sediment and, thus, create a variety of interesting morphologies, including bed forms, meandering, braiding, and alluvial fans. Sand dune is one of the interfacial instabilities due to complex interactions between water flows and sediment transports. Thus, the prediction of sand dune evolutions is still a challenging research topic for river engineers. One of the difficulties widely experienced when investigating dunes in a natural rivers is that the bed material is composed of various sizes of sediment grains, in another word, nonuniform sediment. Under the identical flow conditions, coarse grains respond less strongly to the forces exerted by the water motion than that of fine grains. Therefore, a selective transport of nonuniform sediment causes many morphological processes such as vertical sorting, armouring, and downstream fining.

In the present study, the morphodynamic model for sand dune simulation is developed for the nonuniform sediment transport conditions. The proposed model is designed to deal with the dune geometry, the formation process and grain sorting phenomena predictions. A vertical two-dimensional flow model with non-hydrostatic and free surface flow proposed by Giri and Shimizu (2006) is fully coupled with the sediment transport model during the calculation. The concepts of bed layer model and size fraction transport are employed for nonuniform sediment transport. Nonequilibrium bed load transport is treated using an Eulerian stochastic formula for the sediment exchange process. The numerical model presented herein is focused on lower-flow regime which only bed load transport is dominant. The simulated results of dune geometries and water depth are compared with the experimental data. Moreover, the model is tested with some sensitivity parameters such as domain length, initial perturbation patterns and mean step length values.

Numerical Model

Hydrodynamic model

The governing equations for unsteady two-dimensional flow in the Cartesian coordinate system (x,y,t) are transformed to the moving boundary fitted coordinate system (ξ, η, τ) (Itakura et al., 1986). The transformed equations are solved by splitting them into a non-advection and a pure advection terms. The non-advection term is solved by using central difference method. The pure advection term is calculated by using a high-order Godunov scheme known as the cubic interpolated pseudo-particle (CIP) technique. The pressure term is resolved by using the method of successive over-relaxation (SOR). In the turbulence model, a non-linear k-ε turbulence closure is employed to reproduce turbulence characteristics in shear flow with separation zone. The time-dependent water surface change computation is used for realistic reproduction of free surface flow over migrating bed forms. The kinematic condition is established along the water surface to compute water surface variation.

The governing equations for unsteady two-dimensional flow in the Cartesian coordinate system (x,y,t) can be expressed as follows:

Continuity equation
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

Momentum equation
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial y} \right) - g
\]
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left( u \frac{\partial v}{\partial y} \right) - g
\]

where x and y are coordinates in horizontal and vertical direction respectively; u and v are components of velocity in horizontal and vertical direction respectively; p is pressure; g is gravitational acceleration; \( \rho \) is fluid density; and are Reynolds stress tensor.

Sediment transport model

The sediment transport model is fully coupled with the hydrodynamic model during the flow calculation. The sediment transport model for nonuniform sediment is composed of two submodels which are a nonuniform sediment transport model and a bed layer model. The nonuniform sediment transport model is employed for sediment transport calculation which includes sediment pickup rate, sediment deposition rate, and bed deformation calculations. The bed layer model
is applied for fractional concentration calculations. The total bed material transport is generally modelled by two different approaches, either by dividing the model into bed load and suspended load transport, or by considering only the bed load transport. Presently, the sediment transport computation considers only the bed load transport.

(I) Nonuniform sediment transport model; In the present study, an Eulerian stochastic formulation of sediment transport which was introduced by Einstein (1950) is incorporated in the sediment transport model for each sediment size fraction transport. For uniform sediment, Nakagawa and Tsujimoto (1982) proposed a nonequilibrium transport model characterized by pickup rate and step length, and applied it successfully to explain several alluvial processes such as the micro-scale bed form evolutions. Based on the uniform sediment transport concept, Tsujimoto and Motohashi (1990) advanced the pickup rate formula for nonuniform sediment by using the critical tractive force for nonuniform sediment. The pickup rate for sediment size fraction k can be written as follow:

\[ p_{ak} = \frac{0.03\tau_{ak} \left(1 - 0.7\tau_{ak} / \tau_{ck}\right)}{\sqrt{d_k / \left(\rho_f / \rho_s - 1\right)}}g \]

where \( p_{ak} \) is the pickup rate of sediment size fraction k, \( \rho_f \) and \( \rho_s \) are fluid and sediment density respectively; \( \tau_{ak} \) is dimensionless local bed shear stress of sediment fraction size k, and \( \tau_{ck} \) is dimensionless critical bed shear stress of sediment fraction size k; \( d_k \) is the mean diameter of sediment size fraction k; and \( g \) is gravitational acceleration.

The sediment deposition rate is expressed as:

\[ p_a = \int p_{ak} f_a(x) dx \]

where \( A_k \) is the mean step length and s is the distance of sediment motion from pickup point. On the basis of probability theory, Einstein (1942) proposed \( A_k = \alpha d_k \). The step length of nonuniform sediment was experimentally studied by Nakagawa et al. (1980). They found that the distribution of the step length for each grain size was approximated by an exponential distribution under the flat bed condition. The dimensionless mean step length of nonuniform sediment was almost constant and was proposed to be 10 to 30.

Tsujimoto and Motohashi (1990) developed a nonequilibrium bed load transport model for nonuniform sediment based on the concept of uniform sediment proposed by Nakagawa and Tsujimoto (1980). The nonequilibrium bed load transport potential rate for each sediment size fraction is written as follow:

\[ q_a(x) = \frac{A_2}{A_1} \int_0^x p_{ak}(x - x') f_a(x') dx' \]

where \( A_1 \) and \( A_2 \) are geometrical coefficients of sediment particles and are equal to \( \pi/4 \) and \( \pi/6 \), respectively. Then, the bed deformation can be computed by using the sediment continuity equation which is

\[ \frac{\partial z_b}{\partial t} + \frac{\partial}{\partial x} \left( \lambda \frac{\partial}{\partial x} \left( \sum q_a \right) \right) = 0 \]

where \( z_b \) is the bed elevation and \( \lambda \) is porosity of bed material.

(II) The bed layer model; The bed layer model proposed by Ashida et al. (1992) is employed for calculating the fractional concentration change of each sediment size fraction. In the bed layer model, bed material is divided into sublayers which are a mixed layer, a transition layer and a deposited layer.

The mixed layer represents the exchange layer or top layer containing the bed materials which is active to the transport process. The mixed layer thickness is assumed to be constant and equivalent to the size \( d_{90} \) of initial bed material distribution (Ashida et al., 1992). The transition layer acts as a buffer layer between the mixed layer and the deposited layer. The thickness of transition layer is a function of time and streamwise direction and is restricted between \( 0 < E_t \leq E_d \), where \( E_t \) is the thickness of transition layer and \( E_d \) is the thickness of multiple layers in the deposited layer. The deposited layer is divided into \( N_b \) layers in which the thickness of sublayer is \( E_{d_i} \). Therefore, thickness of the deposition layer is the multiplication of \( N_b \) and \( E_d \).

The bed elevation is calculated by

\[ z_b = E_m + E_t + N_b E_d + z_0 \]

where \( z_0 \) is the bed elevation, \( E_m \) is thickness of mixed layer, \( E_t \) is thickness of transition layer, \( N_b \) is total number of sub-layers in the deposited layer, \( E_d \) is thickness of multiple layers, and \( z_0 \) is the datum elevation.

Model Validation and Discussion

As described earlier, the numerical model is composed of two main models which are the hydrodynamic model and the nonuniform sediment transport model. The initial setting conditions of hydrodynamic model can be described as computational meshes for the flow calculation consisted of 202×22 cells. The grid interval in the streamwise direction is equal to 1.0 cm, whereas the grid interval in the vertical direction is stretched exponentially with the fine grid near the bed. The simulation period is 7200 seconds for all Runs which are designed to exceed the equilibrium time of all experiments. The time step is \( 7 \times 10^{-4} \) seconds. For the initial bed material condition, we set the initial thickness of bed material to 5 cm for all Runs. The thickness of the mixed layer (\( E_m \)) was set equal to \( d_{90} \) (Ashida et al., 1992) of the initial
bed material for all Runs. We set the thickness of sublayer in the deposited layer ($E_d$) to 0.25 cm. The thickness of the transition layer ($E_t$) can be then calculated from the initial set up of the three parameters and is limited between $0 < E_t \leq E_d$. The grain size distribution of bed material is set to be similar to that of the experiments. A random perturbation field with a small amplitude (0.01 cm) is introduced to the initial flat bed. The periodic boundary condition is employed at the downstream end for both the flow and the moveable bed calculations. The model results are then verified with the experimental results conducted by Miwa and Daido (1992) for both dune geometries and water depths. In addition, the grain sorting phenomena is also observed by the proposed model and compared with the experimental data at various locations inside dunes.

**Flow depth**

Bed resistance inducing the flow depth in channel flow consists of two components which are the grain friction and the form drag. In the flat bed condition, the bed resistance is caused by the grain friction only, whereas the form drag becomes more significant than the grain friction when a bed is covered with bed forms. In the present study, both the grain friction and the form drag components affecting the flow depth are considered. The computed flow depths are compared with the observed data in both the temporal variation and the equilibrium stage for all Runs. The temporal variation of the flow depth of Run-3 is depicted in Figure 1. From the simulated results, the flow depth is induced by the bed forms. The water depth is increasing when the bed forms are generated and becomes stable after the bed forms reached to the equilibrium stage. The comparison of the flow depth between the simulated results and the experimental data is shown in Figure 2. The flow depths are measured after the bed forms reach the equilibrium stage. The water depth appears to be underestimated by the numerical model for all simulated cases. However, the model results are in good agreements with the experimental data with the discrepancy $\pm20\%$ in most cases.
Dune geometry

The computed sand dune geometries and the grain sorting are made in the equilibrium stage and compared with the observed data. The simulated dune geometries, wave heights and wave lengths, were measured by using bed form tracking method proposed by Van der Mark and Blom (2007). The simulated data is measured along the domain length, and the mean value is obtained. In Figure 3, the comparison of the wave heights of sand dunes between the simulated results and the experimental results for Runs 1–5 are depicted. The solid line in the figure indicates that the experiment and the simulated result provide an identical value. The proposed model shows good agreement on the wave height predictions with the discrepancy of ±20% in most experimental cases. The comparison of wave lengths of sand dunes between the simulated results and the experimental results for Runs 1–5 is shown in Figure 4. The model performance on wave length predictions are found to be in good agreement in 3 of 5 cases with a discrepancy of ±20%. From the comparison of the results, the proposed model satisfactorily predicts dune characteristics under the sophisticated mechanisms of nonuniform sediment transport.

Grain sorting at various locations inside sand dunes

Among five experiments, the grain sorting is observed in four cases which are Run-2, Run-3, Run-4 and Run-5. The grain size distributions of mixture are compared between the experimental data and the computed results at various locations inside sand dunes for verifying our numerical model on grain sorting predictions. In Figure 5, the comparisons between the experimental results and our simulated results of Run-3 is shown. Layer I denotes the mixture on the upstream part of dune crests, Layer II denotes the mixture in the trough part of dunes, and Layer III denotes the mixture in the substrate layer. From the experimental results, it was found that Layer I provided the finest mixture among three layers. The mixture in Layer II was coarser than Layer I and III, whereas the grain size distribution in Layer III showed no significant changes because it participated weakly in the sediment transport and bed form evolution process.

The simulated results show that the model provides the finest mixture in the upstream part of dune crest among the three layers. In the trough part of dune, the model provided coarser mixture than the upstream part of dune crest and the substrate layer. Whereas the grain size distribution in the substrate layer shows no significant changes because it participate weakly in the sediment transport and bed evolution. The grain sorting mechanism in our simulations is caused by the selective transport mechanism. It is found that the fine and the median grains are transported more than the coarse grains under the same hydraulic conditions. Therefore, most of the coarse grains still remained in the trough area.

Conclusions

The proposed model is developed to simulate dune formation process of nonuniform sediment bed material. A vertical two-dimensional flow model incorporated with a nonequilibrium sediment transport model is applicable to capture
morphodynamic features and flow features. With the use of bed layer model, the grain sorting phenomena inside bed form can be observed. The simulated results are verified with the experimental results in terms of wave heights and wave lengths of dunes, grain sorting inside dunes and flow depth. Some important conclusions can be drawn from this study.

1. A hydrodynamic model with a non-linear k-ε model and a logarithmic expression scale for near-bed region is found to be adequately predicts both the flow depth and temporal water depth variation over the migrating bed forms. Some inconsistency can be noticed in comparison with observed quantities; the water depth appears to be underestimated by numerical model. However, the model is in good agreement to predict flow depth of the sophisticated morphodynamic conditions.

2. With the use of nonequilibrium sediment transport formula proposed by Tsujimoto and Motohashi (1990), the model successfully predicts dune geometries, dune heights and dune lengths, and their evolution process of bed forms.

3. By means of the concept of bed layer model, the mechanism of grain sorting at various locations inside dune can be investigated. The computed results agree well with the observed in the upstream part of dune crest, the trough area of dune and the substrate layer underneath the migrating bed forms. However, some inconsistency of grain size distribution predictions can be found in comparison with observed data. It may caused by the simplified initial distribution of bed material into sediment size fraction method.

### References


